Structural Reliability and the Partial Factors for Materials

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ABSTRACT

Structural Eurocodes introduce an important concept, the “structural reliability”. The Albanian code “*KTP – Kushtet Teknike të Projektimit*”, does not define this term. From this, several difficulties might arise, ranging from the basic understanding of Eurocodes to the practical use of them, including important aspects such as “the determination of partial factors” into the National Annex of a country adopting Eurocodes. The partial factors have a considerable influence on safety and economy, which is the reason why they have historically had and continue to have a sort of political and strategic background.

This paper introduces some aspects of Structural Reliability through a study focused on concrete strength. Test results from already built structures in Tirana have been used for this purpose. How “reliable” are the structures being designed in Albania using the partial factor method with the recommended partial factors? The answer is not easy; a very large number of tests is needed in order to make an assessment towards the answer. Trying to give a contribution to this matter, this paper describes the results obtained for a reinforced concrete beam when structural reliability concepts are considered explicitly in design. Comparison is made with the structural analysis using the partial factors recommended in Eurocode, with focus on partial factor for concrete.

INTRODUCTION

Eurocode 0 (EN 1990) defines reliability as “the ability of a structure or a structural member to fulfill the specified requirements, including the design working life, for which it has been designed; Reliability is usually expressed in probabilistic terms”. Also, it is noted that reliability covers safety, serviceability and durability of a structure (see [1]). EN 1990 presents the index of reliability β as a measure of reliability, in Annex C, through the following expression:

$$P_f = \Phi(-\beta) \qquad (1)$$

where P_f is the "probability of failure" and Φ is the cumulative distribution function of the standardized Normal distribution. The probability of failure P_f can be expressed through a performance function g such that a structure is considered to survive if $g>0$ and to fail if $g\leq 0$. It is noted in EN 1990 that P_f and β are only notional values that do not necessarily represent the actual failure rates but are used as operational values for code calibration purposes and comparison of reliability levels of structures. For structural elements of Reliability Class RC2 (as defined in EN 1990, Annex B [1]), for the ultimate limit state, the recommended value of β is 3.8.

Eurocodes propose the use of "Partial Factor Design". According to Eurocode 0, a design using EN 1990 with the partial factors given in Annex A1 and EN 1991 to EN 1999 is considered generally to lead to a structure with a β value greater than 3.8 for a 50 year reference period.

In Albania, KTP's have been used since the '70s of the last century, and have been updated until the late '80s. KTP's have been successfully used in Albania in a large number of buildings and civil engineering works, but since the time of the KTP's, science and technology has advanced a lot. A reliability based code, such as Eurocode is necessary. In many cases, the Albanian structural engineers of post-'90s have advanced in this direction, by using the Eurocode principles and recommendations by their own initiative.

It is well known among the community of engineers the recommended partial factor γ_C=1.5 for concrete, for persistent and transient design situations. However, γ_C is a Nationally Determined Parameter, which means that another value may be given in the National Annex of the country adopting the Eurocodes. At this point, the basic question is: should Albania adopt this value for γ_C? In general, lack of data is the main issue that makes this question difficult to answer. However, concrete and steel strength test are frequently done, because they are mandatory during construction. What is needed is to bring available data together and to plan special tests for the future. This paper presents a simple example of structural reliability and tends to make a step towards the calibration of design codes in Albania.

Analysis of concrete test results

Statistical analysis of data

Concrete cubic strength test results from a completed building project in Tirana were collected. In total, 176 samples were studied, divided in two sets. Set-1 contains concrete samples taken on the construction site, while Set-2 samples were taken at the concrete factory. This is an important distinction between the sets.

Software Palisade @Risk, incorporated in Microsoft Excel, was used for the analysis of data obtained by test results. The test results were fitted into several known distribution functions. The plots of Figure 2 show only the first distributions chosen from fit ranking by Bayesian Information Criterion (*BIC*).

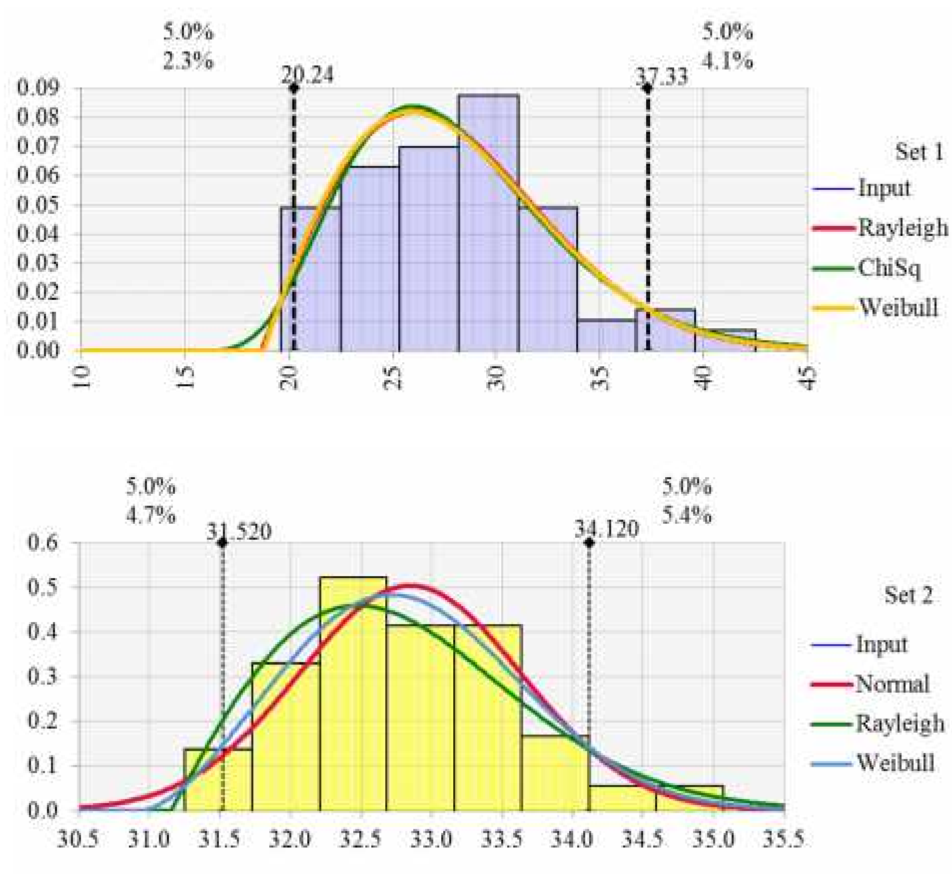

Figure 2 – Fitting comparison for Set 1 and Set 2, Probability Density Functions

For the first set of data, it was found that the distribution function that fits best is a Rayleigh distribution. For Set-2, Normal distribution was found the most appropriate. The following table summarizes the statistical properties of the best fitting functions for each set:

Table 4 - Statistical properties of fitted data

	Set-1	Set-2
	Rayleigh	Normal
Mean (MPa)	27.92	32.85
Std. Dev.	4.84	0.79
5% fractile	21.24	31.52

Analyzing the data of Table 4 it can be seen that concrete strength varies considerably from one set to another. Set-2 has a larger mean value and a smaller standard deviation compared to Set-1. Since concrete was produced at the factory, it went through transport, mixing, waiting on site, sampling procedure and maybe bad weather conditions. Figure 3 allows a visual comparison between distributions corresponding to Set-1 and Set-2.

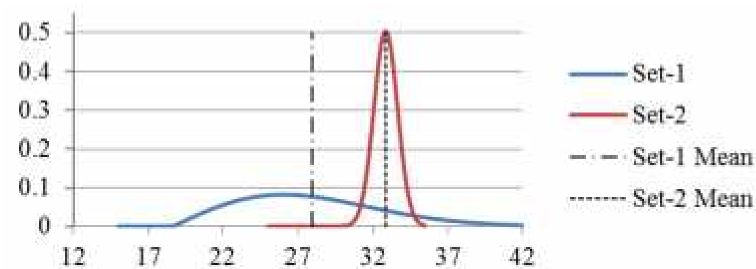

Figure 3 –Comparison of distributions of concrete strength for Set 1 and Set 2

The graph corresponding to Set-2 is far from expectation, even though the 5% fractile corresponds to a high value of concrete strength. The standard deviation of concrete strength is too low compared to international literature, including Eurocodes, so it is not sure that the samples taken at the concrete factory have gone through a completely random procedure of testing.

Assumptions for the concrete compressive strength

Two assumptions were made for the concrete compressive strength measured from available tests. First, it was assumed that the cubic strength and the cylindrical strength have a mathematical relationship. The cylindrical strength was used in the design equations by applying a factor of 0.833 to the cubic strength. Secondly, based on engineering judgment, it was assumed that the partial factor recommended in Eurocode for concrete compressive strength can be expressed as the product of several contributing partial factors:

$$\gamma_C = \gamma_{Rd} \cdot \gamma_c = \gamma_{Rd} \cdot (\gamma_1 \cdot \gamma_2 \cdot ... \gamma_i \cdot ... \cdot \gamma_n) \tag{2}$$

where γ_{Rd} is the partial factor for model uncertainties (see [1]), γ_c is partial factor for concrete strength not taking into consideration the model uncertainties, γ_1 to γ_n are the partial factors that correspond to different influencing factors (such as properties of cement, water, water cement ratio, percentage and properties of aggregates, transport to site, sampling, vibration, conditions on site and other factors).

Reliability analysis

Assumptions for analysis of a RC beam

A reinforced concrete beam is studied in this paper. A few assumptions were made for the analysis, in order to compensate the lack of data. These assumptions are presented here because they influence the results and they must be taken into consideration while interpreting them.

The beam is subject to several statistical uncertainties due to construction process, quality of workmanship, quality of materials, the actual use of the structure etc. The span length, section properties (width, height of section, rebar positioning, rebar diameter), material properties (concrete strength, reinforcement yield strength, modulus of elasticity), the design model and loads are not accurately known. This paper is focused on the concrete compressive strength, while the other factors were considered to be accurately known, except for the actions that were assumed to follow a given distribution function. The ultimate limit state design equations for RC beams were modified in order to consider the fact that concrete strength is probabilistic, while the other factors have deterministic values. For steel, the yield strength was considered to be accurately known, equal to 435*MPa*. Steel in the compression zone was neglected.

Several failure modes may exist for a given structure. For example, for a simply supported beam, failure near supports due to shear or failure in mid-span due to flexure may occur. More detailed analyses can be done considering all the variables and all the failure modes.

Resistance of the beam

It was assumed that concrete from the two studied sets in the previous paragraph was used to build a RC beam, with the cross section shown in Figure 4.

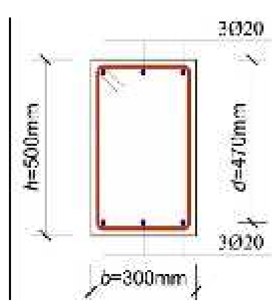

Figure 4 –Cross section of the beam

Failure of the beam will be considered to occur if the beam section reaches the ultimate bending moment. The ultimate bending moment of this beam, considering the assumptions presented above, represents the resistance *R*, as follows:

$$R = M_R = f_{y,det.} \cdot A_{s,det.} \cdot \left(d_{det.} - \frac{S}{2} \right) = f_{y,det.} \cdot A_{s,det.} \cdot \left(d - \frac{f_{y,det.} \cdot A_{s,det.}}{2 \cdot 0.85 \cdot f_{c,prob.} \cdot b_{det.}} \right) \tag{3}$$

In equation (3), the index “det.” refers to deterministic values while “prob.” refers to probabilistic values.

Design value of resistance

The structure is considered to fail if the performance function *g* is equal to or smaller than zero. The target index of reliability is chosen 3.8, in accordance with EN 1990 for RC 2. In order to find the probability of *g* being smaller than zero, several techniques exist. First Order Reliability Method (FORM) is presented in EN 1990 (see references [1] to [6]) as an approach for the determination of design values and it was used in this paper. EN 1990, Annex C gives expressions for Normal, LogNormal and Gumbel distributions of the resistance variable, as shown in Table 5.

Table 5 Design values for various distribution functions, Table C3 of EN 1990

Distribution	Design value
Normal	μ-$\alpha\beta\sigma$
Lognormal	$\mu\exp(-\alpha\beta V)$ for $V=\sigma/\mu<0.2$
Gumbel	$u-\ln\{-\ln\Phi(-\alpha\beta)\}/a$ where $u = \mu - \frac{0.577}{a}$; $a = \frac{\pi}{\sigma\sqrt{6}}$

Factors α (α_E and α_R, with $|\alpha| \leq 1$) are the values of the FORM sensitivity factors. The value of α is negative for unfavorable actions and action effects, and positive for resistances.

Using equation (3), the resistances corresponding to each sample were calculated and for each set they were fitted into one of the three distributions of Table 5. The best fit (among the available distributions) was found to be the Normal distribution for both sets. For the Normal distribution, the following expressions apply:

$$P(R \leq R_d) = \Phi(-\alpha_R \cdot \beta) \tag{4}$$

$$P(R \leq R_d) = P(M - M_d \leq 0) = \Phi(-\alpha_R \cdot \beta) = \Phi(-0.8 \cdot 3.8) = \Phi(-3.04) = 0.0011829 \tag{5}$$

The sensitivity factor α_R is taken 0.8, assuming the ratio of standard deviation of action effects to standard deviation of resistance fulfills the condition: $0.16 < \sigma_E / \sigma_R < 7.6$ (see [1]). The Cumulative Distribution Functions (CDF) are plotted in Figure 5.

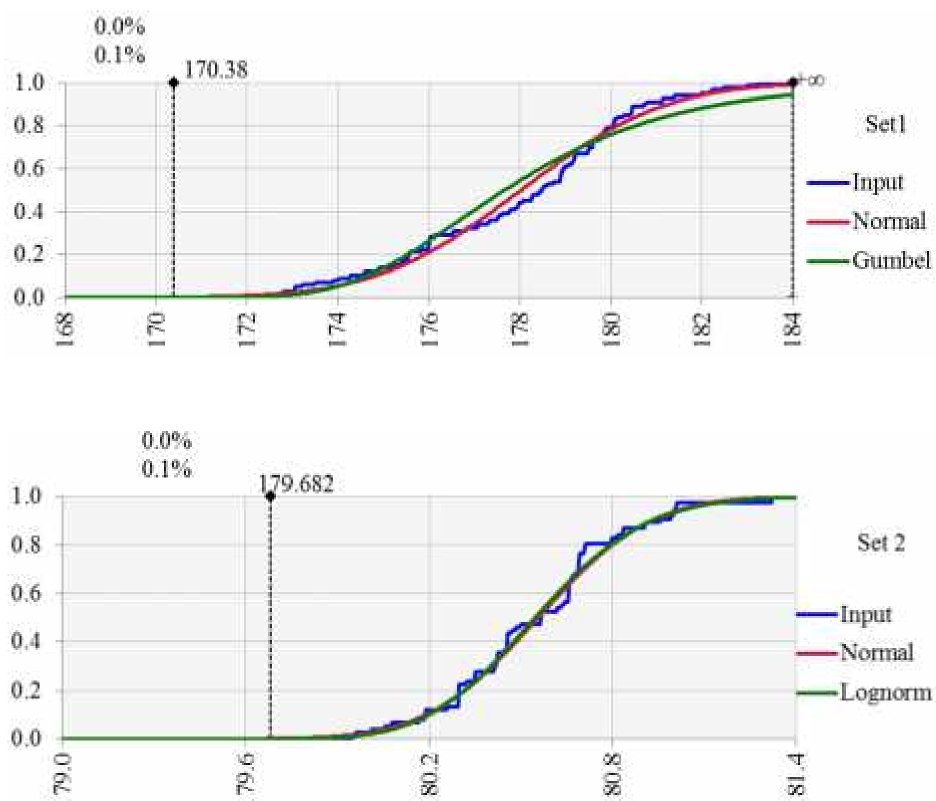

Figure 5 – CDFs for Set-1 and Set-2 and the design values of the resisting bending moment

The vertical lines in CDF in Figure 5 correspond to probability 0.118% (as calculated by expression (5)). The ordinates corresponding to these vertical lines give the design value (which can also be calculated through the expression given in Table 5. For Set-1, the design value of the resisting moment is $M_{R,d,1}$=170.38 *kNm*, for Set-2: $M_{R,d,2}$=179,682 *kNm*.

Comparison with Eurocode design value

In order to make a comparison of the design values calculated above with the design values recommended in Eurocode through partial factors, the graph shown in Figure 6 was built. This graph shows the resistance of the beam for a concrete class C25/30, which is the concrete class specified in the design of the building from where the samples were taken.

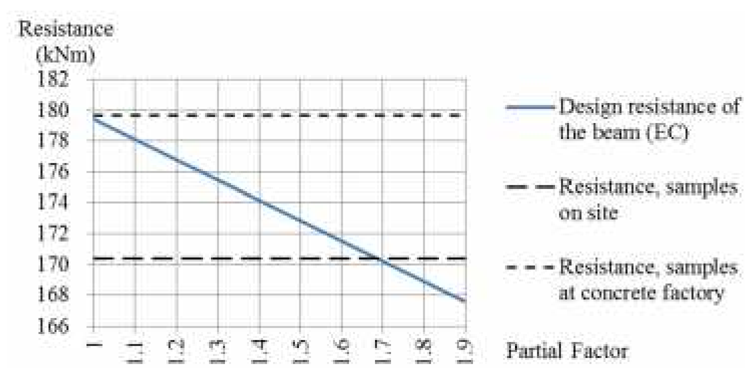

Figure 6 – Relationship of the design values of resistance for concrete class C 25/30 and for actual concrete

Figure 6 demonstrates that, in order to construct a beam that fulfills the Eurocode requirements for reliability, for the studied concrete, a partial factor of 1.68 is the equivalent of γ_c applied to the concrete class C25/30. If the beam was built with the quality of concrete observed at the factory, it would not be necessary to apply any partial factor except for γ_{Rd} (corresponding to model uncertainties). This means that most of the uncertainty that associates concrete strength is due to factors influencing after leaving the concrete factory, i.e. transport, conditions on site, model uncertainties etc. Figure 7 schematically shows the strength variations observed from the concrete recipe to the actual concrete brought on site.

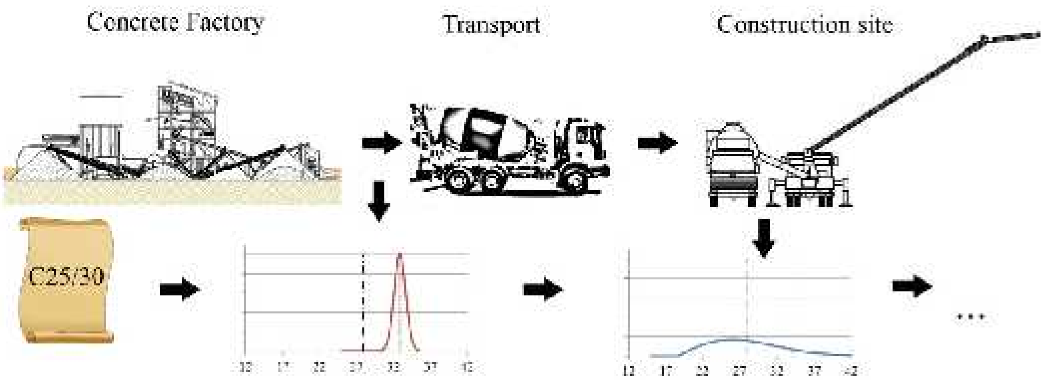

Figure 7 – Schematic view of concrete strength variations

Even after concreting the specific structural element, several other factors influence the actual strength. The low results may have been influenced by bad sampling, treatment and storage of the cubes. The fact that the "equivalent γ_c" resulted larger than 1.5, can be interpreted as an indication that the uncertainties that associate the concrete strength have a larger than expected influence.

Results for another building

Another building was analyzed using the same procedure, with a total of 110 samples, from which 92 were taken on site and the rest were taken at the concrete factory.

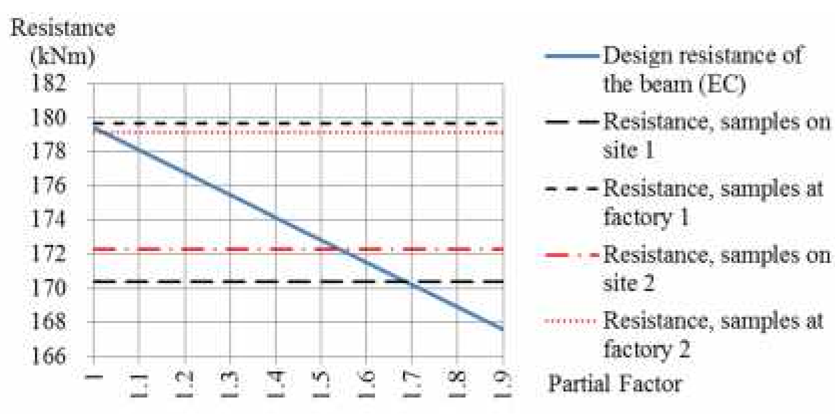

Figure 8 – Resistance of the beam constructed with concrete from another building, compared to first building

Even though the number of samples is smaller in this second building, the same phenomenon was observed for the strength of cubes taken at the factory: their standard deviation is too low compared to literature. The analysis demonstrates that the "equivalent γ_c" is smaller for the concrete corresponding to the second building. Again, there is a big difference between samples taken at the factory and samples taken on site. Analyzing the two buildings, in equation (2), the contribution of transport and sampling would be represented by a relatively large “γ_i” factor.

CONCLUSION

Eurocodes will be fully adopted in Albania in the near future, but before that, special attention should be paid to the Nationally Determined Parameters (as for example the partial factors). Even though the studied data is not sufficient to generalize the conclusions, the simple application in this paper showed that using the recommended partial factor for concrete (1.5) might not always result in a design having the required level of reliability. Due to lack of sufficient data, several assumptions were made in the analysis, which influence the results, but rather than concluding in an actual proposal for the partial factor of concrete, this paper tried to demonstrate the necessity of further studies in the field of structural reliability in Albania. For the concrete samples available, it was shown that transport from factory to site and storage on site have a considerable influence in concrete strength distribution. Further studies with more data, including the probabilistic analysis of all the materials, geometric data, actions, different types of structures etc., are necessary before deciding upon this important factor and other Nationally Determined Parameters of Eurocode.

REFERENCES

[95] EN 1990: Eurocode: Basis of Structural Design

[96] EN 206-1: Concrete - Part 1: Specification, performance, production and conformity

[97] EN 1992-1-1: Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings

[98] Gulvanessian H, Calgaro J-A, Holicky M: Designers’ Guide to EN 1990: Eurocode: Basis of Structural Design. Thomas Telford, 2002

[99] Holicky M, Vrouwenvelder T: Basic concepts of structural reliability

[100] Holicky M, Vrouwenvelder T: Elementary methods of structural reliability