Modifying Ritchie equation for estimation of reference evapotranspiration at coastal regions of Anatolia

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Abstract

Evapotranspiration (ET) is of great importance in many disciplines, including irrigation system design, irrigation scheduling and hydrologic and drainage studies. A large number of more or less empirical methods have been developed to estimate the evapotranspiration from different climatic variables. The Food and Agriculture Organization (FAO) rates the Penman-Monteith equation as the major model for estimation of reference (grass) evapotranspiration (ET_0) because of the fact that it gives more accurate and consistent results as compared to the other empirical models. However, the main disadvantage of this method is that it cannot be used when the sufficient data are not available. The FAO-56 PM equation requires quite a few independent variables such as solar radiation, air temperature, wind speed, and relative humidity in predicting ET_0. Worldwide, the weather stations measuring all these variables are few as the majority measure air temperature only. Therefore, for regions which may not be measuring all these meteorological variables, the temperature based models like Ritchie, Hargreaves-Samani and Thornthwaite equations is necessarily used instead of the FAO-56 PM equation. In this study, the Ritchie equation is applied on the measured data recorded at 158 stations at the Coastal are of Turkey (Mediterranean, Aegean, Marmara and Black Sea regions of Anatolia), and the monthly ET_0 values computed by it are observed to be smaller than those given by the Penman-Monteith equation. Next, average values for the coefficients of the Ritchie equation, which are constants originally developed in [6], are recomputed using the ET_0 values given by the FAO-56 PM equation at all weather stations in coastal regions of Anatolia (Turkey). The Ritchie equation modified in such manner is observed to yield greater determination coefficients (R^2), smaller root mean square errors (MSE), and smaller mean absolute relative errors (MARE) as compared to the original versions of Ritchie equation suggested by [6]. It is concluded that for estimation of reference evapotranspiration at coastal regions of Anatolia where the meteorological measurements are scarce, the modified Ritchie equation can be easily used for estimating the ET_0 values.

Introduction

Evapotranspiration is defined as amount of water, expressed in depth in mm over a certain area in daily, weekly, monthly, or yearly periods, passing from liquid to gaseous state as summation of evaporation from wet soil on which the plants exist plus the transpiration from leaves of the plants controlled by cells called stomata [2], which is an important component of hydrologic cycle. Because it is needed in calculation of crop water requirement, accurate estimation of evapotranspiration is vitally important as it has a significant effect on planning and management of water resources for agricultural irrigation activities, whose economic worth is steadily increasing along with increasing demand for food.

The first stage in estimation of evapotranspiration is determination of the reference evapotranspiration (ET_0). [1] defined ET_0 as the evapotranspiration from the land covered by grass having an average length of 12 mm with a shadow resistance of 70 s m^{-1} and an albedo of 0.23, and they developed the FAO-56 Penman-Monteith method of the World Food and Agricultural Organization (FAO) for its calculation, which is rated by FAO as the major model for estimation of ET_0 because of the fact that it gives more accurate and consistent results as compared to the other models. FAO-56 Penman Monteith (PM) method is described in [1] as:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T+273} U_2 (e_a - e_d)}{\Delta + \gamma(1 + 0.34U_2)} \tag{1}$$

where ET_0 = reference evapotranspiration (mm day^{-1}); Δ = slope of the saturation vapour pressure function (kPa $^\circ C^{-1}$); R_n = net radiation (MJ $m^{-2} day^{-1}$); G = soil heat flux density (MJ m^{-2} day^{-1}); γ = psychometric constant (kPa $^\circ C^{-1}$); T = mean air temperature ($^\circ$C); U_2 = average 24 h wind speed at 2 m height (m s^{-1}), e_a is the saturation vapour pressure (kPa), e_d is the actual vapour pressure (kPa). The FAO-56 Penman-Monteith formula relates the daily ET_0 to solar (extraterrestrial) radiation, relative humidity, and wind speed next to air temperature. In Turkey and in many other countries, the weather stations regularly keep track of air temperature but most of them do not measure all the other three variables. Because of paucity of meteorological data, [6] developed a simpler formula relating ET_0 to air temperature only, known as the Ritchie equation, which is rewritten below:

$$ET_0 = \alpha_1 \{aR_s(0.6T_{max} + 0.4T_{min} + b)\} \tag{2}$$

where, ET_0 is reference evapotranspiration in (mm day^{-1}), T_{max} and T_{min} are maximum and minimum air temperatures in ($^\circ$C), and R_s is solar radiation in (MJ m^{-2} day^{-1}), a and b are the constant values of the original Ritchie equation, 3.87×10^{-3} and 29, respectively. α_1 in this equation are computed as a function of T_{max} as given below:

$$\left.\begin{array}{ll} \alpha_1 = (0.01) \times exp[0.18(T_{max} + 20)] & T_{max} < 5^o C \\ \alpha_1 = 1.1 & 5^o C < T_{max} \le 35^o C \\ \alpha_1 = 1.1 + [0.05(T_{max} - 35)] & 35^o C < T_{max} \end{array}\right\} \tag{3}$$

In this study, the Solar radiation (R_S) data were not available, so the R_S values were calculated with an equation recommended by [6]. If the solar radiation, R_S, is not measured, it can be calculated with the Angstrom formula, which relates solar radiation to extraterrestrial radiation and relative sunshine duration:

$$R_S = k_{Rs} \times R_a \sqrt{(T_{max} - T_{min})} \tag{4}$$

where, R_s is solar or shortwave radiation (MJ m^{-2} day^{-1}); R_a, is extraterrestrial radiation (MJ m^{-2} day^{-1}); T_{max}, is maximum air temperature (°C); T_{min} is minimum air temperature (°C) and k_{Rs} is adjustment coefficient ($°C^{-0.5}$). The square root of the temperature difference is closely related to the existing daily solar radiation in a given location. The adjustment coefficient k_{Rs} is empirical and differs for 'interior' or 'coastal' regions [6]:

• for ‘interior’ locations, where land mass dominates and air masses are not strongly influenced by a large water body, k_{Rs} can be used as 0.16;

• for ‘coastal’ locations, situated on or adjacent to the coast of a large land mass and where air masses are influenced by a nearby water body, k_{Rs} can be used as 0.19.

Materials and method

As shown in Figure 1, there are 158 weather stations at Coastal Regions of Anatolia owned and operated by DMI (www.dmi.gov.tr) which have been measuring at daily, even at hourly, periods all meteorological variables needed by the FAO-56 PM equation. In this study, however, the average values in monthly periods, computed over the length of record, of maximum and minimum temperatures, along with solar radiation, relative humidity, and wind speed, are used as the data material. The study area, Coastal Regions of Anatolia, which is four of the seven geographical regions delineated by topographic and climatic considerations in early 1930s, covers an area of 371.573 km^2 in Turkey.

Figure 1 Distribution of the meteorological station at Coastal Regions of Anatolia in Turkey.

a and *b* coefficients of the Ritchie equation, which is rewritten here as Eqn.(2) above, are recomputed with the help of the Solver menu of Microsoft Excel using the monthly average values of all the pertinent meteorological variables recorded at 158 stations over the period: 1975–2010 and assuming the ET_0 value computed by the FAO-56 PM equation was the true reference evapotranspiration. Excel Solver is part of a broader system of commands called Simulation Solver in Excel, and it is used to determine the most suitable form of the analytical expression chosen in a window known as target cell. The Solver utilizes the data in another cell pertaining to the chosen analytical model input by the user. Constraints for the coefficients of the analytical expression can also be specified [Excel–Help]. Microsoft Office Excel Solver tool has used that nonlinear optimization code was developed the Generalized Reduced Gradient (GRG2) by Leon Lasdon (University of Texas at Austin) and Allan Waren (the Cleveland State University). The algorithm is an improved state of the simplex method for non-linear programming [5]. Generalized reduced gradient algorithm was described in [5] and [3].

Applications and results

Using the monthly average values of the relevant meteorological variables recorded at 158 stations over the period: 1975–2010, the ET_0 values computed by the FAO-56 PM equation were assumed to be the true reference evapotranspirations at those stations. With the help of Excel Solver, suitable magnitudes for the *a*, and *b* coefficients of the Ritchie equation were computed based on the ET_0 values computed by the FAO-56 PM equation. Suitable new values for all the two coefficients were computed for Coastal Regions of Anatolia. The predictive abilities of the Ritchie equation modified in such manner were quantitatively evaluated by the common test statistics of mean absolute relative error (MARE), mean square error (MSE), and determination coefficient (R^2) expressed as:

$$MARE = n^{-1} \times \left| \left(ET_{0,MR} - ET_{0,PM} \right) / ET_{0,PM} \right| \times 100 \quad (5)$$

$$MSE = \left(ET_{0,MR} - ET_{0,PM} \right)^2 / n \quad (6)$$

$$R^2 = 1 - \left(\sum_{i=1}^{p} \left[ET_{0,MR} - ET_{0,PM} \right]^2 \Big/ \sum_{i=1}^{P} \left[ET_{0,MR} - \overline{ET}_{0,MR} \right]^2 \right) \quad (7)$$

where, n is the number of elements (no. of stations), $ET_{0,PM}$ is the reference evapotranspiration computed by the FAO-56 PM equation, and $ET_{0,MR}$ is ET_0 computed by a modified Ritchie equation. is mean value of computed by a modified Ritchie equation.

Table 1 Statistical parameters of long term meteorological data for Coastal Regions of Anatolia

	Variables	X_{ort}	S_x	C_v	C_{sx}	C_k	X_{mak}	X_{min}
The Mediterranean Region	Maximum Temperature	32.29	7.71	0.24	-0.36	-0.84	46.7	12.5
	Minimum Temperature	0.86	10.41	12.12	-0.64	0.42	20.6	-33.5
	Average Temperature	16.35	7.95	0.49	-0.25	-0.79	30.2	-3.8
	Maximum Humidity	73.84	7.49	0.10	-0.43	-0.39	90.1	53.8
	Minimum Humidity	7.92	5.62	0.71	1.27	2.63	38.0	0.0
	Wind Speed	2.03	0.79	0.39	1.39	2.70	5.6	0.7
The Black Sea Region	Maximum Temperature	31.04	6.95	0.22	-0.48	-0.37	45.1	10.3
	Minimum Temperature	-3.04	10.17	-3.34	-0.53	-0.18	17.0	-30.8
	Average Temperature	12.32	7.15	0.58	-0.14	-0.97	25.1	-6.4
	Maximum Humidity	78.77	5.90	0.08	-0.08	-0.66	94.1	64.2
	Minimum Humidity	14.61	9.51	0.65	0.85	0.12	44.0	0.0
	Wind Speed	1.88	0.79	0.42	1.18	2.53	5.4	0.4
The Marmara Region	Maximum Temperature	31.14	7.43	0.24	-0.25	-1.01	45.4	11.6
	Minimum Temperature	-2.18	9.06	-4.17	-0.24	-0.71	15.0	-27.8
	Average Temperature	13.30	7.08	0.53	0.02	-1.20	26.8	-4.0
	Maximum Humidity	81.89	5.64	0.07	-1.32	2.68	91.7	56.0
	Minimum Humidity	16.94	7.77	0.46	0.35	-0.18	40.0	0.0
	Wind Speed	2.37	1.14	0.48	1.40	2.62	7.40	0.6
The Aegean Region	Maximum Temperature	31.98	7.75	0.24	-0.19	-1.08	45.7	14.2
	Minimum Temperature	-0.66	9.03	-13.62	-0.28	-0.32	18.5	-24.6
	Average Temperature	15.29	7.66	0.50	-0.02	-1.08	28.7	0.0
	Maximum Humidity	76.03	7.48	0.10	-0.56	-0.45	89.6	54.3
	Minimum Humidity	11.43	6.52	0.57	0.75	0.27	35.0	1.0
	Wind Speed	2.13	0.75	0.35	0.78	0.53	4.8	0.6

C_v: coefficient of variation, C_{sx}: coefficient of skewness, C_k: the kurtosis coefficient.

Meteorological data used in this model was examined by statistical parameters in *Table 1*. As seen in this table, meteorological data of the four regions indicated similar features. When we examine the skewness and kurtosis coefficients, wind speed is normally distributed with a mean of 2.13 m/s and a standard deviation of 0.75 m/s for Aegean region. The other regions are not normally distributed in terms of skewness and kurtosis coefficients. Temperature and humidity values are generally normally distributed except minimum humidity for the Mediterranean Region, maximum humidity for the Marmara Region.

Table 2 presents the assessment statistics of MARE, MSE, and R^2 for the four regions of the original Ritchie equation. As seen in this table that original Ritchie equation estimates the ET_0 values with high MARE values (about %8) for all regions. *Figure 2* shows the scatter diagram of the ET_0 values computed by the FAO-56 PM equation versus those computed by the original Ritchie equation. The original Ritchie equation underestimates the ET_0 values between 5.0 mm/day and 12 mm/day, and overestimates the higher ET_0 values for all regions of Anatolia.

Table 2 MARE, MSE, and R^2 values computed for Coastal Regions of Anatolia of Original Ritchie equation.

Regions	MSE	MARE	R^2
The Mediterranean Region	32.742	8.221	0.880
The Black Sea Region	34.707	8.277	0.883
The Marmara Region	34.854	8.253	0.901
The Aegean Region	33.094	8.155	0.905

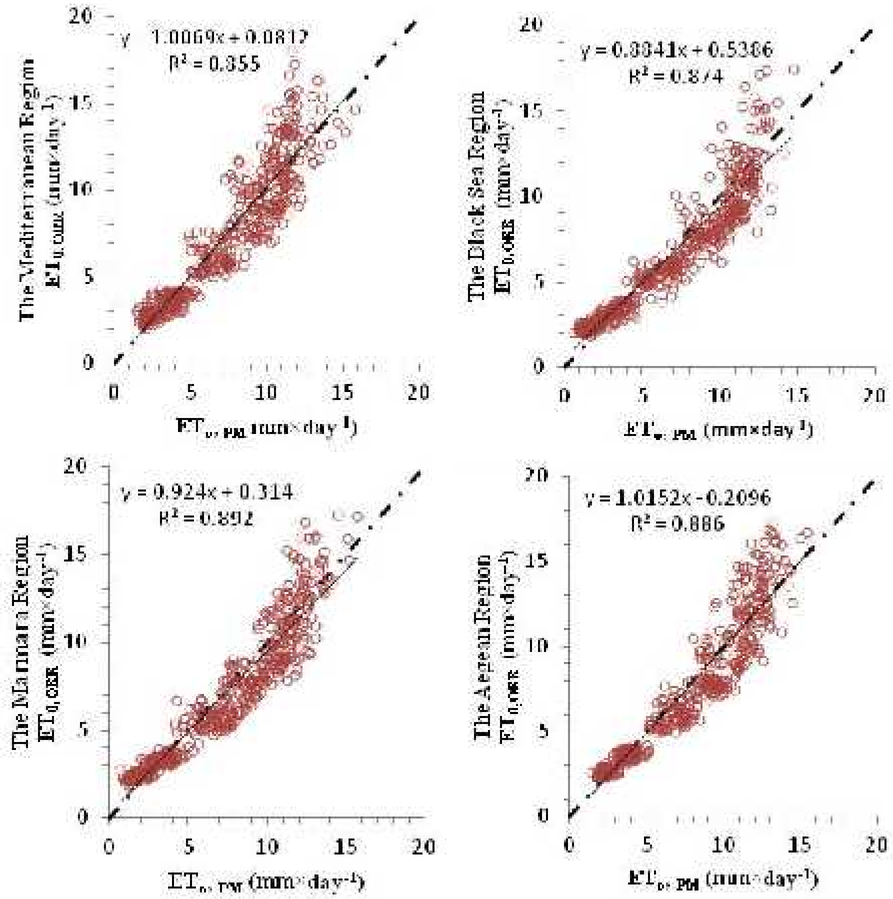

Figure 2 Scatter diagram of ET_0 values computed by the original Ritchie equation versus those computed by the FAO-56 PM equation for Coastal Regions of Anatolia.

Table 3 presents the magnitudes of the coefficients for modified Ritchie equations along with the assessment statistics of MSE, MARE, and R^2. As it can be seen from Table 3, the modified Ritchie equation are closer to the FAO-56 PM equation values. It is concluded from Tables 2 and 3, the modified Ritchie equation is superior than the original Ritchie equation for Coastal Regions of Anatolia according to the MSE, MARE and R^2 statistics. The modified Ritchie equations show an increase in R^2 values and a reduction in the MSE and MARE values compared to the original Ritchie equation for Coastal Regions of Anatolia (Tables 2 and 3).

Table 3 MARE, MSE, and R^2 values computed for Coastal Regions of Anatolia of modified Ritchie equation.

Regions	**a**	**b**	**MSE**	**MARE**	**R^2**
The Mediterranean Region	0.000283	951.09	11.063	1.254	0.895
The Black Sea Region	0.000210	1270.88	12.717	1.238	0.896
The Marmara Region	0.000201	1335.31	10.319	1.125	0.913
The Aegean Region	0.000204	1315.44	9.879	1.047	0.925

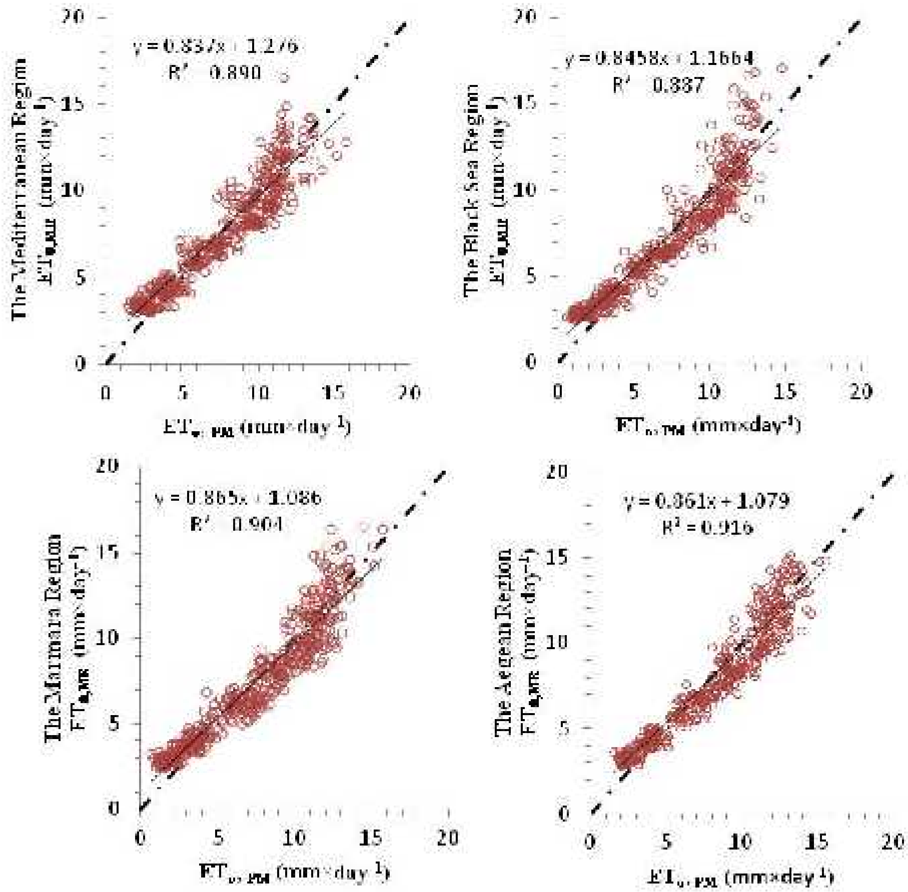

Figure 3 Scatter diagram of ET_0 values computed by the modified Ritchie equation versus those computed by the FAO-56 PM equation for Coastal Regions of Anatolia.

The results presented in Table 3 and Figure 3 show that the computed ET_0 values by the FAO-56 PM equation can be predicted accurately using the modified Ritchie equation based on the temperature data for Coastal Regions of Anatolia. In Figures 2 and 3, it can be seen from the fit line equations (assume that the equation is $y = a_o x + a_1$) that the a_o and a_1 coefficients for the modified Ritchie equation (Figure 3) are respectively closer to the 1 and 0 than those of the original Ritchie equation (Figure 2). This confirms the MSE and MARE statistics, which were evaluated in Tables 2 and 3.

For control the reliability of modified Ritchie equation, four weather stations, namely, Adana, Samsun, Edirne and Denizli, for each regions were used for the period between 2002 and 2011 years. The results of the original and modified Ritchie equations for monthly data were compared in Table 4. It can be seen from Table 4 that an increase in R^2 values and a reduction in the MSE and MARE values are obtained with modified Ritchie equation. In monthly ET_0 estimation, the modified Ritchie equation has not decreased the MSE and MARE statistics' as well as in Table 3. The reason is that monthly meteorological data ranges are not same as the long term meteorological data. This result shows that the validity of the empirical equations are only guaranteed in their valid application range, so the MSE and MARE statistics are not reduced enough for monthly ET_0 estimation with the modified Ritchie equation.

Table 4 MARE, MSE, and R^2 values computed for four Coastal Regions of the Original and modified Ritchie equation.

Regions		MSE	MARE	R^2
The Mediterranean Region	Original	1.861	25.708	0.877
(Adana 2002-2011)	Modified	1.443	22.158	0.895
The Black Sea Region	Original	9.308	51.517	0.751
(Samsun 2002-2011)	Modified	8.501	48.936	0.768
The Marmara Region	Original	3.929	40.096	0.879
(Edirne 2002-2011)	Modified	3.214	36.201	0.902
The Aegean Region	Original	2.759	29.965	0.903
(Denizli 2002-2011)	Modified	1.619	23.595	0.930

Conclusions

The FAO-56 Penman-Monteith equation is the model yielding the most accurate estimate of reference evapotranspiration, ET_0, for any geographical location where the weather data of relative humidity and wind speed are also available next to air temperature and solar radiation and it is formally the accepted and recommended model by the Food and Agriculture Organization (FAO) because of its high precision. However, in many regions of Turkey and also for other countries, recorded weather data of relative humidity and wind speed can be missing and only air temperature data are available because of its easy measurement. Yet, rational estimation of reference evapotranspiration, ET_0, and evapotranspiration specific to another type of cultivation as related to ET_0, are vitally important and gaining significance with time with ever-increasing demand for food and the necessary irrigated farming of all sorts of vegetables and fruits. Therefore, at regions where the required weather data are missing reasonable estimate of ET_0 has to be made by the Ritchie equation which needs only the air temperature as the independent variable. Hence, the two coefficients of the Ritchie equation which originally were suggested as constants valid generally by the developers of this equation, are modified in this study for Coastal Regions of Anatolia. So that the ET_0 values computed by the modified Ritchie equation be as close to the ET_0 value which would be computed by the FAO-56 PM equation in this regions. This modification is done using relevant meteorological data recorded over the period: 1975–2010 at 158 weather stations where the data needed for the FAO-56 PM equation are available. The reliability of modified Ritchie equation was checked in four weather stations with the monthly meteorological data. The study showed that ET_0 values obtained by the modified Ritchie equation were closer to the FAO-56 PM. The results indicate that the modified Ritchie equation give much better estimates of ET_0 at Coastal Regions of Anatolia than the original Ritchie equation. It is expected that suitable magnitudes of the two coefficients of the Ritchie equation can be computed by a similar analysis in the inner regions of Turkey as well. It should not be forgotten that the validity of the empirical equations are only guaranteed in their valid application range.

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References

[1] Allen, R.G., Pereira, L.S., Raes, D., and Smith, M., 1998: Crop evapotranspiration: guidelines for computing crop water requirements. FAO Irrigation and Drainage Paper 56, Rome, Italy.

[2] Unal, B., Çobaner, M., Kişi, Ö., 2010: Yapay Sinir Ağları ile Evapotranspirasyon Miktarının Tahmini, VI. Ulusal Hidroloji Kongresi, 22 - 24 Eylül 2010, Pamukkale Üniversitesi, Denizli.

[3] Ecker, J., ve Kupferschmid, M., Introduction to Operations Research, John Wiley and Sons Ltd, New York, 1991.

[4] Excel-Help, http://office.microsoft.com/tr-tr/excel-help/HP005198368.aspx

[5] Lasdon, L., J. Plummer and A. Warren, "Nonlinear Programming," in M. Avriel and B.Golany (eds.), Mathematical Programming for Industrial Engineers, Chapter 6, pp. 385-485, Marcel Dekker, New York, 1996.

[6] Jones JW, Ritchie JT (1990) Crop growth models. Management of farm irrigation system, G.J. Hoffman, T.A. Howel, and K.H. Solomon, eds., ASAE Monograph No.9, ASAE, St. Joseph, Mich. 63-89.