

## Analysis and design of a pile group using finite element method

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### Abstract

A pile group used as a bridge foundation will be calculated using finite element method. Finding vertical and horizontal coefficient of sub grade reaction, internal forces on piles and displacements.

**Keywords:** pile, finite, element, method, bridge.

### Introduction

Pile foundations are used to transfer loads from the structure in deeper and stiffer soil layers with greater load bearing capacity.

They are required when the load bearing capacity of shallow foundations is insufficient or if the size of the foundation slab would be uneconomically.

We should distinguish between bored and driven piles. Due to their great diameter and reinforcement, bored piles can carry normal forces and bending moments. Driven piles have small bending stiffness and carry vertical loads due to their slenderness. [1]

### Loads acting in pile cap

In fig.1 is given a plan and two views of the bridge foundation. Loads from the upper structure are :

$$N_1 = 7840 \text{ kN}$$

$$M_1 = 5190 \text{ kN}\cdot\text{m}$$

$$Q_1 = 10660 \text{ kN}$$

According to the geological study, we have the below parameters for the soil.

Table 1. Soil parameters

E (daN/cm <sup>2</sup> )	( $\phi$ )	c (daN/cm <sup>2</sup> )	(daN/cm <sup>3</sup> )
200	26	0,4	19.8

Loads above will be transferred at the lower centre of the pile cap . Weight of the pile cap is:

$$N_{pcap} = [(5*5)-(1.7*2.6*0.5)]*8*25 = 4558 \text{ kN}$$

So the loads are :

$$N = 12398 \text{ kN}$$

$$M = M_1 - N_1 * 0.31 + Q_1 * 3.73 = 9697.4 \text{ kN*m}$$

$$Q = 10660 \text{ kN}$$

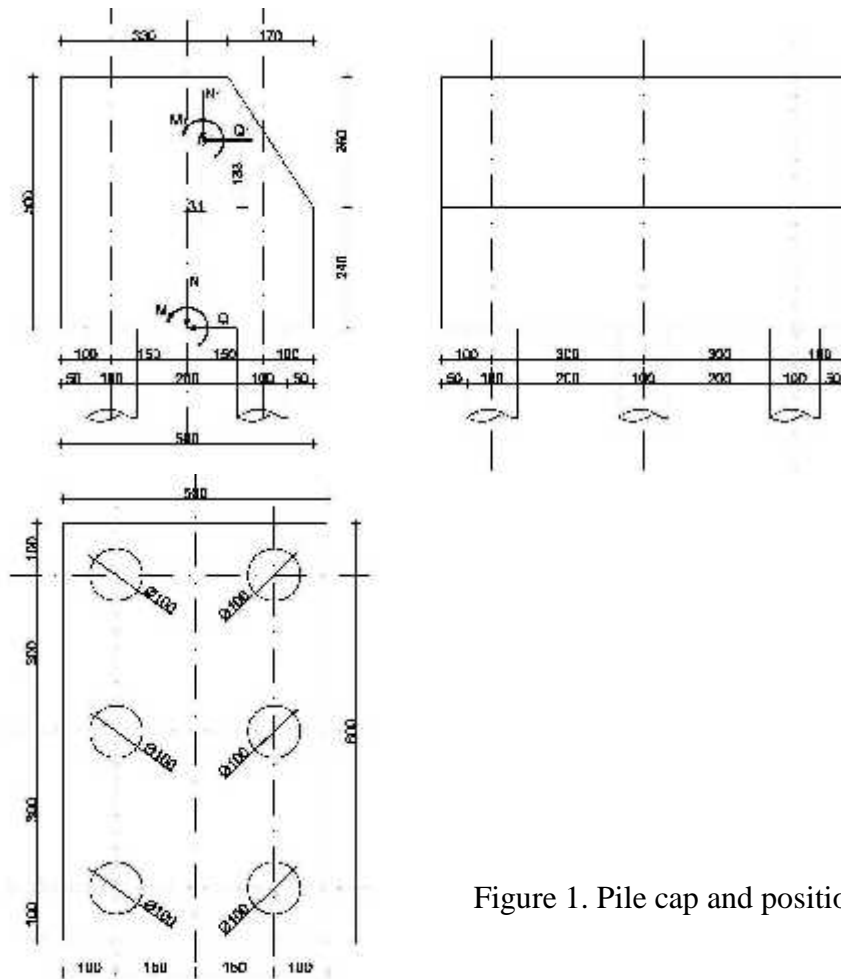


Figure 1. Pile cap and positioning of piles

### Modulus of subgrade reaction.

Modulus of subgrade reaction and the stiffness of the horizontal springs may vary along the length of the pile and its circumference [1] :

$$k_s(z) = k_s(d) * \left(\frac{z}{d}\right)^n \quad (1)$$

- n=0 for cohesive soil under small to medium loads
- n=0.5 for medium cohesive soil and non-cohesive soil above ground level
- n=1 for non-cohesive soil below the ground water level or under greater loads
- n=1.5 to 2 for loose non-cohesive soil under very high loads

In our case, for cohesive soil, n = 0

$$K_s(z) = K_s \cdot d \cdot a \quad (2)$$

If there are no results available from actual pile tests, the subgrade modulus  $k_s$  may be estimated by the following expression:

$$k_s = \frac{E_s}{d} \quad (3)$$

where :

- $k_s$  is the modulus of subgrade reaction
- $E_s$  is the stiffness modulus of the ground
- $d$  is the diameter of the pile

In the calculation, we don't take into account upper structure deformation, pile to pile interaction and friction between pile and soil.

The pile cap is for six piles. Soil is modelled as horizontal springs, which are put every 50 cm. Cross section of pile is constant.

In this study will be taken into account three different pile diameters,  $d=100, 80, 60$ cm.

For each case, horizontal springs are as below:

*For diameter 100 cm*

$$K_s = E/d = 200/100 = 2 \text{ daN/cm}^3 = 20000 \text{ kN/m}^3$$

$$K_{s1} = 20000 \cdot 1 \cdot 0.5 = 10000 \text{ kN/m}$$

$$K_{s2} = 20000 \cdot 1 \cdot 0.25 = 5000 \text{ kN/m}$$

$$K_{s3} = 20000 \cdot 1 \cdot 0.35 = 7000 \text{ kN/m}$$

$$K_{s4} = 20000 \cdot 1 \cdot 0.1 = 2000 \text{ kN/m}$$

*For diameter 80 cm*

$$K_s = E/d = 200/80 = 2.5 \text{ daN/cm}^3 = 25000 \text{ kN/m}^3$$

$$K_{s1} = 25000 \cdot 0.8 \cdot 0.5 = 10000 \text{ kN/m}$$

$$K_{s2} = 25000 \cdot 0.8 \cdot 0.25 = 5000 \text{ kN/m}$$

$$K_{s3} = 25000 \cdot 0.8 \cdot 0.35 = 7000 \text{ kN/m}$$

$$K_{s4} = 25000 \cdot 0.8 \cdot 0.1 = 2000 \text{ kN/m}$$

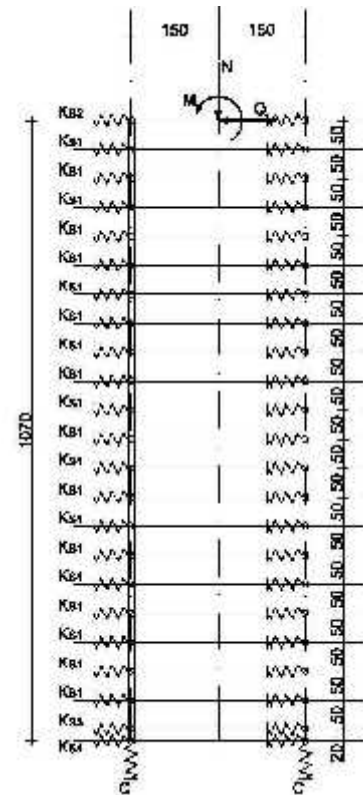


Figure 2. Springs positioning on pile

For diameter 60 cm

$$K_s = E/d = 200/60 = 3.333 \text{ daN/cm}^3 = 33333 \text{ kN/m}^3$$

$$K_{s1} = 33333 * 0.6 * 0.5 = 10000 \text{ kN/m}$$

$$K_{s2} = 33333 * 0.6 * 0.25 = 5000 \text{ kN/m}$$

$$K_{s3} = 33333 * 0.6 * 0.35 = 7000 \text{ kN/m}$$

$$K_{s4} = 33333 * 0.6 * 0.1 = 2000 \text{ kN/m}$$

### Vertical spring stiffness

The stiffness of the vertical spring at the pile toe can only be estimated by tests.

The value of the vertical spring stiffness has great influence on the bending moments of the pile. A fixed vertical support reduces the greatest bending moment by a factor of 2, compared that from an elastic support with  $C = 400 \text{ MN/m}$ . The main reason for this big difference is the rotation of the pile cap [1].

In this study, we will take three values for the vertical spring stiffness:

$$C1 = 400000 \text{ kN/m}$$

$$C2 = 1000000 \text{ kN/m}$$

$$C3 = \text{infinite (fixed vertical support)}$$

### Pile and pile cap design

Pile cap will be modelled with his real dimensions and stiffness.

In this study we will not analyze internal forces of the pile cap, neither his deformations. The pile cap is modelled as shell element, so his deformations are not neglected.

The sign of the bending moment or other internal forces on the pile are not important, because the reinforcement of the pile will be symmetric.

### Bending moments for different pile diameters and different vertical spring stiffness.

With the above values, we make calculation with a finite element program. Form analysis we take below bending moment (values below  $\times 10$  in  $\text{kN}\cdot\text{m}$ )



Fig. 3. Bending moment  
( $C_z = \text{infinite}$ ),  $d = 100 \text{ cm}$



Fig. 4. Bending moment  
( $C_z = 1000000 \text{ kN/m}$ ),  $d = 100 \text{ cm}$

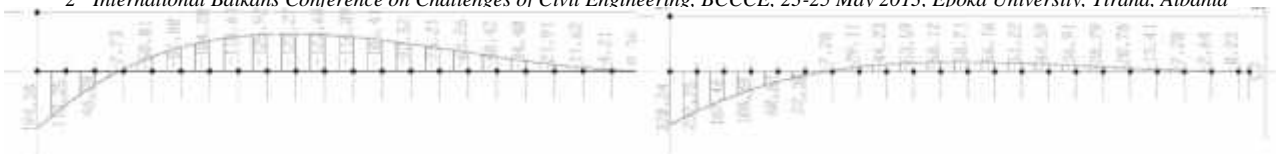


Fig. 5. Bending moment  
( $C_z = 400000 \text{ kN/m}$ ),  $d = 100 \text{ cm}$

Fig. 6. Bending moment  
( $C_z = \text{infinite}$ ),  $d = 80 \text{ cm}$

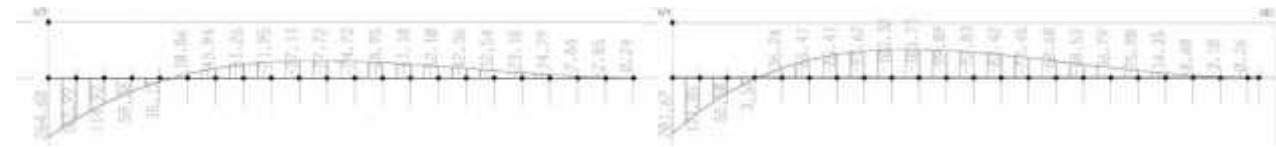


Fig. 7. Bending moment  
( $C_z = 1000000 \text{ kN/m}$ ),  $d = 80 \text{ cm}$

Fig. 8. Bending moment  
( $C_z = 400000 \text{ kN/m}$ ),  $d = 80 \text{ cm}$

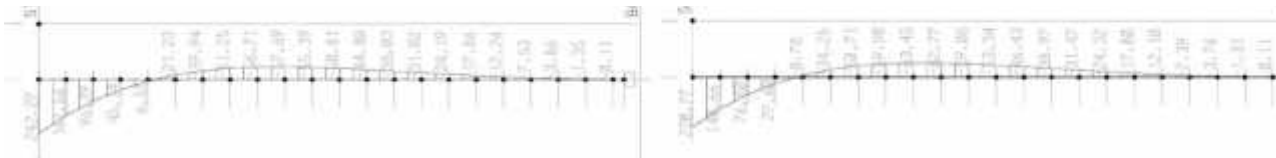


Fig. 9. Momenti perkules  
( $C_z = \text{infinite}$ ),  $d = 60 \text{ cm}$

Fig. 10. Bending moment  
( $C_z = 1000000 \text{ kN/m}$ ),  $d = 60 \text{ cm}$



Fig. 11. Bending moment  
( $C_z = 400000 \text{ kN/m}$ ),  $d = 60 \text{ cm}$

From the above calculations, we see that maximum bending moment is on top of the pile. This values are presented in the table below.

Table 2. Relation of  $M_{\text{max}}$  in pile head with pile diameter

Diameter (cm)	$M_{\text{max}}$ at pile head( $\text{kN}\cdot\text{m}$ )		
	$C_z = \text{infinite}$	$C_z = 106$	$C_z = 4 \cdot 10^5$
100	4112.3	2995.3	1952.0
80	3206.6	2650.2	2020.9
60	2424.6	2209.5	1926.7

With above values we can make the graph below:

Figure 12. Relation between Diameter and  $M_{\text{max}}$  at pile head

In the table and graph below are given bending moments in pile body

Table 3. Relation of Mmax in pile body with pile diameter.

Diameter (cm)	Mmax (kN*m) in pile body		
	Cz = 4*105	Cz = 106	Cz = infinit
100	1272.7	766.2	364.7
Level from pile head (m)	4.5	5	6.5
80	1037.1	777.2	587.1
Level from pile head (m)	4.5	5	5.5
60	723.2	634.5	576.9
Level from pile head (m)	4	4	4.5

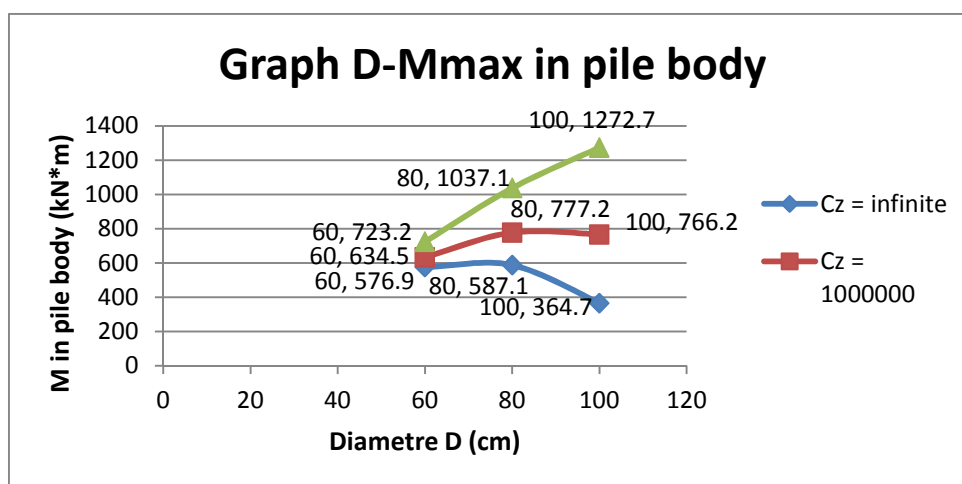


Figure 13. Relation between Diameter and Mmax at pile body

### Displacement calculation and how displacement change with pile diameter.

Maximum pile displacement is at pile head. From the graph and table below, is expressed clearly that the pile with the bigger diameter, has lower head displacements.

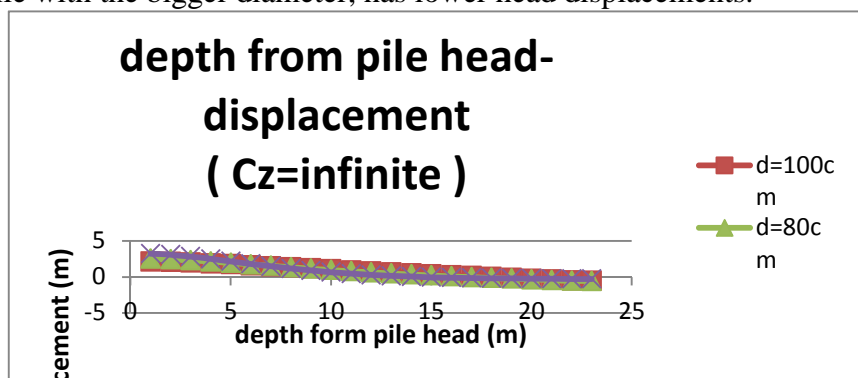


Figure 14. Relation between depth from pile head and displacement

Table 4. Displacement in pile axis

Point	Displacement (cm)		
	D100	D80	D60
1	-2.13	-2.53	-3.24
2	-2.07	-2.44	-3.09
3	-1.98	-2.31	-2.83
4	-1.88	-2.14	-2.51
5	-1.76	-1.95	-2.16
6	-1.62	-1.75	-1.81
7	-1.49	-1.55	-1.48
8	-1.34	-1.35	-1.17
9	-1.2	-1.15	-0.9
10	-1.05	-0.97	-0.66
11	-0.91	-0.79	-0.45
12	-0.77	-0.63	-0.29
13	-0.63	-0.48	-0.15
14	-0.5	-0.34	-0.05
15	-0.37	-0.21	0.04
16	-0.24	-0.09	0.1
17	-0.12	0.02	0.14
18	0.01	0.13	0.18
19	0.13	0.23	0.21
20	0.25	0.33	0.23
21	0.37	0.43	0.25
22	0.49	0.53	0.27
23	0.35	0.54	0.28

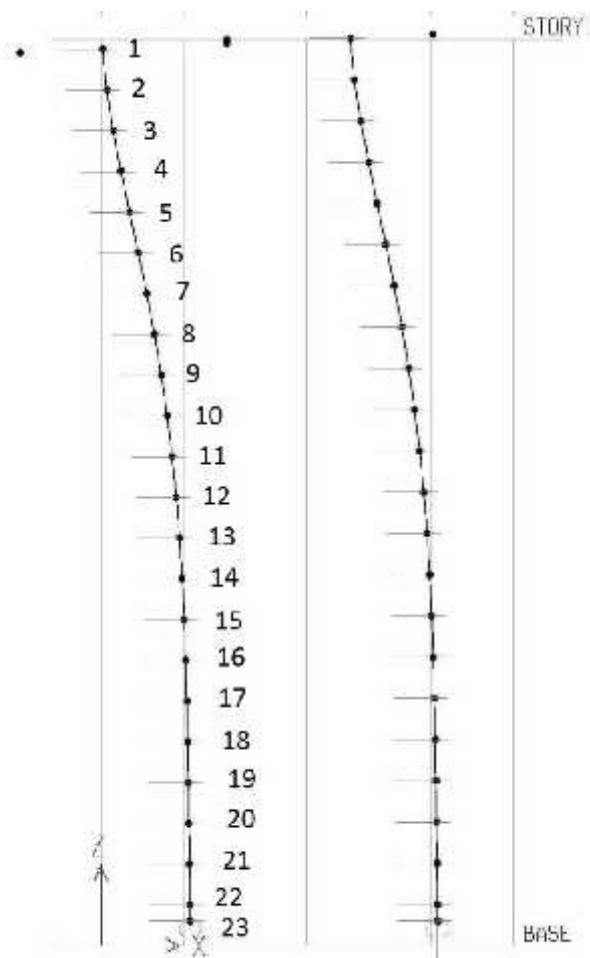


Fig. 15. Pile deformation scheme

## Conclusions

Analysing bending moments at pile head, we can give some important conclusions:

1. Increasing the vertical spring stiffness, can lead to increasing bending moment at pile head, with maximum value for infinite vertical spring stiffness.
2. Bending moment at pile head depends on pile diameter. Increasing pile diameter can increase bending moment at pile head.
3. The greater the value of vertical spring stiffness, the greater is the increase of the bending moment with the increase of the diameter. For  $C_z = \text{infinite}$ , the increase of the bending moment is almost linear with the increase of the diameter.

Analysing bending moments at pile body, we can give this conclusions:

1. Increasing  $C_z$  leads to lower values of bending moments in the pile body.
2. Increasing  $C_z$ , maximum bending moment in pile body is reached for higher depths from pile head.

Comparing bending moment at pile head and at pile body, we can conclude that:

1. Maksimum bending moment at pile body is always lower than maksimum bending moment at pile head.
2. For the same diameter, if we increase  $C_z$ ,  $M_{max}$  at pile head, divided with  $M_{max}$  at pile body increases. So for lower  $C_z$ , we have a better moment distribution in the pile, so a better use of the pile.

## **References**

- [1] Finite element design on concrete structures (G. A. Rombach)