2nd International Balkans Conference on Challenges of Civil Engineering, BCCCE, 23-25 May 2013, Epoka University, Tirana, Albania.

Analysis and Optimum Design of Curved Roof Structures

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ABSTRACT

Curved steel buildings are frequently designed to supply the users of the structure with ordinary light with a sense of capaciousness as well as grandness in public facilities such as stations, buying malls, leisure centres and airports.

This paper presents a method for analysis and optimum design of 2D and 3D curved roof trusses subjected to static loading and specified set of constraints. Here the optimization refers to minimization of total weight of curved roof structures such that they can resist applied forces (stress constraint) and don't exceed certain deformations (displacement constraints). The finite element formulations is developed and implemented for the static analysis of curved roof trusses to determine the stresses and displacements.

The use of a reliable and competitive procedure for finding the optimum solutions for problems involving continuous design variables based on genetic algorithms is demonstrated and used in this study .The performance of genetic algorithms is affected by various factors such as coefficients and constants, genetic operators, parameters and some strategies. Member grouping and initial population strategies are also important factors.

Optimization is an automated design procedure in which the computers are utilized to obtain the best results. The numerical methods of structural optimization with applications of computers automatically generate a near optimal design (converge to solve) in interactive manner. A program was modified and used to automate analysis and optimization of the structure written in FORTRAN language based Finite Element analysis and Genetic Algorithm optimization technique. The developed method is tested on several examples and compared with previous researches or SAP2000 results. It is concluded that this method can serve as a useful tool in engineering design and optimization of curved roofs.

Keywords**:** *curved roof, static analysis, optimum weight, genetic algorithm finite element method.*

INTRODUCTION

Wide-span space structures have been more and more popular in covering large open areas with few intermediate supports. Successful arched structure applications exist all complete the world exhibition canters, bridges, public halls, covering stadiums, and other buildings.

Curved roofs of course have a number of important benefits, as well as they can be a superlative and long-lasting choice. That is where the attractiveness and price of the curved roof come into play. Balanced to the standard flat roof instatement, an arched roof can be far more durable, providing for superior charge for you.

In structural design, it is needed to obtain a suitable form in a structure so that it can carry the required loads safely and profitably. Traditional approaches to the job of discovery such shapes for structures have been using experimental models or by intuition with experience. Arch's structures supply inexpensive results for crossing great spans with bear higher loads for a presumption volume of material when correctly shaped, balanced with beams shorter cross parts can be used in arches, like the membrane forces are dominant, [1].

It is widely impossible to get analytical mathematical results, for problems requiring complex geometries, loadings, plus material properties; analytical solutions are those presumptions by an arithmetical representation that yields the rates of the wanted unknown amounts at any position in an individual with are thus well-founded for an infinite number of positions in the body.

These analytical solutions widely need the solution of regular or partial differential equations that, sense of the difficult material properties, geometries, loadings, and are not generally available. For this reason, we require to depend on numeric approaches, such like the FE approach, for correct solutions. The FE formulation of the problem results in a technique of simultaneous algebraic equations for a result, rather than needing the solution of differential equations. Briefly, the solution to constructional problems normally relates to determining the displacements at each intersection also the stresses inside each member construction up to the building that is subjected to apply loads Logan, in 2007 [2].

Although the subject of truss and arch roof analysis with optimization had been conversed frequently complete current years, this topic was contained to show the validity of an analysis program, which is used in the GA optimization program. Although some structural optimization methods can deal with discontinuous search spaces, they support a native lack of generality and hence, can not be readily extended to different types of structures. The GA, for its part, is a problem independent.

The principal characteristics of a GA are established on the principles of endurance of the fittest with adaptation. Since its establishment liked an intuitive idea, [3].

Many inventors have explored the applicability of GA as well as advanced many applicable supplements such as elitist. GA. Gero, et al[4], improved augmented Lagrangian GA, Adeli, H. and Cheng, N.T, [5], hybrid algorithms of GA with fuzzy system .Tan, L.P.et al [6] and with neural network, Grierson, D.E. and Hajela, P, [7].Various scientists have tried to solve the arch problem by different methods. It seems that FE has been the major tool in this research.

2 Matrix analysis of trusses

Static analysis of trusses can be carried out accurately; also the equations of even complex trusses can be collected in a matrix shape amenable to numeric solution. This approximation, now and then named “matrix analysis,” provided the basis of early FE advancement.

By considering the stiffness of each truss element one at a time matrix analysis of trusses acts, and after that applying these stiffnesses by the displacements of the joints, generally named “nodes” in FE to determine the forces that are set up in the truss. Afterwards noting that the force that is externally contributed by each element to a node must equal the sum of force that is applied to that node, we can assemble a sequence. Of linear algebraical equations in which the applied nodal forces are known amounts, also the nodal displacements are the unknowns. These equations are comfortably written in matrix shape, which gives the system its name.

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \tag{1}$$

Here $\mathbf{F}_i$ and $\mathbf{d}_j$ indicate the force at the $\mathbf{i}^{th}$ node and the deflection at the $\mathbf{j}^{th}$ node (these would actually be vector quantities, with subcomponents along each coordinate axis). The $\mathbf{K}_{ij}$ is global stiffness matrix, with the ij component. The matrix equations can be abbreviated as

$$K_{ij}\, d_j = F_i \tag{2}$$

3 Analysis and optimum design examples

3.1 Static analysis of 2D and 3D truss.

Analysis is done by the FE method coded program for analysis of 2 and 3D curved truss roofing, results are compared with source program (SAP2000).

3.1.1 2D curved truss with 7.32 m height

This example consists of curved truss with 50.8 m span length with 41 elements as shown in Figure 1. The geometry of this example is taken from [8]. The structure is loaded with a point load of 222.41 KN on all upper joints in the Z direction. The members of the structure are divided into 3 groups; first group from element (1-22) and (40, 41), second group from element (26-36) and third group from element (23-25) and (37-39), each group have the same cross sectional area and one design variable. Cross-sectional areas for first group A1=0.04877m^2, second group A2=0.009484 m^2 and third group A3= 0.01290 m^2. The objective function is the weight (or volume) minimized. Maximum tensile stress $\sigma_t = 137.895 \text{Mpa}$, maximum compressive tress $\sigma_c = -103.421 \text{Mpa}$ and maximum u_x and u_z displacement all nodes being 0.05 m. Material properties are: Young's modulus, $E = 199947.96 \text{Mpa}$ and material density, $\rho =$ 7697 kg/m^3.

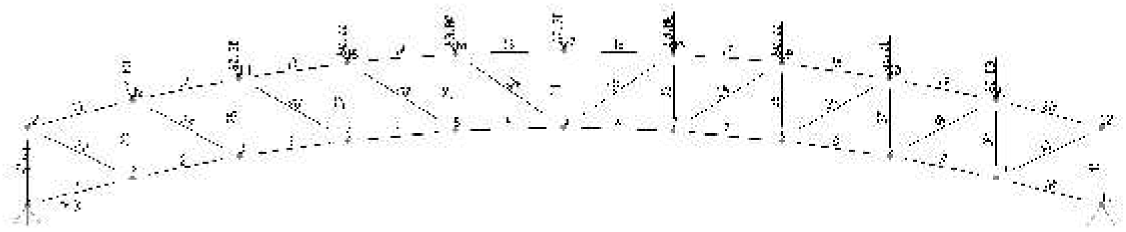

Figure1. 2D curved truss with 7.32 m height

***Discussion of the results**:* Analysis is done by the FE method coded program and source program (SAP2000). Table1. Maximum tension stress occurs in elements (16, 26) and maximum compression stress occurs in elements (1, 10). Table 2 is the result of maximum displacement before and after optimization. Maximum displacement occurs in joints (17, 6). The close agreements between results are seen The GA optimization satisfied displacement constraints. From the results demonstrated in Table 3 it can be observed that the optimum values found for the final weights. The results got applying the GA for continuous design variable quantities. After 65 iterations, minimum weight design was obtained for continuous design variables. The weight of the truss is reduced from 521,195 to 238.962 (54 %reduction) for continuous design variables.

***Table 1** Comparison of stress for 41 bar 2D curved truss with 7.32 m height*

Stress (Mpa))		
Frame NO.	Sap2000	Present work
1	-44.2000	-43.4369

10	-44.2000	-43.4369
16	43.3307	43.3307
26	43.3307	43.3307

Table 2 Displacement result before and after optimization

Displacement (m)				
Joint no.	X-direction		Y-direction	
	Before optimization	After optimization	Before optimization	After optimization
6	0.0000	0.0000	-0.0200	-0.0469
17	0.0000	0.0000	-0.0202	-0.0475

Table3 Initial and optimum design variables of 2D curved truss with 7.32 m height

Design variables	Crosses sectional area (m^2)		Max. Stress (Mpa)	Max. disp. (m)
	Initial	Optimum		
S_1	0.04877	0.02236	103.5695	-0.0475
S_2	0.009484	0.00408		
S_3	0.01290	0.006452		
Weight (kN)	521.195	238.962		
P.R	54 %			

3.1.2 3D dome

This example consists of space dome truss with 120-bars and 49 joints, dimensions of dome are shown in Figure 2 [9]. The members are collected into seven different groups, each group have the same cross sectional area and one design variable. The truss joints are subjected to vertical loading except of supported joints. These are taken as 60.0062 kN at node 1, 30 kN from nodes 2 to 14 and 10 kN at rest of the nodes, the dome span and total height shown in the Figure 3. 3D truss dome example is analysis and optimized under static loads for determine of maximum stress, displacement and minimum weight. In addition to allowable tensile and compressive stresses, an upper limit for the displacement is taken as -0.005 m. at each node. The allowable compressive and tensile stresses are 103.4213, 241.3165 Mpa,

Used material properties are: density, $\rho = 8.027172366$ kg/m^3, Young's modulus $E = 199947.9615$ Mpa . Cross-sectional areas for first group A_1= 0.00394 m^2, second and third group $A_2=A_3= 0.00137\ m^2$, forth group $A_4= 0.00175\ m^2$, fifth group $A_5= 0.0008\ m^2$, sixth group $A_6= 0.00362\ m^2$ and seventh group $A_7=0.00285\ m^2$.

Discussion of the results*:* Maximum compression stress occurred in second group. Table 4 is the result of displacements before and after optimization *x*, *y* and z directions. Maximum displacement occurred in joints (17, 29). The close agreement between results is seen. The GA optimization satisfied displacement constraints. From the results demonstrated in table 5 it can be seen that the optimum values found for the final weights. The results got applying the GA for continuous design variable quantities. After 136 iterations, minimum weight design was obtained. The weight of the truss is reduced from 118.518 to 57.28 (51.6 %reduction).

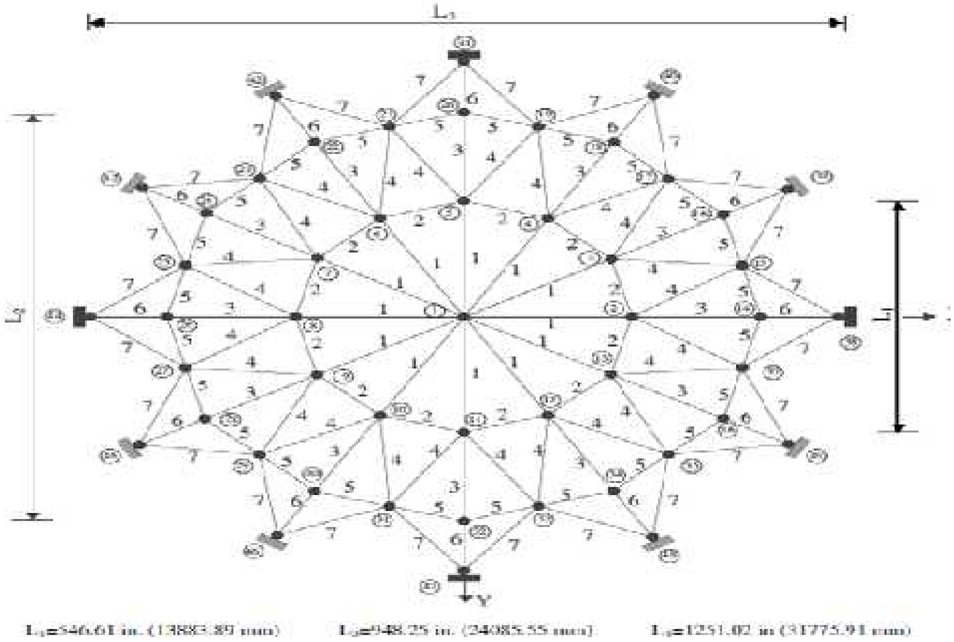

Figure 2 Top wiev120 element 3D curved truss with 7.0 m height

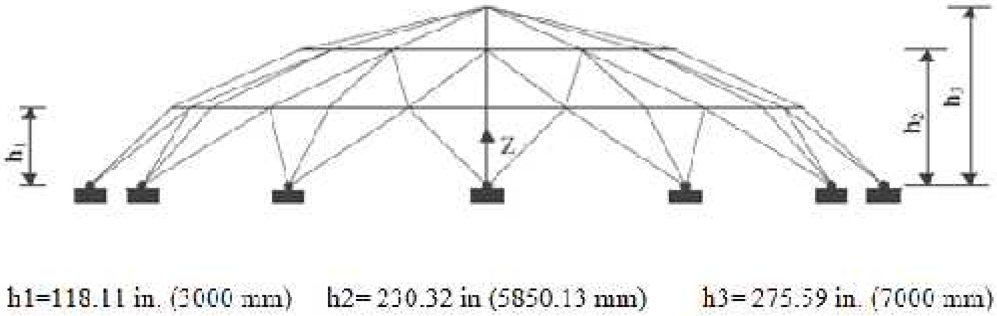

Figure 3 Length and height of 3D space trusses

Table 4 Comparison of stress for 120 bar 3D curved truss with 7.0 m height

Stress Mpa		
GROUP NO.	**Present**	**Sap20000**
2	-32.54325	-33.2327

Table 5 Comparison of displacement for 120 bar 3D curved truss with 7.0 m height

Displacement (m)						
Joint no	**X-direction**		**Y-direction**		**Z-direction**	
	Before optimization	**After optimization**	**Before optimization**	**After optimization**	**Before optimization**	**After optimization**
17	-0.00137	-0.00010	-0.00137	-0.00010	-0.00323	-0.0019
29	-0.00135	-0.00007	-0.00135	-0.00007	-0.00323	-0.0019

Table 6 *Initial and optimum design variables of 3D curved roof dome*

Design variables	**Crosses sectional area (m^2)**		**Max.Stress (Mpa)**	**Max. disp (m)**
	Initial	**Optimum**		
S_1	0.00394	0.00189	-42.5475	0.00454
S_2	0.00137	0.00139		
S_3	0.00137	0.00063		
S_4	0.00175	0.00139		
S_5	0.0008	0.00067		
S_6	0.00362	0.00073		
S_7	0.00285	0.00139		
Weight(kN)	118.518	57.28		
percentage reduction	51.6%			

3.1.3Solid arches under multi point load

This example involves analysis of arches with circle cross-sections, the geometry and a loading of arch which has uniform cross-section with 20 m span is considered shown in Figure. 4. The arches have a radius of curvature R = 11.547 m, the angle $\omega = 2\pi/3$ (span length l = 24.1383 m), the cross-section area= 0.008968 m^2. The following material properties are used: Young's modulus $E = 200\times10^6$ kN/m^2, material density $\rho = 76.9729$ kN/m^3. This example involves analyse and optimization of an arch with maximum tensile stress $\sigma_t = -120\times10^3$ kN/m^3, maximum compressive stress $\sigma_c = -120\times10^3$ kN/m^3 and maximum u_x and u_z displacement all nodes is 0.015 m.

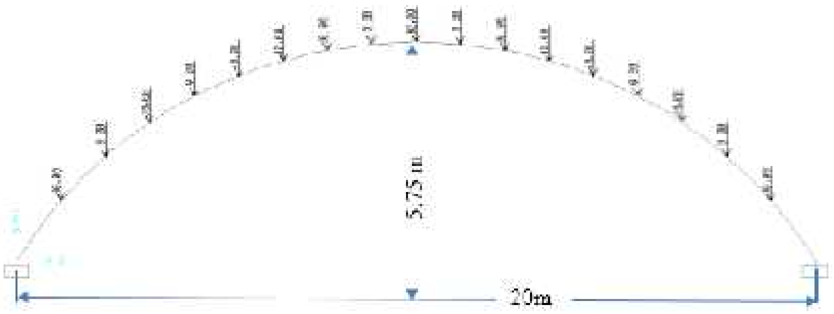

Figure 5.10 Loading condition of arch

Discussion of the results: Analysis is done by source program (SAP2000). Table 7 is the result of displacements in x and z directions. Maximum displacement occurred in the crown.

Table 8 shows the initial and optimal values of design variables and weight. After 42 iterations, minimum weight design was obtained for discrete design variables. The weight of the truss is reduced from 20.618 to 9.698 (53 %reduction).

Table 5.13 *Displacements of uniform cross-section arches*

joint	u_c / l **(Disp. in x-direct.)**	v_c / l **(Disp. in z-direct.)**	θ_c / ω **(Rotation)**
Crown	0.00000	-0.00264	-0.000705

Table 8 *Initial and optimum values of design variables*

Crosses sectional area (m^2)		
Design variables	**Initial**	**Optimum**

S_1	8.968×10^{-3}	5.219×10^{-3}
Weight(kN)	20.618	9.698
P.R	53 %	

Conclusion

A design methodology of 2D and 3D curved roof trusses and solid arch roofing that combines stiffening sizing optimization is an important role in minimizing the amount of steel used in the construction of the structure for economic point of view. The optimization procedure implemented, combined with accurate FE simulation of steel curved roof and solid arch roofing, resulted in a robust and efficient optimization tool.

Optimization algorithm is starts following the implementation of the analysis of the structure. A FORTRAN program which uses the FEMs based numerical analysis was modified. To achieve size optimization based on GA to perform the analysis and design. The problem of choosing the sizes of the bars in order to minimize the weight of the structure while satisfying stress, displacement, stability.

To find the best solution under constrains of allowable displacement and stress GA searches all the available solution among all available results the best solution is selected. Design variables were considered corresponding to the sizing of the cross-sectional areas of the bars.

For all design variables significant decrease in weight of material with respect to the stress and displacement constraints were get. Finally, it must be emphasized that the algorithm proposed is capable of finding the optimum weight or volume with the least number of groups possible to make the design practical. Hence, the solution is feasible and the construction of the structure is easy. The results obtained on these typical problems showed various optimization examples were presented for minimizing the weight of the curved roof structures. Crosses section design variables was used. The influence of the number of design variable employed was also investigated that the accuracy of the concept presented is more than those of the other methods.

Reductions of (54 %, 51.6 % and in arch case 53%) for the three illustrated examples respectively give great encouragement to optimize structures. These reductions are important to save extra materials in construction projects of curved roof structures consequently serving the economical point.

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