

## **Artificial Time Histories of Wind Actions For Structural Analysis of Wind Turbines**

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### **ABSTRACT**

A computational method for generating artificial time histories of wind loads on wind turbine towers is presented, based on wind turbine aerodynamics and a wind field model. First, the forces acting on the blades parallel and perpendicular to the rotor's plane are calculated according to aerodynamics theory for given mean wind velocity. Due to the blades' aerodynamic behavior the inflow wind velocity is transformed into the relative wind velocity, depending on the axial and tangential flow induction factors, the blades' angular velocity and their radius. Then, the forces acting on the blades are calculated combining relative velocity with two-dimensional aerofoil coefficients depending on the blades' geometry and cross-section. The flow induction factors are estimated by an iterative process taking into account the flow angle between the relative wind velocity and the rotor's plane and the aerofoil coefficients. Next, the turbulence component of the wind is determined by the stochastic theory, in order to describe the total wind field model and compute more realistic wind induced actions on the blades. Each fluctuating component is modeled as Gaussian, stationary stochastic process with zero-mean value and is completely characterized by the correlation matrix in time domain or the power spectral density matrix in frequency domain. Wind time histories are simulated by the decomposition of the power spectral density matrix. Then, finite element models of the wind turbine tower are subjected to the load time histories derived above and dynamic analyses are performed. Ultimate objective of this research is to study fatigue at bolted and welded connections between adjacent parts of wind turbine towers and to investigate the importance of dynamic effects on local buckling of the tower shell.

### **INTRODUCTION**

As wind energy is gaining attention as a mature type of cost-effective renewable energy, investigation on wind turbines becomes more and more interesting. Wind turbines are becoming increasingly popular, since, with the progress of technology, such constructions promise to provide more efficient wind power utilization. As wind turbines grow in height, regarding the towers, and in width, regarding the blades, more reliable calculation of forces acting on the wind turbines is required in order to enable a safe and cost-effective design.

The power output of a wind turbine is a function of the power coefficient, the air density, the rotor area and the wind velocity. The power coefficient describes the fraction of wind power that can be converted into mechanical work and varies with the tip speed ratio. Although several attempts have been made in order to increase its value, an upper limit of 0.593 (Betz Limit) cannot be exceeded [1]. As for the air density, its variations are essentially negligible. Thus, major changes in the power output can only be achieved by two means: either by increasing the swept area of the rotor, or by locating the wind turbines on sites with higher wind speeds. More specifically, doubling the rotor diameter leads to a four-time increase in power output. The influence of the wind speed is more pronounced with doubling of wind speed leading to an eight-fold increase in power. This fact has led to today's rotor radius of more than 60m and tower heights of more than 100m, in order to take advantage of the increase of wind speed with height. These characteristics of modern wind turbines, despite the remarkable addition to efficiency, result to much higher wind loads on the blades, tower and foundation. Furthermore, these loads vary with time and in space, which renders their calculation a rather complex procedure. This complexity has two different aspects: firstly, the wind variations are impossible to predict and calculate using deterministic methods. The mean wind speed is subject to changes from one point to another, while the wind turbulence is a continuously varying parameter, which can only be approached by means of stochastic analysis. Secondly, the aerodynamic characteristics of the rotor follow their own, rather complicated physical laws.

Taking into account the above mentioned difficulties and the need of more accurate calculation of wind turbine loads for structural analysis of the tower, the present article aims to produce a user friendly computational tool written in MATLAB, from which time histories of wind loads are derived taking into account the aerodynamic behavior of the blades and the fluctuating component of a wind field. The time histories of wind load generation are initially based on the computation of wind time histories according to the methodology proposed in [2]. Next, the wind produces forces acting on the blades via the methodology proposed in [1,3], accounting for the aerodynamic response of the blades, that sum up to a time history of load acting on the top of the pylon.

The computational tool described in the present article is part of a research aiming to study fatigue at bolted and welded connections between adjacent parts of wind turbine towers and to investigate the importance of dynamic effects on local buckling of the tower shell, particularly in the vicinity of the manhole cutout near the tower base [4-5].

## **Aerodynamics of wind turbines**

### **The actuator disc theory**

The theory describing the function of a wind turbine is founded on the principal concept of the actuator disk. A simplistic assumption is made, that the blades of the wind turbine are expanded to form a uniform disk, called "the actuator disk" (Figure 1). Actuator disc theory assumes a lossless energy converter, operating in a steady, frictionless airflow. The air flow is

considered inviscid, incompressible and only axial. The mass of air passing through the actuator disc and, thus, being affected by it, is theoretically separated from the total mass flow. A boundary surface is, therefore, created in the three-dimensional space, which forms a stream-tube from the upstream to the downstream of the rotor. No mass can be transferred through this surface.



Figure 1: Stream tube of a wind turbine [1]

While the air is approaching the actuator disc, it is gradually slowed down, losing its initial kinetic energy. Moreover, while the air has its velocity reduced, the stream-tube (Figure 1) has to expand, so that the mass flow rate remains the same. The closer the air particles get to the actuator disc, the slower they become, while the cross-sectional area of the stream tube is gradually widened. These simultaneous phenomena reach a peak at a short distance after the contact with the disc. Thereafter, air velocity and stream-tube volume slowly gain their initial values as the air mass returns to its free-flow state. Meanwhile, as the initial velocity of the stream is reduced while approaching the actuator disk, the dynamic pressure is also reduced and therefore, since the total pressure has to be constant along the streamline, the static pressure is forced to increase. On the other hand, right behind the rotor, the reverse procedure takes place until the free-flow values are reached again. The wind velocity variation that the actuator disc theory induces is introduced by the axial induction factor “  $a$  ” as “  $(1 - a)V$  ”, where  $V$  is the wind flow velocity in the far upstream.

However, the energy converter mentioned above as actuator disc is not a static disc but one that rotates with angular velocity  $\omega$ . The air passing through the rotor exerts a torque on the rotor and, as a result, the rotor imposes on the air an equal and opposite torque and the air rotates in a direction opposite to that of the rotor (Figure 2).

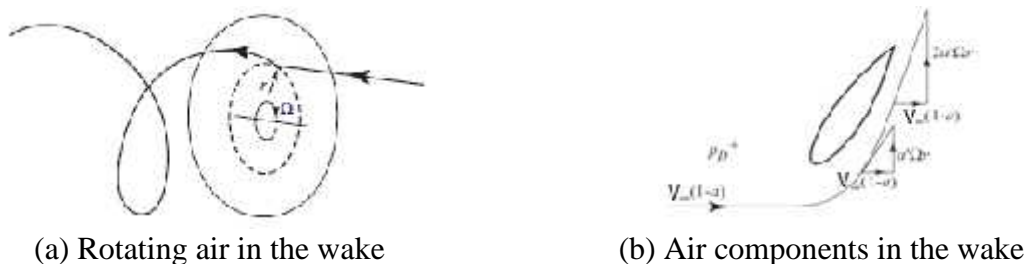


Figure 2: Air components in the wake [1]

The wind flow  $V$  entering the actuator disc has no rotational motion, while the wind flow exiting the disc has an axial component and a tangential to the rotation of the disc component (Figure 2). The axial component is afore mentioned as “  $(1 - a)V$  ”, whereas the change in the tangential velocity is expressed in terms of a tangential induction factor “  $a'$  ” as “  $a'V$  ”.

### The rotor blade theory

The aerodynamic lift and drag forces are responsible for the rate of change of axial and angular momentum of the air passing through the annulus swept by the blades. The total force acting on a blade derived by the wind flow is the integral of pressure and frictional forces and can be calculated by two-dimensional aerofoil coefficients based on the blade geometry and the angle of attack determined from the incident resultant velocity  $W$ .

Consider a wind turbine with  $N$  blades, tip radius  $R$  (finite radius  $dr$ ), chord length  $c$  and pitch angle (angle between the between the aerofoil zero lift line and the plane of the disc).

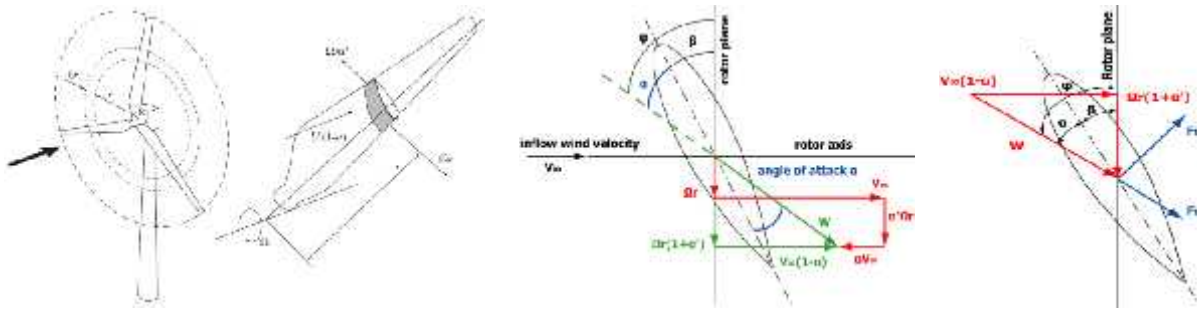


Figure 3: Wind turbine geometry [1], incident wind velocity  $W$ , drag and lift forces acting on the blade

As mentioned above, when the flow wind velocity passes through the rotor plane, an axial component  $V$  and a tangential component  $r$  are generated. However, together with the two components, a variation is introduced (Figure 3). As the wind flow interacts with the moving rotor, a tangential flapstream “  $\alpha' V$  ” is introduced while due to axial interference, the axial component is reduced by “  $-\alpha V$  ”. Finally, the relative wind velocity  $W$  acting on the blade is the resulting force after reduced velocity  $V$  and tangential flapstream takes place and is equal to

$$W = \sqrt{V_{\infty}^2 (1 - \alpha)^2 + \Omega^2 r^2 (1 + \alpha')^2} \quad (1)$$

The relative wind velocity  $W$  acts at an angle (Figure 3) to the plane of rotation and as a result the angle of attack can be determined as “  $\alpha = \beta - \phi$  ”. Respectively, the lift and the drag forces on a span-wise  $dr$  of the blade can be calculated by

$$\delta L = 1/2 \rho W^2 c C_L \delta r, \quad \delta D = 1/2 \rho W^2 c C_D \delta r \quad (2)$$

where  $C_L$  and  $C_D$  are aerofoil coefficients depending on blade’s geometry and the angle of attack and are obtained from the blade manufacturer. It is noted that the lift and drag forces can be transformed to the axial (along the rotor axis) and to the tangential component (along the rotor plane) forces.

### The blade element momentum theory (BEM)

According to the blade element momentum theory, the flow area swept by the rotor is divided into a number of concentric ring elements which have no radial dependency between them. Each ring element is divided into a number of tubes which are also assumed to be independent and wind speed is assumed to be uniformly distributed over them. The forces from the blades on the flow through each ring element are assumed constant, thus assuming that the rotor has

an infinite number of blades. We can now abstract an annular ring of length  $r$  and obtain the lift and drag forces as well as the axial and tangential forces.

The axial force is equal to the rate of change in axial momentum and the additional axial force caused by the drop pressure in the wake. Respectively, the rotor's torque is equal to rate of change in angular momentum. Solving these two equations and taking into account the blade solidity  $\sigma_r$  and the cord solidity  $\sigma_c$ , the flow induction factors  $\alpha$  and  $\alpha'$  are obtained depending on the aerofoil coefficients using an iterative method:

$$\frac{\alpha}{1-\alpha} = \frac{\sigma_r}{4\sin^2\phi} \left[ C_x - \frac{\sigma_r}{4\sin^2\phi} C_y^2 \right] \text{ and } \frac{\alpha}{1+\alpha'} = \frac{\sigma_r C_y}{4\sin\phi \cos\phi} \quad (3)$$

The assumptions made for the BEM theory are corrected by applying two correction factors. The first, called "Prandtl's tip-loss factor" [1], was proposed by Prandtl and corrects the assumption of infinite blades. Glauert's correction factor [1] is applied when the axial induction factor  $\alpha > 0.3$  in order to compute correctly the induced velocities for small wind speeds. Moreover, the tower of the wind turbine becomes a significant obstacle for the wind's free flow, especially in the case of tubular towers. Thus, in order to produce a realistic model which reflects the behavior of free wind stream approaching a wind turbine, the velocity deficits upwind of a tubular tower have to be taken into consideration. The effect of tower shadow on blade loading can be estimated by setting the local velocity component at right angles to the plane of rotation equal to  $V(1-\alpha)$  instead of  $V_\infty(1-\alpha)$ , and applying blade element theory as usual.

### artificial wind time histories generation

The wind time histories generation is based on [2].

### The wind field model

The wind velocity  $V(x,y,z,t)$  (Figure 4) consists of the mean value wind velocity  $V_m(z)$  and its fluctuating components  $u(t)$ ,  $v(t)$  and  $w(t)$  along the  $x$ ,  $y$ ,  $z$  axes. As a result

$$V(x,y,z,t) = V_m(z) + [u(t) + v(t) + w(t)] \quad (4)$$

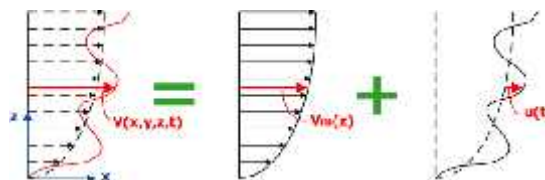


Figure 4: The wind field model in xz plane

The mean value of wind speed is calculated by the EC1-Part 4 [6] as

$$V_m(z) = k_r \ln \left( \frac{z}{z_0} \right) u_b \quad (5)$$

where  $k_r$  the terrain factor depending on the roughness length  $z_0$ ,  $z$  the height of the reference point and  $u_b$  the basic wind speed. However, the fluctuating components of the wind field are

not so easy to determine. They are modeled as normal stochastic processes which are completely characterized by the power spectral density (PSD) function.

### The wind field model as a stochastic process

In the case of a wind field model, the stochastic process is a 4D-dimensional ( $x, y, z$  coordinates in space and time  $t$ ) and 3V-variate (three fluctuating components) model. These components are assumed to be independent of each other, thus the [4D-3V] model becomes a 3[4D-1V] model. The structure is discretized in  $n$  nodal points, for which wind time histories are calculated, and each fluctuating component is represented as a vector of  $n$  points. Thus the 3[4D-1V] model becomes a 3[4D- $n$ V] model. Moreover, in case of wind turbulence, the stochastic process is stationary (time independent), Gaussian (the stochastic moments of order more than 2 are equal to zero so that the process is characterized by the mean value and variance), and with zero mean value. Hence, the complete characterization of the fluctuating components of a wind field as stochastic processes is ensured by the knowledge of the power spectral density (PSD) function in the frequency domain or the correlation function in the time domain. In the present article, the characterization of the turbulent wind components is modeled in the frequency domain and the process is characterized by the PSD matrix  $S_{vv}$ :

$$S_v(\omega) = \begin{bmatrix} S_{v_1v_1}(\tau) & S_{v_1v_2}(\tau) & \cdots & S_{v_1v_n}(\tau) \\ S_{v_2v_1}(\tau) & S_{v_2v_2}(\tau) & \cdots & S_{v_2v_n}(\tau) \\ \vdots & \vdots & \cdots & \vdots \\ S_{v_nv_1}(\tau) & S_{v_nv_2}(\tau) & \cdots & S_{v_nv_n}(\tau) \end{bmatrix} \quad (6)$$

in which the diagonal terms are the auto-spectrum and the rest are the cross-spectrum terms. The auto-spectrum terms of the PSD matrix can be easily determined from the bibliography [1-2-3-6]. Here, the Kaimal spectrum, used also in EC1-Part 4 [6], is applied

$$S_{vv}(\omega) = \frac{6.868\sigma_v^2 f L_u/h}{(\omega/2\pi)(1 + 10.302 f L_u/h)^{5/3}} \quad (7)$$

where  $f = \omega h / 2\pi \bar{V}(h)$  is the Monin coordinate,  $L_u$  the internal length scale of turbulence,  $\sigma_v^2$  the variance of the longitudinal component of the velocity fluctuations and  $\bar{V}(h)$  the mean wind velocity. The cross-spectrum terms of the PSD matrix are determined by [2] as

$$S_{v_i v_j}(\omega) = \sqrt{S_{v_i v_i}(\omega) S_{v_j v_j}(\omega)} \exp(-f_{ij}(\omega)), \quad f_{ij}(\omega) = \frac{|\omega| \sqrt{C_y^2 \eta^2 + C_z^2 \zeta^2}}{2\pi [\bar{V}(z_i) + \bar{V}(z_j)]} \quad (8)$$

where  $\exp(-f_{ij}(\omega))$  is the coherency function,  $C_y$  and  $C_z$  are appropriate decay coefficients and  $\eta, \zeta$  are distances between the points  $i$  and  $j$ . The wind time histories generation is based on the decomposition of the PSD matrix  $S_{vv}$ . Two methods are well known in the bibliography, the Cholesky Decomposition and the Proper Orthogonal Decomposition (P.O.D.). Here P.O.D. is employed, according to which the PSD matrix is decomposed in its eigenvectors  $k(i, j)$  and eigenvalues  $\lambda_k(i, j)$  so that

$$Y^T(\omega) \cdot S_{VV}(\omega) \cdot Y(\omega) = \Lambda(\omega) \quad (9)$$

The vector process  $V(t)$  is written as a summation of independent fully coherent stochastic processes:

$$V(t) = \sum_{k=1}^n Y_k(t), \quad Y_k(t) = 2 \sum_{j=1}^N \Psi_k(\omega_j) \sqrt{\Lambda_k(\omega_j) \Delta\omega} g_j^{(k)}(t) \quad (10)$$

where  $k$  are the points of the structure,  $j$  are the frequencies and  $g$  is a function equal to

$$g_j^{(k)}(t) = R_j^{(k)} \cos \omega_j t + I_j^{(k)} \sin \omega_j t \quad (11)$$

$R_j$  and  $I_j$  are zero-mean normal random numbers obeying orthogonality relationships.

### The algorithm

The algorithm for simulating the aerodynamics and generating the wind time histories was coded in Matlab language. The basic steps of the algorithm are: i) selection of wind field characteristics, ii) selection of wind turbine characteristics, iii) generation of artificial wind time histories for points along the blades of a wind turbine for several rotated positions along the rotor disc, iv) implementation of wind turbines blade aerodynamics and v) generation of time histories of wind load on the top of the pylon. The accuracy of the proposed algorithm was considered as satisfactory by means of comparison of representative results with those of the literature [1], as shown in Figure 5.

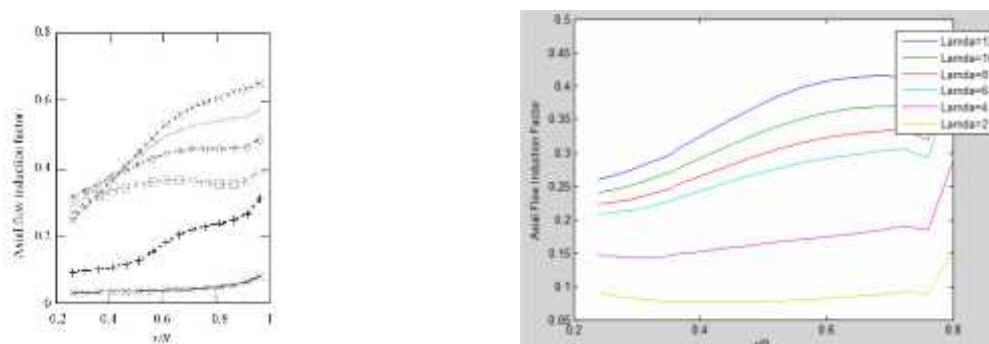


Figure 5: Axial induction factor from literature [1] (left) and proposed algorithm (right)

### RESULTS

Results are presented for a wind turbine with a 120m tall pylon and three 61.5m long blades. The lift and drag coefficients along with the attack angle to which they refer have been obtained from tables, using linear interpolation for intermediate values of the angle of attack. The tower diameter at the top and the clearance between the blades and the tower were both assumed equal to 2m. Six aerofoils were examined but only results for one are displayed here.

The code takes into account points that cover the entire rotor disk area, according to a grid density defined by the user. In this example points were considered along each blade with an azimuth angle step of 20 degrees. The edge points and a representative wind time history are shown in Figure 6 for a duration of 250 sec and a time step of 1 sec.

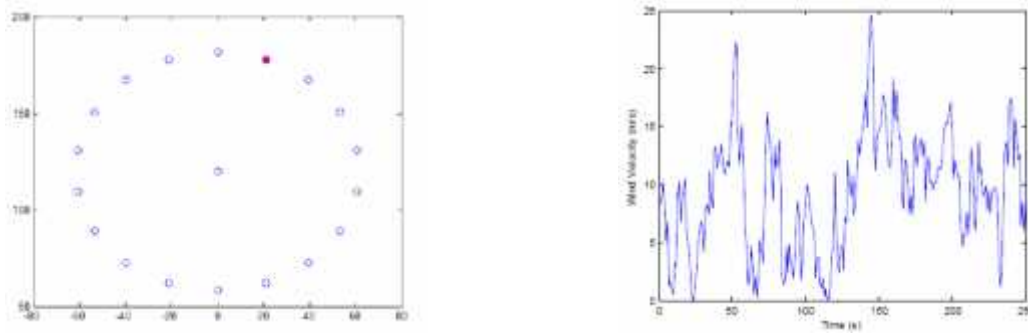


Figure 6: Artificial wind time history for an blade edge point

In Figure 7 the distribution of the out-of-plane force along a blade is presented for different tip-speed ratios. The wind turbine operates at a rotation velocity of 8.6, 12.5, 16.6, 21.5 and 25.0 rounds per minute and the mean wind speed is 10m/s. The out-of-plane force is increasing approximately linearly with radius, in spite of the reducing blade chord, until the effects of tip loss are felt beyond about 75 percent of tip radius.

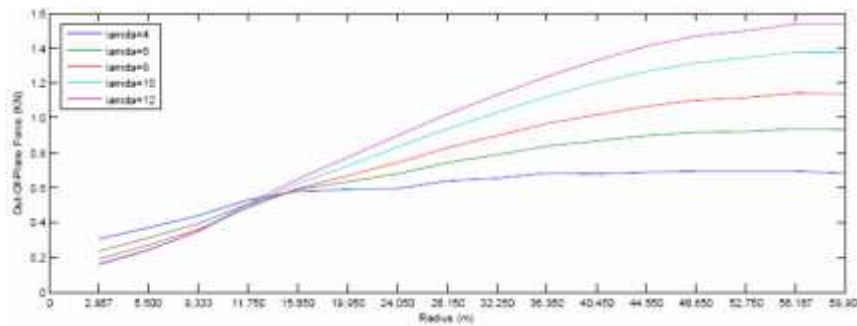


Figure 7: Out-of-plane force distribution for a DU40-A17 blade

Representative time histories of the out-of-plane-load are shown in Figure 8 for three different values of rotational velocity of the blades. It is interesting to note that the load values are of the same order of magnitude as the load obtained assuming a solid disk with radius 61.5m and a constant wind speed of 10m/s (728kN).

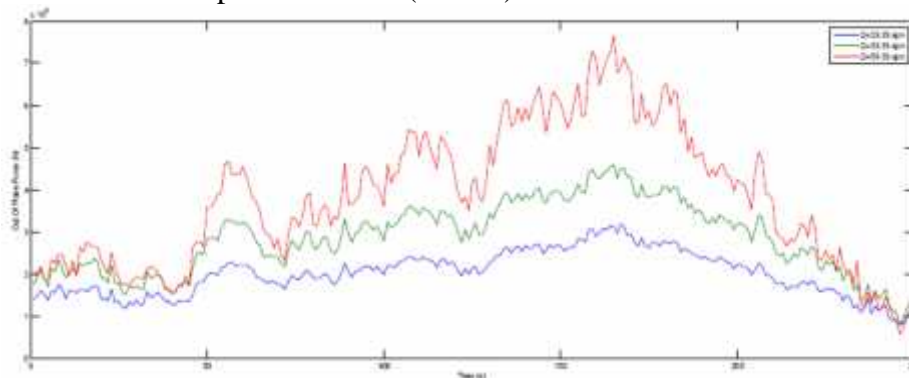
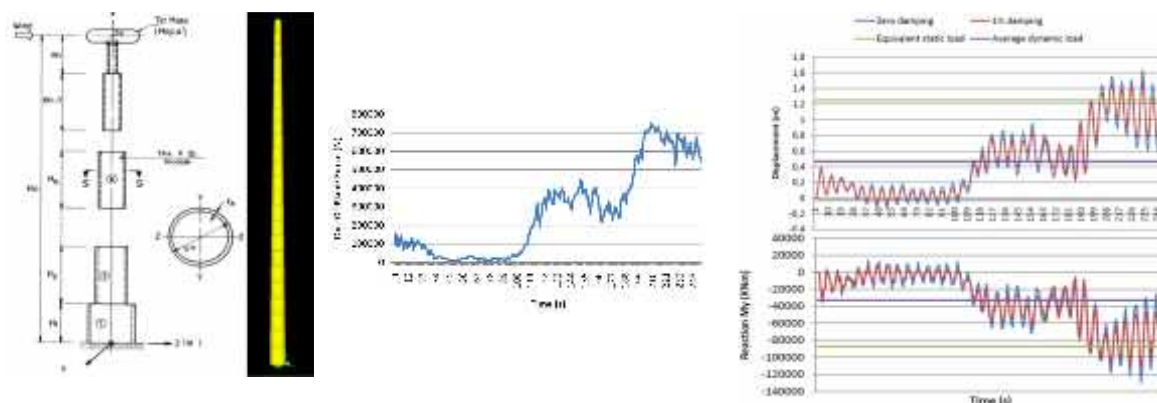


Figure 8: Time histories of wind load on the top of the pylon

Moreover, a simple model of a wind turbine tower was built with beam finite elements in the finite element software SAP2000 NonLinear. The wind turbine tower geometry and the corresponding model are displayed in Figure 9a. A wind load time history (Figure 9b) was imposed on the top of the tower of steel grade S235 and dynamic analysis was performed.





(a) Geometry and model (b) Time history of wind load (c) Top displacements - base moments  
Figure 9: Dynamic analysis of wind turbine tower

In Figure 9c, the displacements at the top and the bending moments at the base of the wind turbine tower are displayed and compared with the corresponding quantities of the equivalent static method (from load obtained assuming a solid disk) and the average values from dynamic analyses.

## SUMMARY AND CONCLUSIONS

In order to provide an easy and understandable computational tool for generation of artificial time histories of wind loads, the basics of blades aerodynamics have been presented and the wind field model has been described via stochastic process analysis. The proposed methodology has been programmed in MATLAB, and preliminary results showed satisfactory comparison with results from the literature. Dynamic analyses of a wind turbine tower subjected to generated wind load time history has been performed in a first attempt to highlight the differences between static and dynamic analyses.

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