Kinetic energy and momentum correction factors in a stream

[1]Mehmet Ardıçlıoğlu, [1]Mücella Vapur, [2]Onur Genç,

[1]University of Erciyes, Department of Civil Engineering, Kayseri, Turkey
[2]Melikşah University, Department of Civil Engineering, Kayseri, Turkey

Abstract

In this study, discharge, kinetic energy, α, and momentum, β , correction coefficients in a stream were calculated precisely. For this purpose the point velocities of a section were measured using an Acoustic Doppler Velocimeter (ADV). For a subcritical uniform flow condition, cross-section was divided in different slices, discharge, kinetic energy and momentum correction coefficients were investigated depending on the slice number. According to the measured and calculated verticals point velocities, 9 slice number was accepted as optimum for discharge, α and β coefficients

***Keywords**: Stream, Discharge, Kinetic energy and Momentum correction coefficients*

Introduction

Investigations of flow discharge and velocity distributions over the river cross-section are very important for purposes such as water management, water supply, irrigation, flood control, etc. Velocity distributions are especially necessary for open-channel conditions in order to calculate important parameters such as shear stress distributions, energy loss, sediment, discharge, kinetic and momentum correction factors.

The discharge of a stream usually is calculated from a series of measurements of width, depth and velocity along a cross section of the stream. Theoretically, the true discharge would be an integration of the velocity and area throughout the cross section. Discharge is expressed in volume of water per unit time, usually liter per second or cubic meters per second in the metric system. Discharge measurements may be conducted by several methods given in the literature [1].

The velocity-area method is most commonly used in natural streams. In this method cross section is divided into slices according to the width of the section as shown in Figure 1. For each slice, velocities are measured from closed bed till the water surface. For each vertical point velocities (v_i) small area (a_i) were defined and for this area discharge can be calculated by Eq. (1). Total discharge is also calculated with Eq. (2) where n is the number of small areas.

$$q_i = a_i \; v_i \tag{1}$$

$$Q = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} a_i v_i = \tag{2}$$

In river flow, generally the velocity distributions are not uniform over the cross-section; and so, the velocity head and the momentum flux are generally greater than the values computed by using the average velocity. These values may be corrected by using the so-called energy and momentum correction coefficients, which are always slightly greater than the limiting value of unity [2]. The kinetic energy head was corrected by Coriolis as $\alpha V^2/2g$, where α = kinetic energy coefficient, V= average velocity and g= gravitational acceleration. Similarly, the non-uniform velocity distribution at any section influences the linear momentum flux and should be computed as $\beta \rho Q V$, where β is the Boussinesq (momentum) coefficient [3]. Kinetic energy, α and Momentum, β, correction coefficients, are often assumed to be unity when the momentum and energy principles are used in the computations as presented by many authors [3, 4, 5, 6].
Kinetic energy and momentum correction coefficients, α and β are computed using equations (3) and (4) for a single and compound cross-sectional areas of a river.

$$\alpha = \frac{1}{V^3 A} \int_A v^3 \, dA \tag{3}$$

$$\beta = \frac{1}{V^2 A} \int_A v^2 \, dA \tag{4}$$

where v = point velocity at each point in the cross section, V = cross-sectional mean velocity, A = whole water area, and dA = an elementary area in the whole water area.

The objective of this study is to investigate the slice number for discharge calculation for a river cross-section. Also the magnitudes of α and β in a flow condition were determined for different slice numbers.

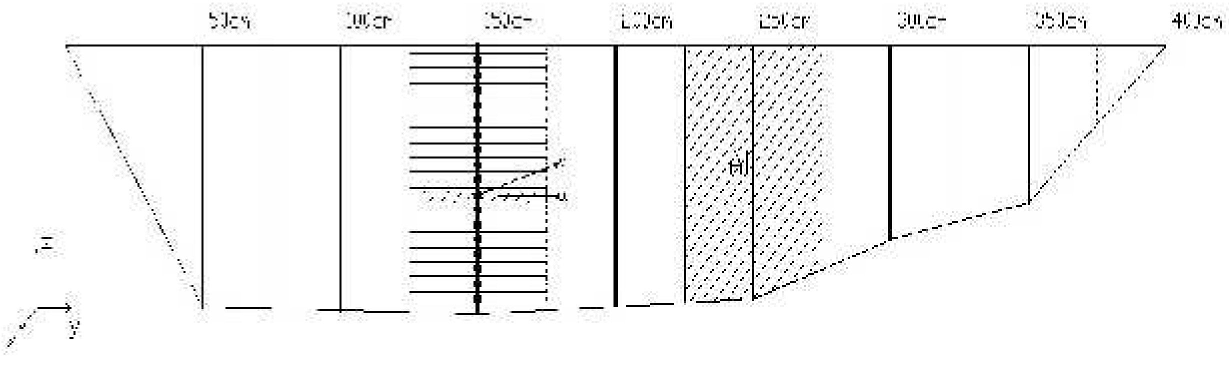

Figure1. Bunyan station cross-section and measured slices and verticals

Field study

Field measurements were undertaken on Kızılırmak basin, which is situated in the central Anatolia in Turkey. Turkey has a semi-arid climate with some extremities in temperature. Central Anatolia has a Steppe climate with little precipitation and daily and yearly temperature values differing significantly. Field measurements are achieved on a branch of Kızılırmak River named Sarimsakli stream. Velocity measurements were taken at Bunyan station in 24 June 2009 as shown in Figure 2.

The velocity measurements were undertaken through the use of Acoustic Doppler Velocimeter (ADV). The SonTek/YSI FlowTracker Handheld ADV was used for field measurements. ADV measures three-dimensional flow velocities (u, v, w) in a sampling volume using the Doppler shift principle and consists basically of a sound emitter, three sound receivers and a signal conditioning electronic module. The ADV sampling volume is located 10 cm in front of the probe head. Therefore the probe head itself has a minimal impact on the flow field, surrounding the measurement volume. Velocity range is ±0.001 m/s to 4.5 m/s, resolution 0.0001 m/s, accuracy ±1% of measured velocity, ±0.001 m/s [7].

Cross-section was divided into seven slices according to the water surface width as shown in Figure 1. Point velocity measurements were made at different positions in the vertical direction starting 2cm from the bed for each vertical. Free surface velocity was then estimated by regression of the upper two measurements. Discharge was calculated using Eq. (1) and (2) as Q=0.788 m^3/s for performed measurements. Mean velocity (V) was calculated using integrated discharge and cross-sectional area as V (=Q/A) =0.354 m/s where A is the cross-sectional area. For measurement; Reynolds number Re $(= 4VR/\upsilon)$ was calculated as 709448 and Froude number Fr $(= V/\sqrt{gH_{max}})$ was calculated as 0.133 where R is the hydraulic radius, υ is the kinematic viscosity, g is the gravitational acceleration and H_{max}=72cm is the maximum flow depth at the cross-section. As shown in these values flow is sub-critical and turbulent.

Figure 2. A picture from measuring station at Bünyan

Analysis of measured data

Discharge is the volume of water flowing through a cross-section of a stream in a given amount of time and it is very important information for understanding hydrological processes The precision in the velocity and discharge measurements taken in the stream flow are important for using the limited water resources in a correct and suitable manner. Therefore, the number of measurements and the method for the cross section are needed to be determined correctly. Discharge through a cross-section in a natural channel is estimated from the mean velocity in the section and the cross-sectional area. In this method cross section was divided into slices according to the width of the section as shown in Figure 1. The verticals should be spaced in a way that no subsection exceeds 10 percent of the total discharge [8].

In this study slice number effect for discharge calculation is investigated. For this purposes cross-section was divided in different number of slices as given in Table 1. Flow measurements were taken at seven different verticals along the cross-section. In this situation discharge was obtained as Q_7=0.788 m^3/s. When only one vertical measurement for y=200cm was considered discharge was found as Q_1=0.878 m^3/s. Similarly for 3 slice y=100cm, 200cm and y=300cm verticals measurements were used and discharge was found as Q_3=0.808 m^3/s. For 9, 15, 23, 31 and 39 slice, linear interpolation formula from MATLAB package was used and point velocities on intermediate verticals were also computed. Using these new vertical velocities discharges were calculated with Eq. (2) and (3). Variation between previous and next discharges was given in the last column in Table 1. As shown in this column, after the 9th slice discharges were closed to each other. That means 9 slices are suitable for this cross-section. Literature also says that similar slice number is enough for stream measurements [8]. In Figure 3 the relation between discharge and slice number was also given. As shown in this figure discharge is constant after the 9th slice.

Table 1. Slice numbers and discharge variation used for the measurement

Slice Number	Discharge	Variation %
	m^3/s	$\varepsilon=[(Q_p-Q_n)/Q_p]$
1	0.878	
3	0.808	-8.60
4	0.821	1.59
7	**0.788**	**-4.24**
9	0.773	-1.89
15	0.775	0.26
23	0.772	-0.45
31	0.769	-0.40
39	0.760	-1.15

Q_p=previous discharge
Q_n=next discharge

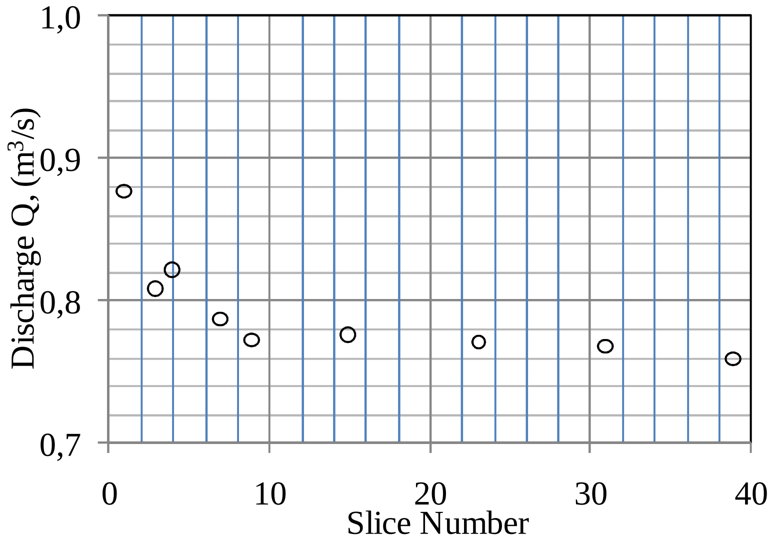

Figure 3. Relation between discharge and slice number

Using the velocity distribution data, the kinetic energy and momentum correction coefficients were computed using equations (3) and (4) for the Bünyan station. For seven slice α and β coefficients were calculated as 1.4586 and 1.1732 respectively. For different slice numbers α and β were also computed and given in Table 2. In this table variation between previous and next coefficients α and β are also given in the next two columns. As shown in the Table 2 these values after the 9th slice these variations differ slightly, which means that 9 slices are enough for these calculation. In Figure 4 the relation between correction coefficients and the slice number was also given. As shown in this figure these coefficients do not change after 9 slices.

Seçkin et all. [6] found that there is a large difference in velocity distribution between main channel and floodplains for compound channels. They calculated the correction coefficients for asymmetrical and symmetrical compound channels, the values of α and β averaged at 1.156 and 1.056 respectively, while for the single channels they averaged at 1.0604 and 1.0222 respectively. This means that α and β for single channels yield lower average values than those of compound channels. For natural stream α and β are higher than prismatic channels.

Table 2. Slice number and variation of Alfa, Beta

Slice Number	Alfa α	Beta β	Variation % $\varepsilon=[(\alpha_p-\alpha_n)/\alpha_p]$	
1	1,1116	1,0406	α	β
3	1,4005	1,1331	20,6	8,2
4	1,3227	1,1099	5,9	2,1
7	1,4598	1,1741	9,4	5,5
9	1,3857	1,1307	5,4	3,8
15	1,4422	1,1489	3,9	1,6
23	1,4422	1,1501	0,0	0,1
31	1,4436	1,1504	0,1	0,0
39	1,5136	1,1848	4,6	2,9

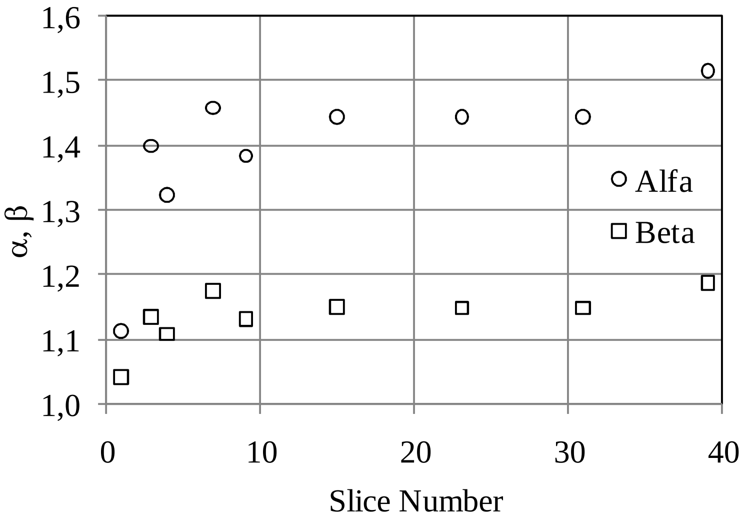

Figure 4. Relation between slice number and Alfa, Beta

Conclusions

Kinetic energy and momentum correction coefficients, α and β, are often used to be unity when the energy and momentum principles are used in the hydraulic computations. However, because of non-uniform distribution of velocities over a channel section, α and β, are generally greater than unity. Depending on non uniform cross sections and surface roughness, velocities change point to point, and correction factors, α and β, become higher than in prismatic channels. Nine slices were found to be enough for the calculation of discharge, and correction coefficients in this cross-section. For nine slices α and β were found as 1.41 and 1.14 respectively.

References

[1] Rantz S., E., 1982-a,b, Measurement and Computation of Streamflow, Vol. 1,2, Measurement of Stage and Discharge. Geological Survey Water-Supply Paper 2175, U.S. Government Printing Office: Washington, DC.

[2] Al-Khatib, I. A., 2013, Investigation of momentum and kinetic energy correction coefficients in asymmetric compound cross-section flumes. Turkish J Eng Env Sci, 37, 69 – 78

[3] Chow, V. T., 1959, Open Channel Hydraulics. McGraw-Hill Book Co., New York.

[4] French, R.H., 1987, Open-Channel Hydraulics, McGraw-Hill, Singapore. 2. Edition.

[5] Chen, C. L., 1992, Momentum and energy coefficients based on power-law velocity profile. Journal of Hydraulic Engineering, ASCE, Vol. 118, No. 11, pp. 1571-1584.

[6] Seçkin, G., Ardıçlıoğlu, M., Çagatay, H., Çobaner, M. and Recep Yurtal, R., 2009, Experimental Investigation of Kinetic Energy and Momentum Correction Coefficients in Open Channels, Scientific Research and Essay, 4, 473–478.

[7] SonTek 2002: Flow Tracker Handheld ADV, Technical Documentation.

[8] U.S. Department of the Interior Bureau of Reclamation, 1984, Water Measurement Manual. Water Resources Technical Publication, Third edition.