Implementation of Topology Optimization on Building Design-Study Case

Ludian Komini[1], Blerta Dyrmishi[2], Niko Pojani[2]

[1]Department of Civil Engineering, University of Maribor, Slovenia

[2] Department of Technical Appraisal, Construction Institute, Albania

[2] Department of Mechanics of Structures, Polytechnic University of Tirana, Albania

ABSTRACT

Structural Optimization, in general, can be interpreted as "the point" where the required structural performance meets the minimal usage of resources and other constraints. The limited material resources, which have concerned researchers and professionals over the last decades, should not affect the quality and the performance a structure must reach. Topology Optimization is a Finite Element based Method, which basically consists in distributing the material in a given volume or in a design domain, so this will lead to designing the structural element that is excepted to satisfying the boundary conditions, achieves the best performance under given solicitations in respect to some design criterion. This paper will briefly present the concept of one of the most successful Optimization Methods ESO/BESO (Evolutionary Structural Optimization/Bi-directional Evolutionary Structural Optimization), its provision of the optimum shape of the structures, reaching the target volume under certain conditions, by adding and taking off elements in the 3D Finite Element Model used as the design domain, illustrated with a short practical example.

Keywords: *Topology Optimization, ESO/BESO Method, Structural Performance.*

INTRODUCTION

The aim of the most researchers in the field of structural optimization is to produce a structural optimization method that can become a useful tool for engineers and designers. Topology optimization of continuum structures is the most general type of structural optimization, being performed in the initial phases of the design. So far it has proved that it is the most challenging technically and at the same time the most rewarding economically.

The philosophy of topology optimization for the structure is to produce, or finding the most appropriate design, with the optimum geometry according to the objective function, considering one or more such criteria. Basically the entire feasible domain is considered, the aim being to find the most advantageous material distribution inside this domain, with respect to the design objectives, respectively the mean compliance (Strain Energy).

The topology optimization, instead of limiting the changes to the sizes of structural components, provides much more freedom and allows the designer to create highly efficient conceptual designs for continuum structures and boosts the opportunity in arriving at a more realistic structure. After the results came out from the topology optimization procedure, structural engineers have to modify the structure to make it buildable or easily manufactured if it is a prepared structure, among other things.

One of the most efficient methods in structural optimization is Bi-directional Evolution Structure Optimization (BESO). This method was developed in 1993 by Xie and Steven (1993), and the implemented computer code known as EVOLVE (Querin el al. 1996)[8]. The definitive form of this method was developed by Huang and Xie (2007) [3] which addresses many issues related to topology optimization of continuum structures such as a proper statement of the optimization problem, checkerboard pattern, mesh-dependency and convergence of solution.

BESO is a numerical method that is integrated to finite element analysis (FEM) where the design domain is discretized into a fine mesh of elements. In such a setting, the optimization procedure is to find the topology of a structure by determining for every point in the design domain if there should be material (solid element) or not (void element), based on sensitive numbers. (Cazacu, and Grama, 2010) [6] It obtains the desired optimal design from an oversized structure by removing the elements of the domain design with low sensitive numbers and vice versa. Each time elements are added and removed, the structure is then re-analyzed to obtain the new load paths. (Huang X and Xie YM, 2007)[5]

For BESO method it is obligatory that the initial physical structure has to contain the minimum number of elements necessary to support all the load cases and support cases. It obtains the desired optimal design from the structure by removing the elements of the domain design with low sensitive numbers and vice versa. Each time elements are added and removed, the structure is then re-analyzed to obtain the new load paths. This is repeated until the result is a fully stressed design - all the members support the same maximum stress. (T. Nguyen el al 2013) [7]

BESO *HARD-KILL* procedures

The maximum stiffness of a structure with volume of material constraint is pursued for our case of structural optimizations. One method to achieve that is by minimizing the mean compliance or Strain Energy of the structure that is defined with the area below the load deflection curve and is equal to the external work in quasi-static condition (Huang and Xie 2010) [3]. Thus, the optimization problem is stated as:

$$\text{Minimize} \quad C = \frac{1}{2}\{F\}^T\{U\} \tag{1}$$

$$\text{Subject to} \quad V^* - \sum_{i=1}^{N} V_i x_i = 0 \tag{2}$$

$$x_i = 1 \;\; or \;\; 0 \tag{3}$$

Where $\{F\}$ and $\{U\}$ are the applied load and displacement vectors and C is known as the mean Compliance. V_i is the volume of an individual element and V^* the prescribed total structural volume. N is the total number of elements in the system. x_i is the design variable of element i. x_i =0 means the absence of an element or (0) or x_i =0 presence of an element based on the sensitive number of the element (Querin el al. 1996)[8].
When the i-th element is removed from a structure, the overall stiffness of the structure reduces and, consequently, the total strain energy increases. Beside this, the change of the mean compliance or total Strain energy is equal to the elemental strain energy. This change is defined as the elemental sensitivity number:

$$\alpha_i^e = \Delta C_i = \frac{1}{2}\{U\}_i^T[K]_i\{U\}_i \tag{4}$$

U_i is the nodal displacement vector of the i-th element; K_i is the elemental stiffness matrix. When a non-uniform mesh is assigned the sensitivity number (or strain energy density) of the i-th finite element can be defined by its strain energy divided by its volume:

$$\alpha_i^e = \alpha_i = \left(\frac{1}{2}\{u\}_i^T[K]_i\{u\}_i\right)/V_i \tag{5}$$

Approaches to overcome checkerboard, mesh independence and numerical instabilities

To avoid the typical checkerboard during the optimization process, we implement a filter scheme proposed by Bendsøe and Sigmund (2000).[1]

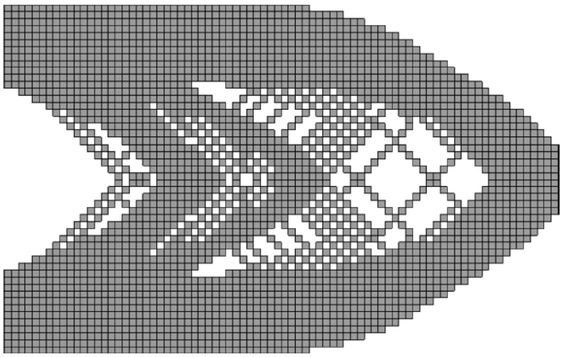

Figure 1 *Typical Checkerboard (source Bendsøe and Sigmund, 2000)*

$$\alpha_j^n = \sum_{i=1}^{M} w_i \alpha_i^e \tag{6}$$

M denotes the total number of elements connected to the j-th node. w_i is the weight factor of the i-th element and $\sum_{i=1}^{M} w_i = 1$.

$$w_i = \frac{1}{M-1}\left(1 - \frac{r_{ij}}{\sum_{i=1}^{M} r_{ij}}\right) \tag{7}$$

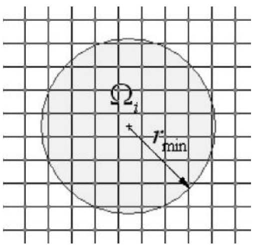

Figure 2 Nodes located inside the circular sub-domain Ω_i are used in the filter scheme for the i-th element (source Bendsøe and Sigmund, 2000).

where r_{ij} is the distance between the center of the i-th element and the j-th node and the value of r_{min} should be big enough so that Ω_i covers more than one element. Nodes located inside Ω_i contribute to the computation of the improved sensitivity number of the i-th element as:

$$\alpha_i = \frac{\sum_{j=1}^{K} \omega\left(r_{ij}\right)\alpha_j^n}{\sum_{j=1}^{K} \omega\left(r_{ij}\right)} \tag{8}$$

where "*K*" is the total number of nodes in the sub-domain Ω_i , $w(r_{ij})$ is the linear weight factor defined as

$$\omega\left(r_{ij}\right) = r_{\min} - r_{ij} \quad (j = 1, 2, ..., K) \tag{9}$$

If a chaotic behavior of the mean compliance occurs due to discrete value of the variables x_i, 0 or 1, the optimization process will not be stable. However, averaging the sensitivity number with its historical information is an effective way to solve this problem and to make the objective function convergence easier. (Huang and Xie, 2010).[3]

$$\alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2} \tag{10}$$

where *k* is the current iteration number.

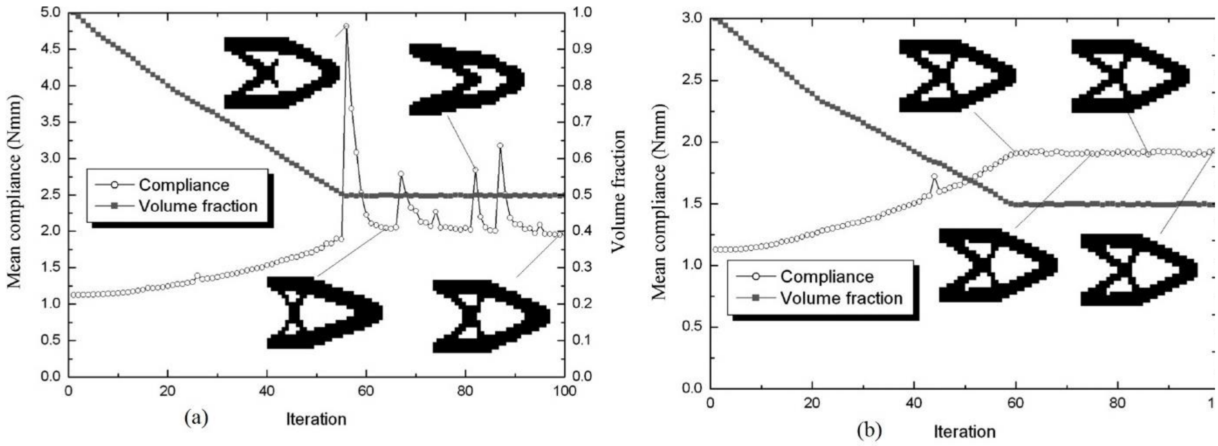

Figure 3 (a) Unstable optimization process; (b) Stable optimization process using the average sensitive numbers (source Huang and Xie, 2010).

Then, the sensitivity numbers of all elements, both solid and void, are calculated as described in the previous sections. The elements are sorted according to the values of their sensitivity numbers (from the highest to the lowest). (J. Banchak el al, 2013)[9] For solid element (1), it will be removed or switched to 0 if

$$\alpha_i \le \alpha_{th} \tag{11}$$

For void elements (0), it will be added a solid element or (switched to 1) if

$$\alpha_i > \alpha_{th} \tag{12}$$

where α_{th} is the threshold of the sensitivity number determined by the target material volume in each iteration V_{k+1}. For example, there are 1000 elements in the design domain and

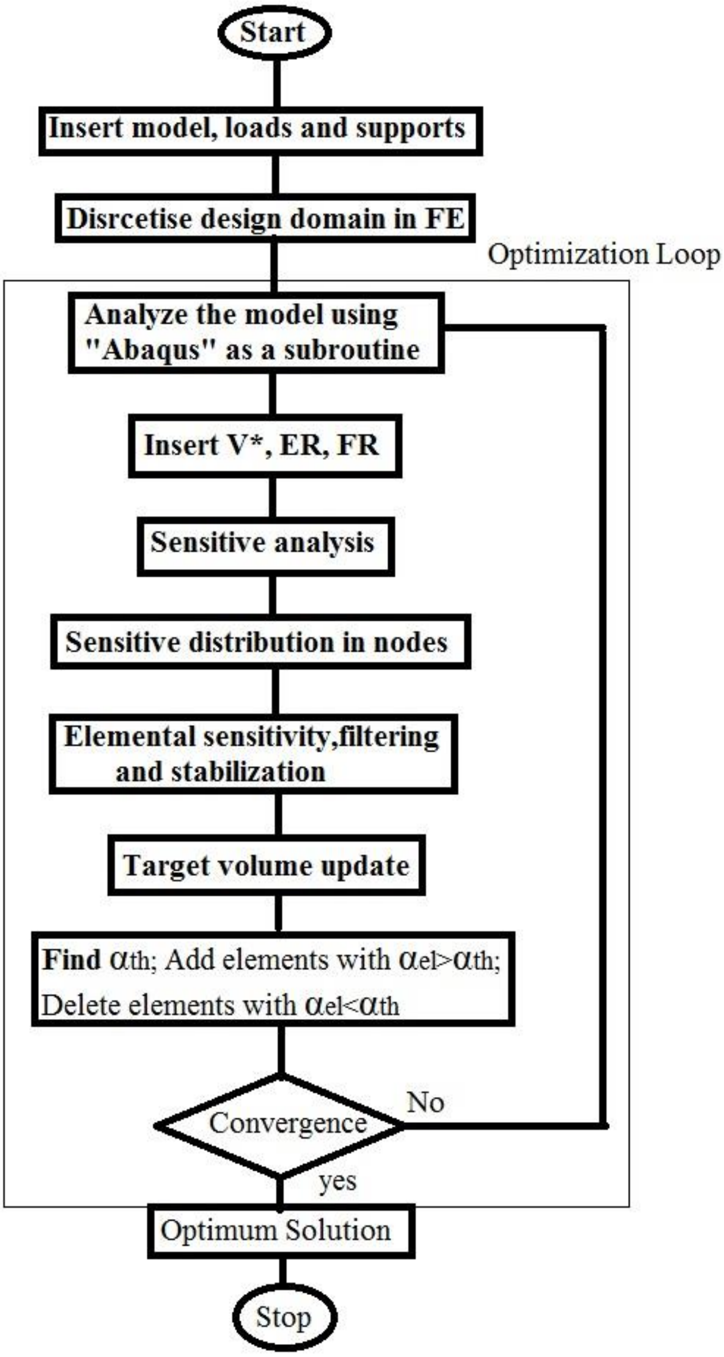

Figure 4 Flowchart for BESO Method

$\alpha_1<\alpha_2<\ldots\ \alpha_{1000}$ and if V_{k+1} corresponds to a design with 600 solid elements then $\alpha_{th}=\alpha_{600}$ (X. F. Sun el al, 2011). [4]

The cycle of Finite Element Analysis and element removal and addition will be repeated until the objective material volume V^* is reached

$$V_{k+1} = V_k(1 \pm ER) \quad (k = 1, 2, 3, \ldots) \tag{13}$$

where *ER* is the evolutionary volume ratio. Once the volume constraint is satisfied, the volume of the structure will be kept constant for the remaining iterations and the termination criterion defined in the later section is satisfied. (Huang and Xie, 2010) [3]

$$V_{k+1} = V^* \tag{14}$$

So the performance index of two successive designs is used to define the termination criterion:

$$error = \frac{\left| \sum_{i=1}^{N} C_{k-i+1} - \sum_{i=1}^{N} C_{k-N-i+1} \right|}{\sum_{i=1}^{N} C_{k-i+1}} \leq \tau \tag{15}$$

where k is the current iteration number, τ is an allowable convergence tolerance assumed 0.01% and N is an integer number normally, N is selected to be 5. (Huang X and Xie YM, 2007)[5]

Example

We will implement this method on optimization of the bridge with L=100m W=15m.

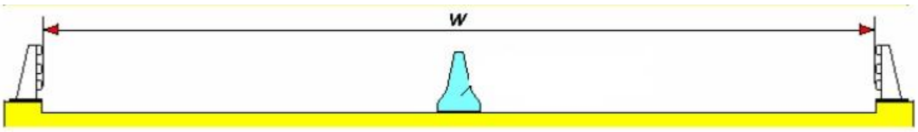

Figure 5 Secion of the bridge

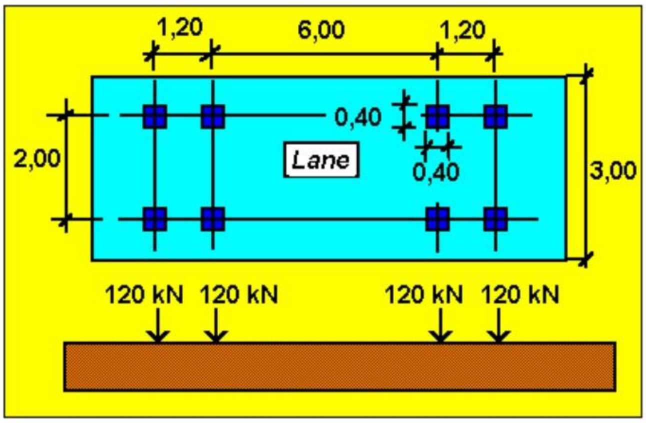

Figure 6 Load scheme of the bridge

Material used is steel S355 E = 210 GPa; v = 0.3; ρ = 7850 kg/m^3.

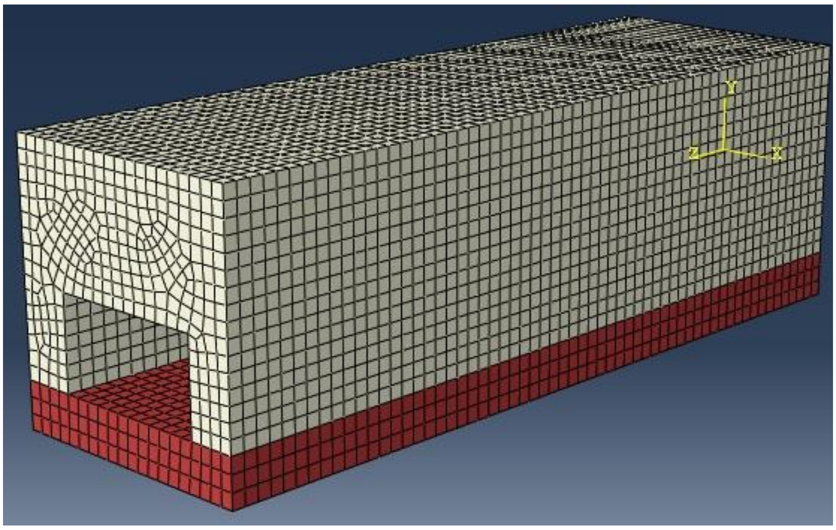

Figure 7 Model

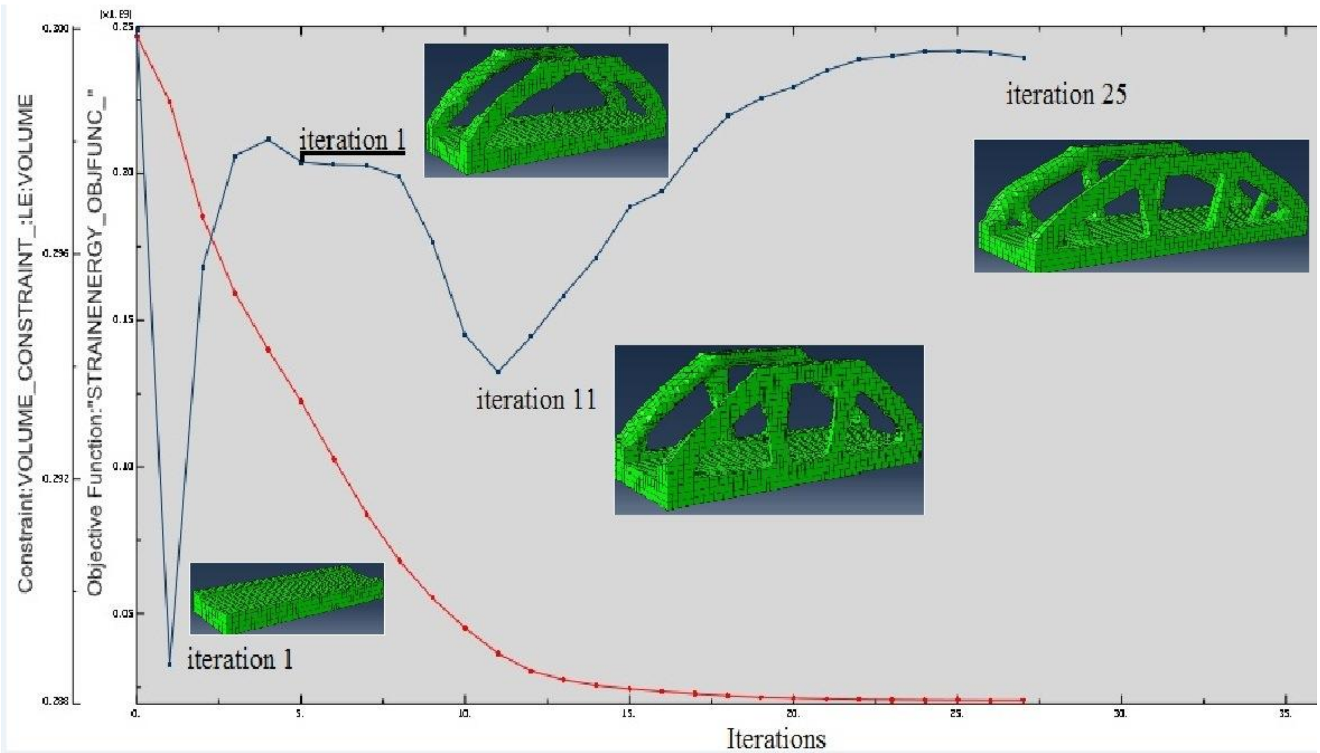

Figure 8 Iterations of optimization

After 25 iterations the structure will satisfy all the conditions required.

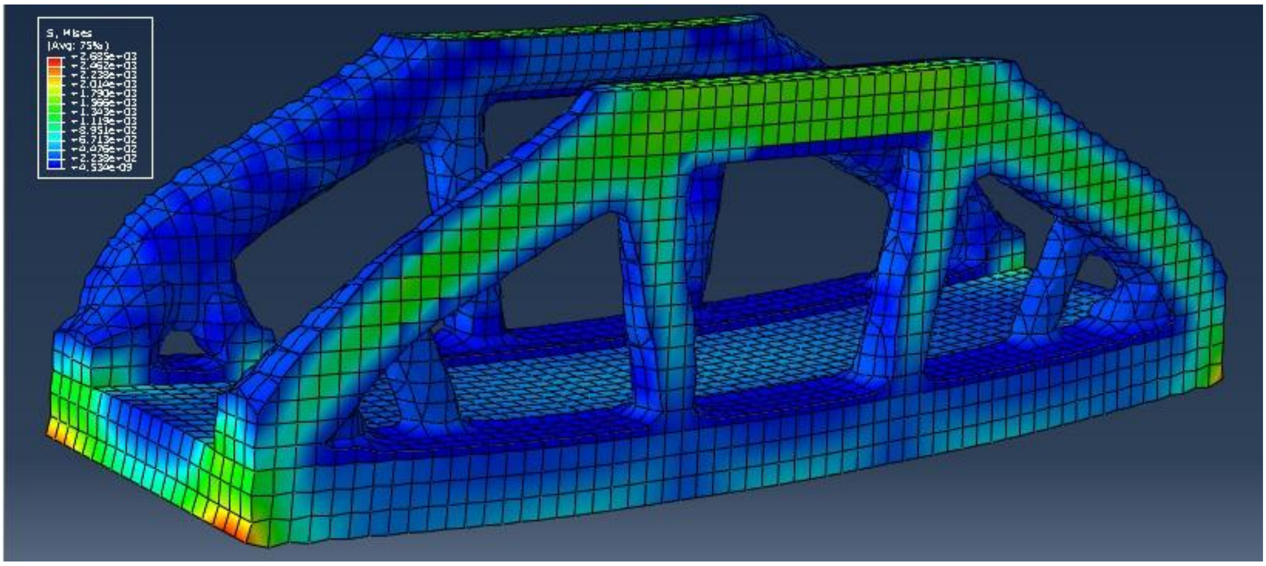

Figure 9 *Final step of optimization proccess*

CONCLUSIONS

This paper presents BESO method for stiffness optimization based on compliance minimization with a constraint on the volume of the optimal solution. The current 'hard-kill' BESO method removes an element (as opposed to changing it into a very soft material) if the certain variable is 0 and 1 for vice versa. The variables that are based on sensitive numbers 0 or 1 are discrete.
The sensitivity numbers of elements base on the elemental strain energy density. The sensitivity numbers used for material removal and addition are modified by a mesh-independency filter scheme which smooths the sensitivity numbers throughout the design domain that avoids checkerboard problems.
The procedure requires just one additional constraint to converge and convergence is usually reached with a small number of iterations (10–40 iterations depending on the optimization problem).
The main advantage of the hard-kill approach is that the computational time can be significantly reduced, especially for large 3D structures, because the eliminated elements are not involved in the finite element analysis.
The proposed method is effective in reducing the chance of structural failure due to high stress concentration by providing optimal design of the structures and producing shape design with uniform stress distribution.
Moreover, one of the aims of BESO methods is to give a useful idea of the structure for the project idea and is a helpful tool for Architectures to design an organic building.

Acknowledgement
This work was fully supported by CAESS (Center of Advance Engineering Software and Simulations). We would like to thank the reviewers for their helpful comments

REFERENCES

[1] P.Bendsøe O. Sigmund 2010 Topology Optimization Theory, Methods and Applications *ISBN 3-540-42992-i Springer-Verlag Berlin Heidelberg New York*

[2] Martin P. Bendsøe 2010 Optimization of Structural Topology, Shape, and Material *Springer-Verlag Berlin Heidelberg GmbH ISBN 978-3-662-03117-9*

[3] X. Huang and Y.M. Xie 2010 Evolutionary Topology Optimization of Continuum Structures: Methods and Applications *John Wiley & Sons Ltd. ISBN: 978-0-470-74653-0*

[4] X. F. Sun, J. Yang, Y. M. Xie, X. Huang, Z. H. Zuo Procedia Engineering 14 (2011) Topology Optimization of Composite Structure Using Bi-Directional Evolutionary Structural Optimization Method *The Twelfth East Asia-Pacific Conference on Structural Engineering and Construction*

[5] Huang X and Xie YM (2007*).* Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method. Finite Elem Anal Des, *doi: 10.1016/j.finel.2007.06.006*

[6] Razvan Cazacu, Lucian Grama (December 2010) Overview of structural topology optimization method for plane and solid structures. *Annals of the University of Oradea, Fascicle of Management and Technological Engineering*

[7] T. Nguyen, K. Ghabraie, T. Tran-Cong (2013) Applying bi-directional evolutionary structural optimization method for tunnel reinforcement design considering nonlinear material behavior *Computational Engineering and Science Research Centre (CESRC), Faculty of Engineering and Surveying (FoES), University of Southern Queensland (USQ), Toowoomba, QLD 4350, Australia*

[8] V. Young, O.M. Querin, G. P Steven, Y.M.Xie 3D and multiple load case bi-directional evolutionary structural optimization (BESO) Structural Optimization 18, 183-192 Springer-Verlag 1999

[9] Ronald J. Banchak, Markus Stephan and Michael Böhm, (2012) Topology Optimization – Driven Design Development for Automotive Components, *Automotive Simulation World Congress*