THE DESİGN OF SLENDER RC COLUMNS

Diana Lluka[1], Merita Guri[2], Veronika Hajdari[3], Alketa Ndoj[3]

[1]*Department of Building Constructions and Transport Infrastructure, Faculty of Civil Engineering, Polytechnic University of Tirana, Albania*

[2]*Department of Applied and Human Sciences, Faculty of Architecture and Design, Polis University, Albania.*

[3]*Department of Building Constructions and Transport Infrastructure, Faculty of Civil Engineering, Polytechnic University of Tirana, Albania.*

[3]*Department of Building Constructions and Transport Infrastructure, Faculty of Civil Engineering, Polytechnic University of Tirana, Albania.*

ABSTRACT

The paper presents the design method for slender reinforced concrete column based on design code EC2. Rarely, when the column height is longer than typical story height and/or the column section is small relative to column height, secondary stresses become significant, especially if end restraints are small and/or the columns are not braced against side sway. The expressions given in this paper for the additional moments were derived by studying the moment and curvature behavior for a member subject to bending plus axial load. The equations for calculating the design moments are only applicable to columns of a rectangular or circular section with symmetrical reinforcement. The slender column should be designed for an ultimate axial load plus an increased moment.

The slenderness effect must be considered in design, over and above the sectional capacity considerations incorporated in the interaction diagrams. Results indicate the behavior of slender columns and the difference with short columns.

Keywords: Reinforced concrete column, slenderness ratio, eccentricity, creep ratio, capacity reduction factor.

INTRODUCTION

Columns are defined as structural elements that carry loads in compression. Usually they carry loads in compression and bending moments as well about one or both axes of the cross sections. Two types of columns can be classification according to EC2. A short column is one in which the ultimate load at a given eccentricity is governed only by the strength of the materials and the dimensions of the cross-section.

A slender column is one in which the ultimate load is governed not only by the strength of the materials and the dimensions of the cross section but also by the slenderness, which produces additional bending moment due to lateral deformations. For eccentrically loaded the short columns behavior will follow the linear path until intersect the interaction diagram. For eccentrically loaded slender columns, the column will follow a non-linear path until intersects the interaction diagram. This means that, due to the non-linear effects the actual moment on the column is greater than the linear moment. In designing eccentrically loaded slender columns, the second-order effects are very important parameters

Slender columns can be defined as columns with small cross sections compared to their lengths. Generally, slender columns have lower strength when compared to short columns, for a constant cross section, increasing the length causes a reduction in the strength.

THE BEHAVIOUR OF SLENDER COLUMN

The slenderness of a column may result in the ultimate load being reduced by lateral deflections of the column caused by bending.

This effect is illustrated in Fig. 1 for a particular case of an initially straight column with bending in single curvature caused by load N applied with equal eccentricity e=ee+ei at both ends. The bending deformation of the column causes the eccentricity of the load at the critical section to become (e + e2), where e2 is the additional eccentricity due to lateral deflection at that section. Hence, the maximum moment increases to N (e + e2).

This is commonly referred to as the P-Δ effect. A short column is defined as one in which the ultimate load is not reduced by the bending deformations because the additional eccentricities Δ=e2 are negligible. A slender column is defined as one in which the ultimate load is reduced by the amplified bending moment caused by additional eccentricity.

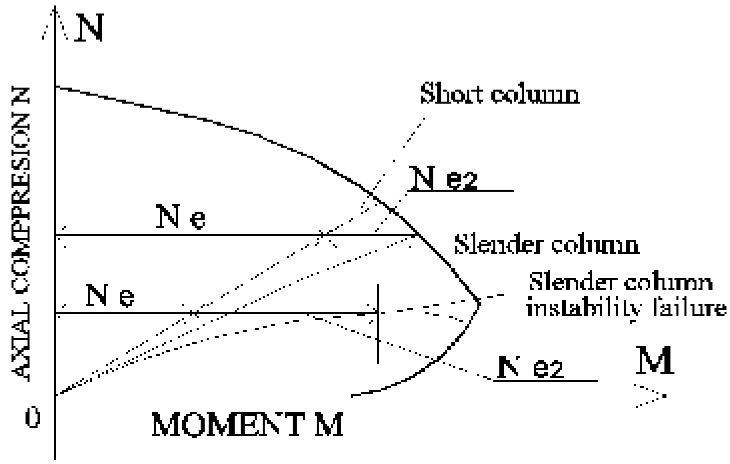

FIG. 1.INTERACTION DIAGRAM FOR A COLUMN SECTION ILLUSTRATING SHORT AND SLENDER COLUMN N-M BEHAVIOR UP TO FAILURE.

1. NUMERICAL SOLUTION ACCORDING TO EUROCODE 2

A column is classified as slender if the slenderness ratio $\lambda = lo / i > \lambda lim$, where $lo = \pi\sqrt{EI/NB}$ is the effective length, NB is buckling load and i is the radius of gyration of the uncracked concrete section. If $\lambda \leq \lambda_{lim}$ then the column may be classified as short and the slenderness effect may be neglected. A slender column with $\lambda > \lambda_{lim}$ must be designed for an additional moment caused by its curvature at ultimate conditions. There are two methods for calculation of reinforcement concrete column, (a)Nominal Stiffness and (b)Nominal Curvature.

Method (a) Nominal Stiffness may be used for both isolated members and whole structures, if nominal stiffness values are estimated appropriately. It is based on calculation of nominal stiffness and moment magnification factor.

Method (b) Nominal Curvature is mainly suitable for isolated members. But it can also be used for structures. It is based on calculation of bending moment and the curvature. For calculating of the design value of eccentricities for concentric loading we will apply the method (b) of “Nominal Curvature”.

The total moment for calculation of slender columns is:

$$Mt = NEd * etot \quad or \quad Mt = M0Ed + M2 \tag{3.1}$$

NEd is the design value of axial force ,

M_{0Ed} is the first order moment, including the effect of imperfections.

M_2 is the nominal second order moment.

Differing first order end moments M_{01} and M_{02} may be replaced by an equivalent first order end moment M_{0e}:

$$M_{0e} = 0{,}6\, M_{02} + 0{,}4\, M_{01} \geq 0{,}4\, M_{02} \qquad (3.2)$$

M_{01} and M_{02} should have the same sign if they give tension on the same side, otherwise opposite signs. Furthermore, $M_{02} \geq M_{01}$. The nominal second order moment $M_2 = N_{Ed}\, e_2$

The eccentricities according to EC2 are used in the cross-sectional design of columns. The total eccentricity e_{tot} is:

$$e_{tot} = max\begin{Bmatrix} ee + ei + e2 \\ e0 \end{Bmatrix} \qquad (3.3)$$

$ee + ei + e2$ is sum of eccentricities

$e0$ is minimal value of eccentricities

Where:

ee is the first order eccentricity of the normal force on the undeformed column

ei is due to the imperfections

e2 is the second order imperfection due to the deformations of column.

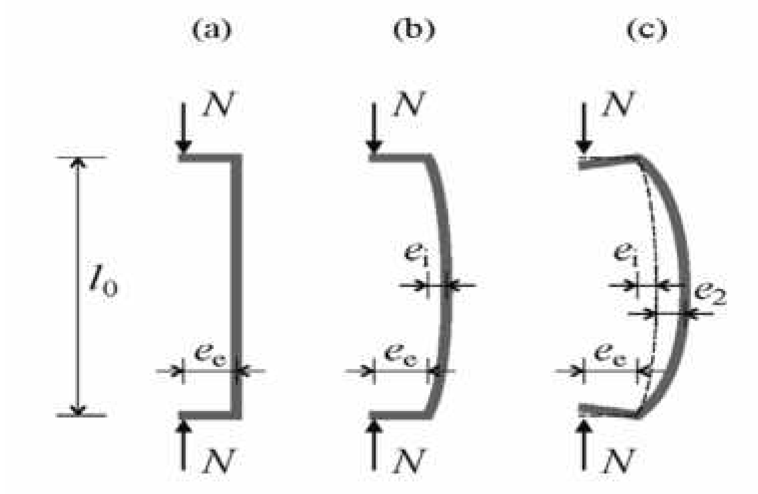

Fig. 2

The accidental eccentricity *ei* is given by the equation:

$$e_i = v\frac{l_0}{2} \tag{3.4}$$

*l*0 is the effective column height about the axis considered, which depends on the length *l*, and on the boundary conditions of the column.

$$\mathbf{v} = \frac{\mathbf{1}}{\mathbf{100\sqrt{l}}} > \frac{\mathbf{1}}{\mathbf{200}}$$

L **IS THE HEIGHT OF THE COLUMN IN METERS. A CONSERVATIVE ESTIMATE OF e_i CAN BE GIVEN BY:**

$$\mathbf{e_i} = \mathbf{v}\frac{\mathbf{l_0}}{\mathbf{2}} = \frac{\mathbf{1}}{\mathbf{200}}\mathbf{x}\frac{\mathbf{l_0}}{\mathbf{2}} = \frac{\mathbf{l_0}}{\mathbf{400}}$$

The nominal second order moment *M*2 = *N*Ed *e*2

The second order eccentricity or the deflection *e2* is calculated as:

$$e2 = l0^2/[c * r] = l0^2/[\pi^2 \cdot r] = l0^2/[10 \cdot r] \tag{3.5}$$

$c = \pi^2 \approx 10$ is a factor depending on the curvature distribution, *1/r* is the curvature: $\frac{1}{r} = Kr * K\varphi * \frac{1}{r0}$ (3.6)

The basic value of the curvature is:

$$\frac{1}{r0} = \frac{fyd/Es}{0.45*d} = \frac{\epsilon yd}{0.45d} \qquad (3.7)$$

Es is the elastic modulus of steel, $d = (h/2) + i$s and is is the radius of gyration of the total reinforcement area.

The second-order eccentricity e_2 is an estimate of the deflection of the column at failure and is given by the equation:

$$e2 = \frac{Kr \square K\varphi * \epsilon yd * l0^2}{0.45d\pi^2} \qquad (3.8)$$

Kr is a correction factor depending on axial load

$$Kr = (nu - n) / (nu - nbal) \leq 1 \qquad (3.9)$$

$n = NEd / (Ac\ fcd)$ is the relative axial force; *N*Ed is the design value of axial force

$nu = 1 + \omega$, where $\omega = As\ fyd / (Ac\ fcd)$

*n*bal is the value of *n* at maximum moment resistance; the value 0,4 may be used (EC2)

*A*c is the gross area of the concrete section

*A*s is the area of longitudinal reinforcement

$$Kr = [1 + \frac{As\ fyd}{Ac\ fcd} - \frac{NEd}{Ac\ fcd}] / [1 + \frac{As\ fyd}{Ac\ fcd} - 0.4] \leq 1 \qquad (3.10)$$

$Kr = \frac{Ac\,fcd + As\,fyd - NEd}{Ac\,fcd + As\,fyd - 0.4} \leq l$

NRd = Ac$f$cd + As$f$yd, or NRd = 0.567$f$ck Ac+ 0.87$f$yk As is the design axial resistance of section.

$$Kr = \min\left\{\frac{NRd - NEd}{NRd - Nbal}; 1\right\} \tag{3.11}$$

depends on the applied normal force NEd. Nbal is the value of the normal force at maximum moment resistance, as shown in *Fig. 3*. Nbal=0.29fckAc.

$K\varphi$ is a factor for taking account of creep

$K\varphi = \max\{1 + \beta\varphi ef; 1\}$
(3.12)

where φef is the effective creep coefficient

$\beta = 0.35 + \frac{fck}{200} - \frac{\lambda}{150}$ (3.13)

λ is the slenderness ratio . $\varphi_{ef} = \varphi(\infty, t_0) \times M_{0Eqp}/M_{0Ed}$ is the effective creep ratio

$\varphi(\infty, t_0)$ is the final creep coefficient

M0Eqp is the bending moment in the quasi-permanent load combination at the SLS

M0Ed is the bending moment in the design load combination at the ULS

$\boldsymbol{\varphi_{ef}}$ **MAY BE TAKEN AS ZERO IF** $\boldsymbol{\varphi(\infty, t_0) \leq 2}$ **AND** $\boldsymbol{\lambda \leq 75}$ **AND** $\boldsymbol{M_{0Ed}/N_{Ed} \geq h}$

Where M0Ed is the first order moment and h is the cross section depth in the corresponding direction.

According to EC2 a cross-section without cracks should be taken into consideration. In most practical cases the above equation may be simplified to

$$e_2 = \frac{K_\varphi K_r l_0^2 f_{YK}}{\pi^2 \cdot 103\,500d}$$

In Figure 3 Nu is the ultimate load of the centric loaded cross-section according to EC 2 is calculated as: Nu = $f$cd Ac + min {400; $f$yd As}.

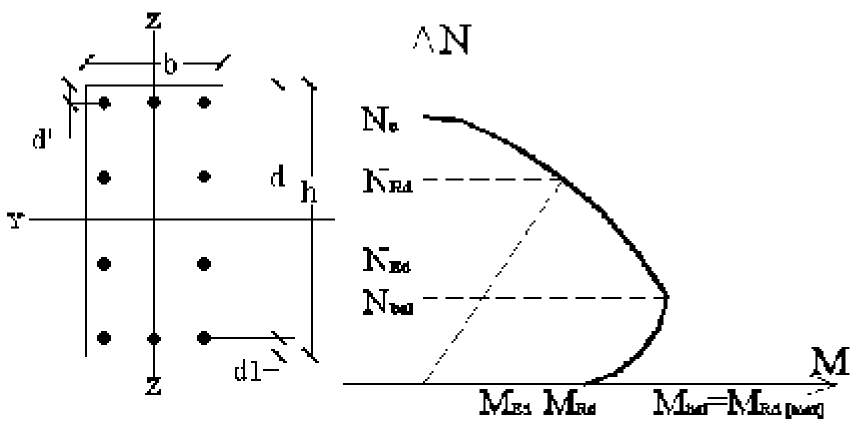

Fig. 3. Doubly symmetric cross-section and the interaction diagram with eccentricity in direction z

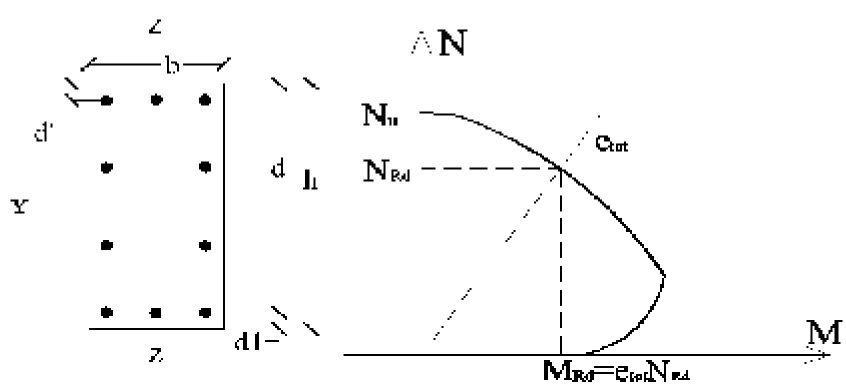

Fig. 4. Doubly symmetric cross-section and the interaction diagram with eccentricity etot in direction z (For the load NEd ≤ NRd with the eccentricity e_{tot}, the cross-section is safe)

The minimum value of the eccentricity (Eq.3.3) is:

$$e0=\begin{Bmatrix} 20mm, \\ h/30 \end{Bmatrix}$$

The eccentricity of the normal force depends on:

- the value of the axial load, NEd,
- the amount and arrangement of the rebars,
- the effective length of the column,
- the concrete class.

The minimum reinforcement according EC2 is: *As,min*≥{ 0.1NEd/fyd and 0.002Ac}

A slender column should be designed for an ultimate axial load (N_{Ed}) plus an increased moment given by $M_t = N_{Ed} e_{tot}$

2. NUMERICAL EXAMPLE

Design of a slender column A column of 300x450 cross-section resist, at the ultimate limit state, an axial load of 1700kN and end moments of M01=20kNm and M02=-70kNm causing double curvature about the minor axis YY as shown in figure. The column's effective heights are $l_{0y} = 6.75\text{m}$ and $l_{0z} = 8.0\text{m}$ and the characteristic material strengths $f_{ck} = 25\text{N/mm}^2$ and $f_{yk} = 500\text{N/mm}^2$. The effective creep ratio $\varphi_{ef} = 0.89$.

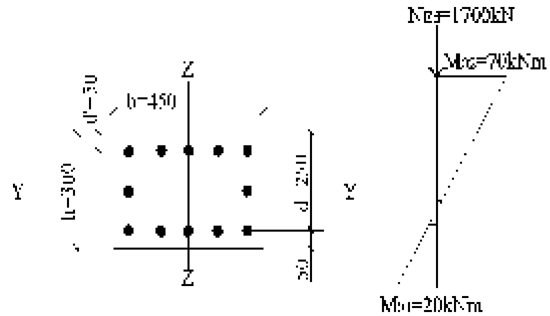

Fig. 5.Rectangular cross-section with uniform rebar arrangements along the circumference.

Eccentricities are:

$$e_{01} = \frac{M_{01}}{N_{Ed}} = \frac{20x10^3}{1700} = 11.76mm$$

$$e_{02} = \frac{M_{02}}{N_{Ed}} = \frac{-70x10^3}{1700} = -41.18mm$$

Where e_{02} is negative since the column is bent in double curvature.

$$\lambda_{lim} = 20 \text{ x } A \text{ x } B \text{ x} C/\sqrt{n}$$

The limiting slenderness ratio can be calculated from EC2:

$$A = 1/(1 + 0.2\varphi_{ef}) = 1/(1 + (0.2x0.88)) = 0.85$$

$B = 1.1$ as the default value

$$C = 1.7 - \frac{M_{01}}{M_{02}} = 1.7 - (-20/70) = 1.99$$

Therefore $\lambda_{lim} = 20 \text{ x } A \text{ x } B \text{ x} C/\sqrt{n} = 20\text{x } 0.85 \text{ x } 1.1 \text{ x } 1.99/\sqrt{n} = 37.213/\sqrt{n}$

$$n = \frac{N_{Ed}}{A_c f_{cd}} = \frac{1700 \text{ x } 10^3}{(300x450)x0.567x25} = 0.89$$

Therefore:

$$\lambda_{lim} = \frac{37.213}{\sqrt{0.89}} = 39.45$$

The slenderness ratio are $\lambda = lo/i = lo\sqrt{12}/h = lo x3.46/h$

$$\lambda_y = \frac{l_{0y}}{i_y} = \frac{6.75}{0.3}x3.46 = 77.85 > 39.45; \ \lambda_z = \frac{l_{0z}}{i_{yz}} = \frac{8.0}{0.45}x3.46 = 61.51 > 39.45$$

The column is slender and λ_y is critical.

Equivalent eccentricity $e_e = 0.6e_{02} + 0.4e_{01} \geq 0.4e_{02}$

$$0.6e_{02} + 0.4e_{01} = 0.6 \text{ x } 41.18 + 0.4 \text{ x } (-11.76) = 20\text{mm}$$

$$0.4e_{02} = 0.4 \text{ x } 41.18 = 16.47\text{mm}$$

The equivalent eccentricity $e_e = 20\text{mm}$

The accidental eccentricity is: $e_i = \nu\, l_{0y}/2 = 6750/400 = 16.88mm$

The second-order eccentricity is

$$e_2 = \frac{K_\varphi K_r l_0^2 f_{yk}}{\pi^2 \text{ x } 103\,500d}$$

where :$K_\varphi = \max\{\left[1 + \left(0.35 + \frac{f_{ck}}{200} - \frac{\lambda_y}{150}\right)\varphi_{ef}\right\}; 1\}$

$K_\varphi = max\{\left[1 + \left(0.35 + {}^{25}/_{200} - {}^{77.85}/_{150}\right)\text{x}0.89 = 0.96\right]; 1\}$

$e_2 = [K_r K_\varphi l_0^2 f_{yk}]/[\pi^2 \text{ x } 103\,500d] = [1 \text{ x } 1 \text{ x } 6750^2 \text{x } 500]/[\pi^2 \text{ x } 103\,500 \text{ x } 250]$

$$e_2 = 89.2\text{mm}$$

with $K_r = 1.0$ for the initial value.

For the first iteration the total eccentricity is

$$e_{tot} = e_e + e_i + e_2 = 20 + 16.88 + 89.2 = 126.08\text{mm}$$

And the total moment is

$$M_t = N_{Ed}e_{tot} = 1700x126.08x10^{-3} = 214.34kNm$$

$$\frac{N_{Ed}}{bhf_{ck}} = \frac{1700 \times 10^3}{450 \times 300 \times 25} = 0.504; \frac{M_t}{bh^2 f_{ck}} = \frac{214.34 \times 10^6}{450 \times 300^2 \times 25} = 0.211$$

From the design chart : $A_s f_{yk}/bhf_{ck} = 0.75$ and $K_2 = 0.78$

This new value of K_2 is used to calculate e_2 and M_t for the second iteration. The design chart is again used to determine $A_s f_{yk}/bhf_{ck}$ and a new value of K_2 as shown in table below.The iterations are continued until the value of K_2 in columns (1) and (5) of the table 1 are in reasonable agreement, which in this design occurs after two iterations.

$$e_2 = 69.58mm$$

$$e_{tot} = e_e + e_i + e_2 = 20 + 16.88 + 69.58 = 106.46mm$$

$$M_t = N_{Ed}e_{tot} = 1700x106.46x10^{-3} = 181kNm$$

TABLE 1

1	2	3	4	5
Kr	Mt	Mt/(b·h²·fck)	As·fyk/(b·h·fck)	Kr
1	214.34	0.211	0.8	0.78
0.78	181	0.179	0.65	0.75

So that the steel area required is

$$A_s = \frac{0.65bhf_{\mathrm{ck}}}{f_{\mathrm{yk}}} = \frac{0.65 \times 450 \times 300 \times 25}{500} = 4387.5\mathrm{mm}^2$$

and $K_2 = 0.75$.

As a check on the final value of K_2 interpolated from the design chart:

$$N_{\mathrm{bal}} = 0.29f_{\mathrm{ck}}A_{\mathrm{c}} = 0.29 \times 25 \times 300 \times 450 \times 10^{-3} = 979\mathrm{kN}$$

$$N_{Rd} = 0.567f_{ck}A_c + 0.87f_{yk}A_s$$

$$= (0.567 \times 25 \times 300 \times 450 + 0.87 \times 500 \times 4387.5)10^{-3} = 3822\mathrm{kN}$$

$$K_2 = \frac{N_{\mathrm{Rd}} - N_{\mathrm{Ed}}}{N_{\mathrm{Rd}} - N_{\mathrm{bal}}} = \frac{3822 - 1700}{3822 - 979} = 0.75$$

which agrees with the final value in column 5 of Table 1.

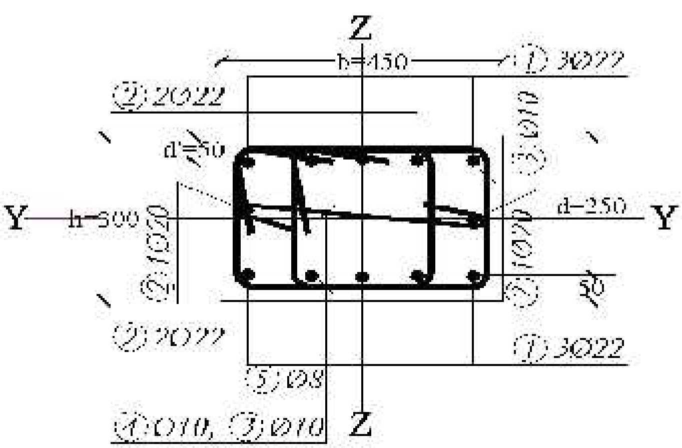

Fig.6. Rectangular cross-section with uniform rebar arrangements

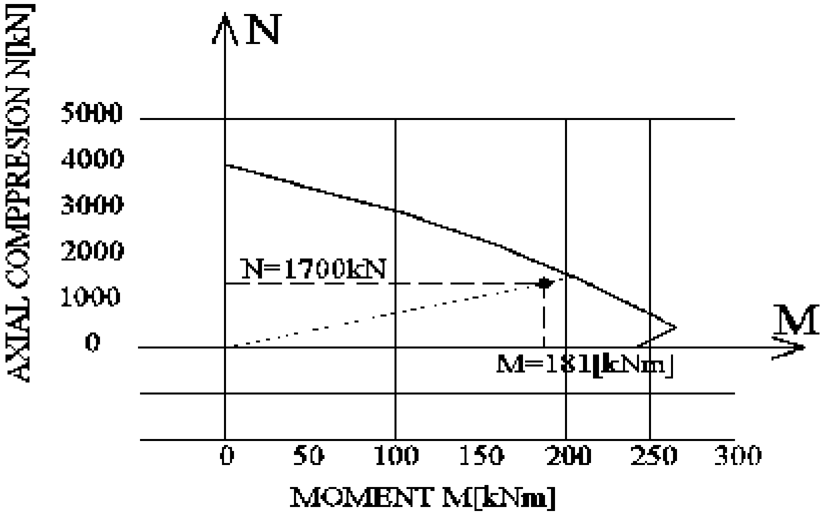

Figure 7. The column interaction diagram is for Ned=1700kN, M=Mt=181kNm

The difference between short and slender column is given in Fig.8, according Fig.1.

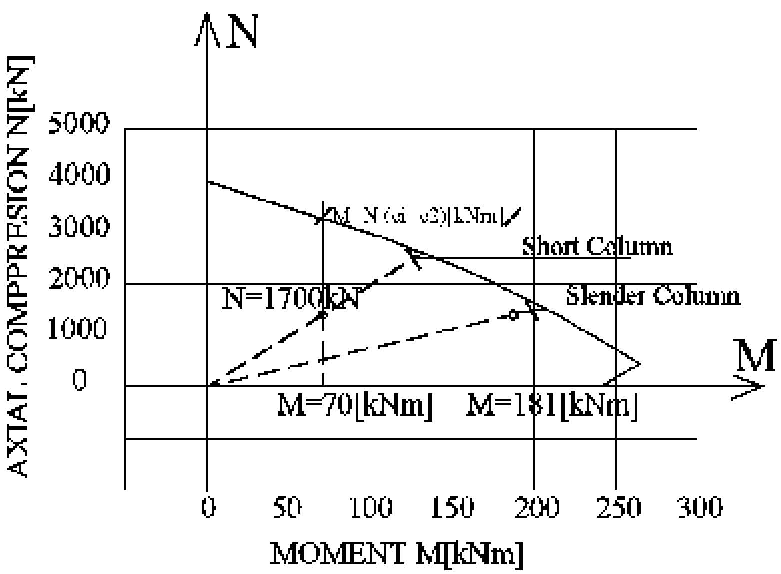

Figure 8. The column interaction diagram is for M-N.

The interaction M-N diagrams can be constructed for any shape of cross-section which has an axis of symmetry by applying the basic equilibrium and strain compatibility equations with the stress-strain relations.

CONCLUSION

A slender column must be design for an additional moment caused by its curvature at ultimate condition. The expressions given in EC2 for the additional moments were derived by studying the moment/curvature behavior for a member subject to bending plus axial load. The equations for calculating the design moments are only applicable to columns of a rectangular or circular section with symmetrical reinforcement.

There are four different approaches to designing slender columns accoding to EC2:

A general method based on a non-linear analysis of the structure and allowing for second-order effects that necessitates the use of computer analysis.

A second-order analysis based on nominal stiffness values of the beams and columns requires computer analysis using a process of iterative analysis.

The method of Nominal Stiffness may be used for both isolated members and whole structures. It is based on calculation of nominal stiffness and moment magnification factor.

The method of Nominal Curvature is mainly suitable for isolated members but it can also be used for whole structures. It is based on calculation of bending moment and the curvature.

These second-order moments are added to the first-order moments to give the total column design moment.

3. REFERENCES

[1] Eurocode 2 (2004), „Design of concrete structures – Part 1-1: General rules and rules for buildings", EN 1992-1-1.

[2] Design of Concrete Structures, A. Nilson, D. Darwin, Ch. Dolan, New York, NY (2010).

[3] ACI 318-08, Building Code Requirements for Structural Concrete and Commentary, (2007).

[4] Design of Reinforcement Concrete, J.C.McComac, J.K.Nelson. Seventh Edition, (2004), USA.

[5] Nonlinear Mechanics of Reinforcement Concrete, Makeawa, Pimanmas and Okamura, Spon Press, London and New York (2003).