Topology optimization of double-curved double-layer grids

Mustafa Sonmez[1], Ozer Sevim[1], Mete Kılıc[2]

[1]Department of Civil Engineering, School of Engineering, Aksaray University, Turkey
[2]Ministry of Environment and Urbanization, Aksaray, Turkey

Abstract

A topology optimization procedure for double layer grids which have square-on-square configuration and top and/or bottom layers are curved in one or two directions is presented. The number of nodes is treated as design variables during optimization process; as a result, the number of members varies. The configuration of grid structures is generated using the graph product operations. The coordinates of nodes and the cross sections of members are taken as continuous and discrete design variables, respectively. As an optimizer, a new swarm based optimization algorithm called Artificial Bee Colony algorithm is used. A design example is included to show the applicability of the presented procedure for optimization of double-curved double-layer grids.

Introduction

The double layer grids (DLGs) which are a special type of space frame structures have become more popular because of a need for large column-free areas. They are used to build for covering large areas such as hangars, exhibition or sports halls and many other purposes. DLGs are three dimensional lattice type structures that provide great rigidity and inherent redundancy. DLG structures which are composed of two planer grids forming the top and bottom layers and connected by diagonal members are the most common type of the space frames. Although a variety of DLG systems are used in the practice, the square grids offset over bottom layer grids may be the most used form of DLG systems [1-5].

At the preliminary design of a DLG structure, a number of parameters such as the module size which is distance between two nodes at a layer, the depth-to-span ratio *d/l*, and connection type must be decided because all these parameters directly affect the cost and speed of the construction. Generally the depth of a DLG is selected as 4% to 8% of its span and the module size is varied from 1 to 2 times larger than the depth of a DLG. [2] stated that module number increases with the span length. In some cases, in order to obtain large inner space and/or architectural concerns, the upper and lower layers may be arched in one or two directions. DLGs curved in on direction are called as “double layer barrel vault” while the DLG curved in two directions may be called as double-curved double-layer grids [3]. Figure 1 shows such a structural system whose layers are curved in two directions and which is used to cover the rectangular or square areas. In this paper, a design procedure for finding the optimal design of such structures is presented. Artificial Bee Colony algorithm [6,7] which is a direct search method such as genetic algorithm is used as an optimization algorithm. The algorithm has been used in the optimum design for truss structures and viscous damper in planer building steel frames in [8,9] and [10].

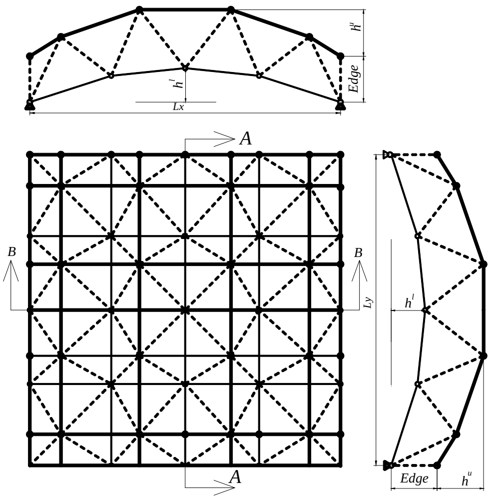

Figure 1 Plan and side views of a double-curved double-layer grid

The presentation of the double layer grid optimization problem

DLGs are composed of a number of straight members jointing together. Obviously, the main factor which affects the cost of these structures excepting the roof cover and joints is weight of the straight members. The goal of the optimization is to minimize the cost of structures while providing the design provisions. It may be expressed mathematically as:

$$\textit{Minimize} \quad W = \sum_{j=1}^{n} A_j L_j \circ_j \tag{1}$$

where A_j, L_j and $\circ_j$ are the cross-sectional area, length and unit weight of j-th gird member, respectively; n is the total number of grid members. While minimizing the weight of structure, the provisions provided by codes also must be satisfied. Stress ($s_{j,l}$), buckling ($b_{j,l}$) and displacement ($d_{i,l}$) constraints are used after normalized in the optimization process.

In order to solve a constrained optimization problem, it is necessary to transform an unconstrained optimization problem to a constrained optimization. The following penalty function is used to transform the unconstrained optimization problem to constraint optimization problems [8];

$$\Phi = W\left[1 + r(t)\sum_{l=1}^{n_c}\left(\sum_{j=1}^{n}\left(s_{j,l} + b_{m,l}\right) + \sum_{i=1}^{n_n} d_{i,l}\right)\right] \tag{2}$$

In Eq. (2), W: unconstrained objective function, Φ: constraint objective function, $r(t)$: the penalty coefficient which is used to tune the intensity of penalization and a function of iteration. When $r(t)$ is set to a predefined static value such as r=1 for all cycles, the resulting function becomes a static penalty function.

Graph theory for generating of configuration of double layer grid

Extensive information related to graph theory and its application to the configuration processing of mesh generation for finite element analysis have been presented by Kaveh and his co-workers [11-13]. They used the graph related theories effectively in the configuration processes of regular structures.

Before explaining the configuration procedure of DLGs, some definitions related to the graph theory must be given. Typically a graph, *S(N,M)*, consists of a finite set of dots (called vertices or nodes, *N*) connected by link (called edges or member, *M*). A one-dimensional graph with *n* nodes and *n+1* member and lie on a straight line, is called a path graph P_n. Two nodes of a graph, *S*, are called *adjacent* if they belong to the same member. Nodes of a member are said to be *incident* to that member. A path graph *S(N,M)* have nodes labeled as $u_1, u_2, u_3, \dots, u_n$ and two adjacent node u_i and u_j forms a member denoted as $u_iu_j \vee M(S)$. Simple graphs such as path graphs are used in the formation of more complex graphs using graph operations. For instance, top and bottom layers of a DLG may be derived using Cartesian product of path graphs. To do so, consider that two path graphs, $P_x(N_x, M_x)$ and $P_y(N_y, M_y)$, have disjointed point sets $N_x=\{x_1, x_2, x_3,...,x_n\}$ and $N_y=\{y_1, y_2, y_3,...,y_m\}$ and edge sets $M_x=\{x_1x_2, x_2x_3,..., x_{n-1}x_n\}$ and $M_y=\{y_1y_2, y_2y_3,..., y_{n-1}y_n\}$. The Cartesian product of these graphs is the graph with the point set $N_x \times N_y$, and $p_1(x_i,y_j)$ connects $p_2(x_l,y_k)$ to form a member whenever

$$
\begin{gathered}
(x_i = x_l)\ \textit{and}\ (y_j\ \textit{adjacent}\ y_k) \\
\textit{or} \\
(y_j = y_k)\ \textit{and}\ (x_i\ \textit{adjacent}\ x_l\ )
\end{gathered}
\tag{3}
$$

The product of P_x and P_y are depicted in Figure 2.

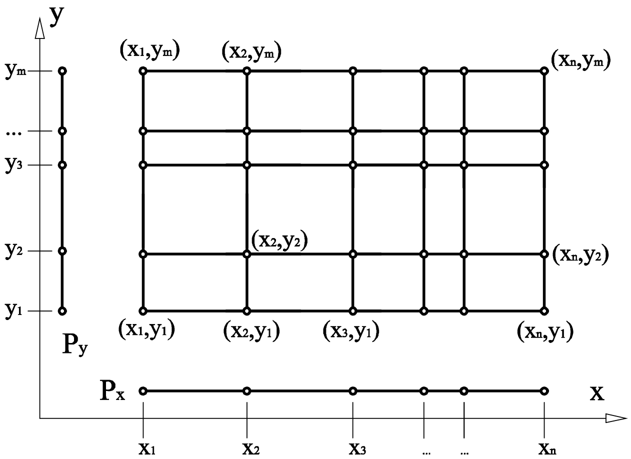

Figure 2 Cartesian product of two path graph

Before the upper and lower layer of a DLG are constructed using the theory of graph product mentioned above, two simple path graphs lied on the x- and y-axis must be determined. For instance, if span lengths at *x* and *y* directions (L_x and L_y) are given, the number of node laid at the *x-axis* and their coordinates, $N^l_x=\{x^l_1, x^l_2, x^l_3,...,x^l_{nx}\}$ is calculated using following mathematical relation;

$$x^l_i = \frac{L_x}{nx-1}(i+\text{¤}) \qquad 1 < i \le n/2 \tag{4}$$

Where ¤ is a random number between -0.5 to 0.5. *nx* is for the total number of nodes laid on *x-axis* and vector N^l_x represents the coordinates of nodes. Coordinates of nodes at y-axis, $N^l_y=\{y^l_1, y^l_2, y^l_3,...,y^l_{ny}\}$ are determined as:

$$y^l_j = \frac{L_y}{ny-1}(j+\text{¤}) \qquad 1 < j \le m/2 \tag{5}$$

After creating two simple path graphs ($N^l_x \times N^l_y$), the lower layer of DLGs is constructed using the Cartesian product of these paths. Then, the heights of nodes on the lower layer are determined using hyperbolic equations as;

$$z_{ij}^{l} = \frac{16\,h^{l}}{L_x\,L_y}\left(x_i^l - \frac{x_i^{l\,2}}{L_x} \right)\left(y_j^l - \frac{y_j^{l\ 2}}{L_y} \right) \tag{6}$$

in which h^l, the rise of lower layer, is taken as design variable.

The upper layer gird members and their coordinates are calculated usingthe nodes on the lower layer. The path graphs $N^u{}_x$ and $N^u{}_y$ are determined following equations:

$$x_i^u = x_i^l + \ddot{}\ (x_{i+1}^l - x_i^l), \quad 1 < i \le (n+1)/2 \tag{7}$$

and

$$y_i^u = y_i^l + \ddot{}\ (y_{i+1}^l - y_i^l) \quad 1 < i < (n+1)/2 \tag{8}$$

After computing *x*- and *y*- coordinates of nodes on the lower chord,*z*-coordinates of the node on upper layer are determined by using the following formulation;

$$z_{ij}^{u} = edge + \frac{16\,h^{u}}{L_x\,L_y}\left(x_i^u - \frac{x_i^{u2}}{L_x} \right)\left(y_j^u - \frac{y_j^{u2}}{L_y} \right) \tag{9}$$

where *edge* is the distance between upper and lower layer at the supports and h^u is the rise of upper layer. Both edge and h^u are considered as design variables in this study.

Artificial bee colony (abc) algorithm

In topology optimization of DLGs, there are several design variables such as number, coordinates and cross-sections area of members. Some of these design variables may be discrete such as steel sections which are selected form available lists and continuous such as node coordinates. Hence, the optimization algorithm indented to use in the topology optimization must handle both types of design variables. The direct search techniques, in which the gradient of the objective function and design variables are not required, are generally more preferable than the conventional gradient based optimization algorithms for the problems having mix design variables. There are various powerful direct search techniques available in the literature such as Genetic Algorithm, Evolutionary Algorithm, Ant Colony Algorithm, Particle Swarm Optimization and Artificial Bee Colony (ABC) algorithm for structural optimization problems. Among them, the ABC algorithm [12] which is a new approach for structural design inspired by forging behaviour of honeybee has already applied to different optimization problems [6-10]

Bees behaviour

Honeybees like other social insects are only able to survive as a member of a community known as a colony. The colony inhabits a hive, an enclosed cavity. A typical bee colony contains 3 types of bees; a queen bee, a few thousand fertile males (called drones) and thousands of sterile female worker bees. The task of queen is to lay eggs and start new colonies by mating with drones. The female workers must perform the all the other task associated with colony life such as; building the honeycomb and storing food, cleaning cells, feeding the queen and drones, guarding the entrance and finding and bringing the food to their hive. The female workers which are about three weeks old are performing most tasks within the hive. After three weeks, they become foragers for the rest of their lives. Forager honeybees search for promising flower nectar from different food sources in multiple directions up to 12 km from the hive, but they mostly fly within a 3 kilometre radius.

Frisch [14] discovered that when honeybees find a food source they advertise their findings by performing a “round dance” or “waggle dance” based on the distance of the food source from the hive. The round dance gives the information that the discovered food source is in the vicinity of the hive and the waggle dance informs the other bees that the food source is situated more than 100 m from the hive. The direction and duration of dances are closely correlated to the direction and distance of the patch of flowers. All forager bees, of course, can find differing quality food sources during their trip. Hence, after unloading the nectar, each bee can follow one of three options: (a) abandon the food source and search for another promising flower patch, (b) continue to forage at the food source without recruiting nest mates, or (c) perform the one of the bee dance to recruit nest mates before returning to the food source. The option selected is based on the food level of the nectar source the individual bee had found. If a bee has found a nectar source that is above a certain limit, she follows option (c). If the nectar source is average, the bee goes to forage at the food source without recruiting nest mates (option b). Otherwise, the bee continues to search for promising nectar sources (option a). The main goal of the bees is to locate the most abundant nectar source.

Modeling of Bee behavior

The food sources and their nectar qualities represent possible solutions to given optimization problem. The location of food source corresponds to the design variables. In the first step of the ABC, all worker bees (BN) are considered to leave the hive to search for promising flower patches. The location of a food source, *s*, can be determined as:

$$ {}_s x_j^{new} = {}_s x_j^{low} + ¥\left({}_s x_j^{up} - {}_s x_j^{low}\right), \qquad x^{low} \leq x_j \leq x^{up} \tag{10} $$

${}_s x_j^{low}$ and ${}_s x_j^{up}$ are the lower and upper bounds of the j^{th} design variable and ¥ is a random number between 0 and 1. After the bees return to the hive with a certain amount of nectar (Φ), the first half of the bees which found the best food sources become “employed bees.” The remainder of the bees watch the dancing bees to decide one of which employed bee may be followed. These bees, which watch the dance, are called “unemployed bees” or “onlooker bees.” It means that unemployed bees select a food source according to a probability proportional to the amount of nectar to be found at that food source.

After an employed bee has recruited unemployed bees, if any, she leave hive to find a better food source (called candidate food sources) in the neighbourhood of the previous food sources discovered by her and other employed bees. This means that the ABC algorithm uses the previous food source (${}_s x_j^{old}$) to search for a candidate food source (${}_s x_j^{new}$). Numerically, the location of a candidate food source, *s,* is determined as:

$$ {}_s x_j^{new} = {}_s x_j^{old} + ¤\left({}_s x_j^{old} - {}_s x_k^{old}\right) \tag{11} $$

where ¤ is a random number between -1 and 1. ${}_s x_j^{new}$ is an updated design variable. The left hand subscripts represent the solution number (food source, $s = 1, 2, ..., SN$) while the right hand script denote the design variable number. *k* is a randomly chosen integer number but cannot be equal to *s*. ${}_s x_k^{new}$ plays an important role in the ABC convergence behaviour since it is used to control the exploration abilities of the bees. It directly influences the location of the new food source, which is based on the previous location of other food sources. If the level of food in the new location is better than old one, the new position becomes the food source; otherwise, the old location is maintained as the best food source.

Numerical examples

The topology optimization algorithm presented is used to determine the following optimum values of; *i*) Edge distance between layers at the supports, *ii*) the height of lower and upper layers at the middle of the DLGs, *ii*) the number and location of nodes, and *iv*) the circular steel hollow section designations for DLGs. During optimization process, DLGs are assumed to be subjected dead load and snow load only. Dead load and Snow load are assumed to be 1.8 kN/m^2 and 1.0 kN/m^2, respectively. For the proposed algorithm, a bee colony is assumed to consist of 50 bees (N). The number of cycle (MNC) is set to 1000 and LIMIT=MNC/3. The following data is used to construct the sample double-curved and double-layer grid:

- Span length on x and y directions are 1000 cm and 2000 cm
- Max. and min. edge distance are 200 cm and 10 cm
- Max. and min. value of lower layer at the apex of grid are 200 cm and 0 cm
- Max. and min. value of upper layer at the apex of grid are 200 cm and 0 cm
- Max. and min. node number in x-direction are set to 10 and 5
- A set of 11 discrete values are used for the possible cross-sectional areas and second moment of inertia for each member L = { 2.69 (1.56), 3.37 (1.99), 4.24 (3.13), 4.24 (3.71), 4.83 (4.27), 6.03 (5.4), 7.61 (6.89), 8.89 (10.67), 10.16 (12.26), 10.8 (16.18), 11.43 (20.41), 12.7 (22.81)}

The displacements at all nodes in x-direction and y-direction are set to less than L/360 set. The allowable stresses for all members were ±240 MPa. Buckling resistance of members are calculated with according to AISC-ASD [15]. All members were assumed to be constructed from a material with an elastic modulus of E = 205 GPa and a mass density of

ρ =7.85 gr/cm^3. Due to the symmetry of the, 22 independent design variables on lower layer, 30 variables on upper layer and 12 variables on web are used for linking.

The best design developed by the ABC algorithm gives a grid weighing 36897.74 N without any constrain violation. ABC finds 75.40 cm, 147.24 cm and 112.88 cm for edge, the height of lower layer and the height of upper layer. The convergence rate of 50 bees for the sample problem double layer grid is given in Fig. 3.

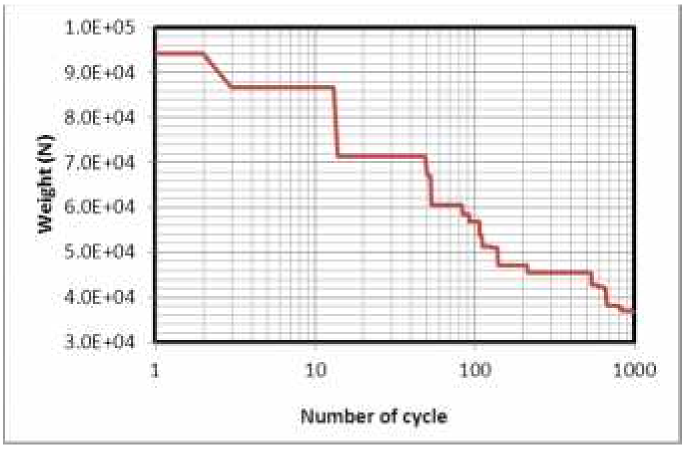

Figure 3 Convergence rate of 50 bees for the sample problem double layer grid.

Fig. 4 to Fig. 6 show final configuration of the lower layer, upper layer and web members. Numbers on the members in Fig. 4 to 6 are index of cross-section area in the listed in L.

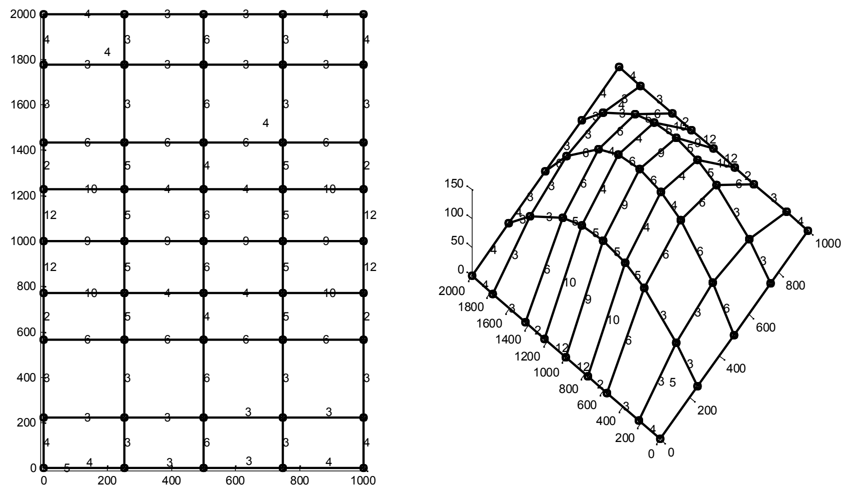

Figure 4 Final configuration of the lower layer a) plan view b) 3d view

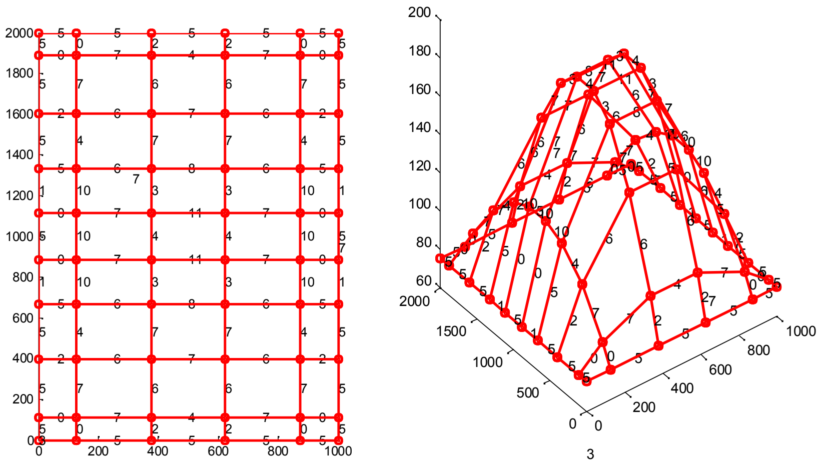

Figure 5 Final configuration of the upper layer a) plan view b) 3d view

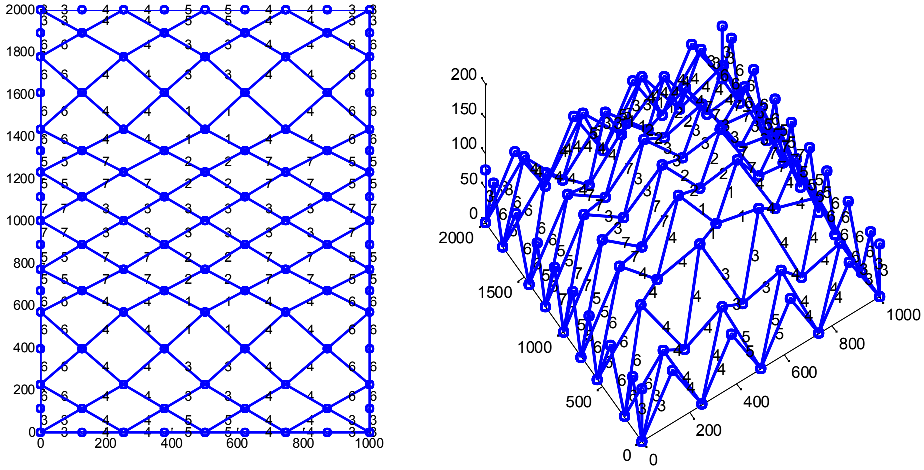

Figure 6 Final configuration of web layer a) plan view b) 3d view

Conclusion

In the present work, the artificial bee colony (ABC) algorithm and graph theory were employed to establish a new methodology for topological studies of double layer grids. The fundamental procedure of the ABC was explained. The steps for developing the methodology and applying it for studying a double layer grid were presented. Studies show that the ABC algorithm is a robust technique that can deal with topology optimization to double layer grids.

References

[1] Daniel A Grids (1997), Cuoco, Guidelines for the design of double-layer grids, ASCE, Task Committee on Double-Layer.

[2] Lan T. T, (1999) Space frame structures, Structural engineering handbooks, editors W.F. Chen, E.M. Lui, CRC USA.

[3] John Chilton (1999), Space Grid Structures, Architectural Press, Jordan Hill, Oxford.

[4] Farsangi H.E. (2002), "Topological optimization of double layer grids using genetic algorithm, University of Surrey, Civil Eng. Dept. Ph. D. Thesis, UK.

[5] E. Salajegheh, M. Mashayekhi, M. Khatibinia and M. Kaykha, (2009) Optimum shape design of space structures by genetic algorithm, International Journal of Space Structures Vol. 24 No.1, pp:45-57.

[6] Karaboga D (2005) An idea based on honey bee swarm for numerical optimization. Technical Report, TR06, Erciyes University.

[7] Karaboga D and Basturk B. (2008), On the performance of artificial bee Colony (ABC), Applied Soft Computing 8-1, 687-697

[8] Sonmez M. (2011), Artificial Bee Colony Algorithm for Optimization of Truss Structures, Applied Soft Comput. J., 11:2406-2418.

[9] Sonmez M.(2011), Discrete Optimum Design of Truss Structures Using Artificial Bee Colony Algorithm, Struct Multidisc Optim, 43: 85-87.

[10] Sonmez M, ·E. Aydin E ·Karabork T (2013), Using an artificial bee colony algorithm for the optimal placement of viscous dampers in planar building frames, Struct Multidisc Optim, DOI 10.1007/s00158-013-0892-y

[11] Kaveh, A.(1992) Structural Mechanics: Graph And Matrix Methods, John Wiley and Sons Inc, England

[12] Kaveh A. and Nouri M. (2009), Weighted Graph Product for Configuration Process of Planar and Space Structures, International Journal of Space Structures, Vol. 24, No:1.

[13] A. Kaveh K. Koohestani (2008), Graph Products for configuration of space structures, Computers and Structures 86, 1219-1231.

[14] Frisch K (1967), Dance Language and Orientation of Bees, Belknap Press of Harvard Uni. Press. Cambridge. Massachusetts.

[15] American Institute of Steel Constructions (2005), Allowable Stress Design Manual, Chicago, Illinois.