Optimization of Particle Size Distribution of Concrete Aggregates

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ABSTRACT

This paper investigates the optimization of concrete aggregate particle size distribution under a dense arrangement of a group of sphere particles with maximum diameter of 25 m. The proposed sphere model reflects the particles that constitute 'aggregate' in concrete. In scope of this, the volume proportions on some regular close-packed arrays of spheres, such as face center cubic (FCC) lattice packing will be discussed. Such space filling between this regular crystalline system is taken into consideration as it suggests a regular arrangement and the possibility to attain a dense mixture of different aggregate sizes is considerable. The main objective is to compare the generated particle size distribution and its corresponding density with those proposed by European standards, referring EN 933-2 and EN 1097-3. The concept of gradation, compacted density, packing and Fuller distribution are also discussed.

Keywords: *aggregate, FCC packing, gradation, compacted unit weight, Fuller Distribution*

INTRODUCTION

Aggregates form the body of the concrete and occupy approximately three-quarters of total volume of it. This high volume percentage has a considerable influence on major concrete properties and make aggregates a major contributor to the compressive strength of concrete.

To create dense mixtures, aggregates are used in two major different sizes, fine and course aggregates. According to BS EN 12620: 2008 [1], aggregates passing through a 4 mm sieve are called fine aggregate and those retained on such sieve are called coarse aggregates. Coarse aggregates consists of gravels or crushed stones with particle size more than 4 mm up to 63 mm. The coarse aggregate form the main matrix of concrete and the fine aggregate from the filler matrix between the coarse aggregate [2].

Aggregates help to produce an engineering material in two important ways: reduce the cost as far as being the cheapest constituent and provide rigidity to the material necessary for engineering use [3]. For that reason, the role that aggregates play needs to be emphasized.

One of the most important parameters of aggregates affecting the properties of concrete is the particle size distribution and their corresponding packing density. The particle size distribution in an aggregate sample is determined by a sieve analysis with reference to EN 933-2 [4] and is known as gradation. Good aggregate gradation corresponds to high packing density, workable and uniform concrete, without any segregation of the particles [5].

Obtaining models for dense packing using spherical particles is an old problem but still interesting. The first efforts to provide the ‘best’ optimal particle size distribution were based on trials with spheres of different diameters. These experiments resulted in optimal distribution curves which are currently recognized as standards todays [6].

At the same time, limit gradation curves with upper and lower boundaries are placed by standards. Based on these restrictions, a suitable aggregate sample is one that falls between these two curves.

It is well-known that aggregates may be selected and combined for the maximum packing density (fraction of the occupied volume) using ideal particle size distributions or mathematical packing models [10]. Classics studies in this area are in particular those established by Fuller and Thompson [6], [10]. The idea behind this ideal particle distribution is that the maximum aggregate size in a mixture influences the resulting gradation significantly.

This ideal particle size distribution is a series of parabolic curves used for optimization of asphalt and concrete aggregates. It is expressed as:

$$P_i = 100\left(\frac{d_i}{D_{max}}\right)^n,$$

where P_i is the percent of aggregates finer than size d_i, D_{max} is the maximum aggregate size and n is the Fuller exponential in a range of 0.33-0.5. The value of the exponent for optimizing packing density was found to vary with the packing density of the individual size fractions and the degree of compaction [10].

Using ideal particle size distribution to obtain optimized gradation is significant and results in more well-graded mixtures and enhanced concrete properties like workability, strength, durability, water requirements and cement requirements.

METHODOLOGY

3.1 Approach

Space filling between regular crystalline systems was taken into consideration as they suggest regular arrangements and the possibility to attain a dense mixture of different aggregate sizes is considerable. The packing density of the combined sphered- agfgregates aggregates must be at maximum. Aggregates must be distributed throughout such that they must be dimensionally arranged in order to fit well enough into the spaces between particles. As a result, a more particle-to particle contact is created and the void space is reduced. Therefore we start to discuss the volume fractions of some regular close-packed arrays of spheres, such as face center cubic (FCC) lattice packing.

3.2 Development of the model

The approach discussed on the section 3.1 refers to the 3-D arrangements of sphere-aggregate particles in a FCC lattice unit. For such lattice configuration the packing factor is approximately 0.74, implying that the lattice suggests a hard dense optimization and with interest to be studied.

The idea is to establish new aggregate proportions (compatible with the standardized particle distribution) having some aggregate size limitation. For practical convenience, the maximum size of the sphere-aggregate diameter is taken as D = 25 mm, and the minimum one as d = 0.20 mm. That is, how to fill the FCC lattice void volumes with different sizes of “aggregate spheres” under the specified restrictions for the diameter sizes.

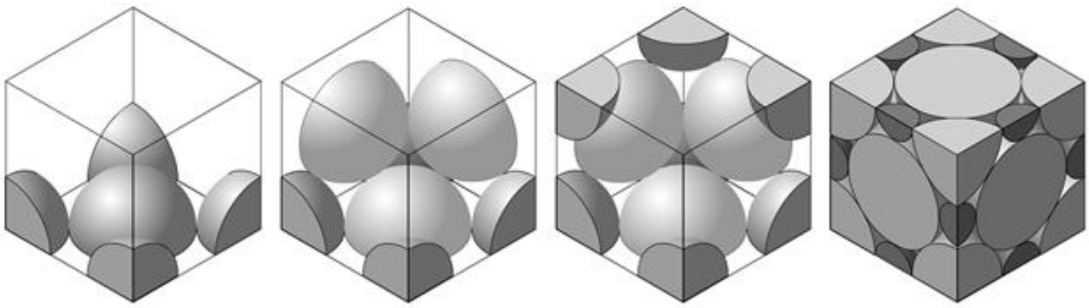

Figure 1-Face Centered Cubic structure when the interstitial spaces are packed with smaller spheres on a unit volume

For an optimized reduced void content the approach is treated as a mathematical packing problem. In other words, the objective is to attain such configuration of aggregate particles that is based on geometrical relationship of the FCC particle arrangement.

Filling the gaps between large particles (D = 25 mm) with the smaller ones turns out to be a three-dimensional cubic-close packing problem, if we take advantage from the fact that in such lattice arrangement the particles touch each other at least once (tangential spheres). This means that it is possible to define the diameters of spheres that fit exactly on this void space.

In order to do so, a 2D representative unit of aggregate arrangement in a FCC pattern is considered for practical purposes. This 'unit', simplifies the problem as far as it is repetitive in the FCC lattice. Like so, the problem becomes the filling of the empty space between three tangential spheres sections of D/2, D/2 and (0.414D)/2 radius respectively, such that the 'filling' spheres will be closely arranged in a plane of an equilateral right triangle of (D+0.414D)/2 adjacent that passes through their centers.

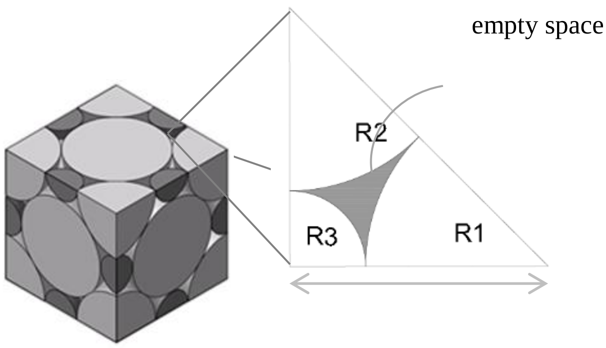

Figure 2– Top view of the repetitive unit

R3 is the sphere placed on the center of the lattice, which fills the space between four tangential spheres on the lattice. A top view of such configuration defines the radius of this sphere as equal to $\sqrt{2}-1$ for a unit sphere.

Hence, R3 = [D ($\sqrt{2}-1$)]/2.

Back to the 'unit' plane, inner Soddy circles are placed within each triple of mutually tangential circles and continue to be placed. The repetition ends when the restrictions as being specified are satisfied. The radius of the fourth circle is found given the radius of the other three, following the relationship:

$$R_i = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3 \pm \sqrt{R_1 R_2 R_3 (R_1 + R_2 + R_3)}} \quad [1]$$

By obtaining the diameters of first closed arranged sphere-aggregates, the corresponding volume proportions in their densest state can be obtained under the 25 mm maximum size.

In order to define the volume proportions under the 25 mm maximum size a Fuller Distribution was followed with a selected n = 0.45, as it is proved that for such grading type factor the highest density for sphere particles (as the developed ones) is reached as stated in section 2.4.

Volume percentages of sphere aggregates were then defined for the maximum packing density condition. Using Fuller Distribution with a maximum diameter D = 25 mm the volumes were calculated for each of the sphere particles obtained geometrically and generated by the close-packed arrangement in FCC lattice. The distribution equation for this model became:

$$P_i = 100 \left(\frac{d}{25}\right)^{0.45} \quad [2]$$

Based on equation 2, the cumulative volume proportions of the model were calculated for each size arranged to the ideal cubic-close-packed FCC lattice according to equation 3:

$$p_{i+1} = P_{i+1} - P_i \quad [3]$$

3.3 Experimental study

The experimental part of this study was conducted at the Epoka University Civil Engineering Research Laboratory. Two types of aggregates were investigated in this study, which have characteristics as provided in table 1 .

Table 1–Tested aggregates characteristics

Aggregate type	**Specific gravity**	**Water absorption (%)**
SAMPLE T	2.85	35
SAMPLE V	2.55	39

The SAMPLE T was a natural crushed stone (4-31.5 mm in size) and SAMPLE V was natural river stone (4-31.5 mm in size). River sand (0.25-4 mm in size) was also used as fine

Testing comprised of three parts: grading, determination of dry rodded unit weight for each grading with reference to EN 1097-3 (8) and sieve analysis for the graded mixtures according to the suggested particle size distribution referring EN 933-2 (4).

Aggregates samples were previously washed and soaked for 24 h, then drained and oven dried for 24 h at 100 ± 5°C.

Four different grading G1, G2, G3 and G4 were investigated for an amount of 5500 gr of sample for each of two aggregates type. The three first grading G1, G2, G3 corresponds to the standardized particle size distributions following EN 933-2 representing lower limit grading curve distribution, upper limit grading curve distribution and intermediate grading curve distribution respectively, for a maximum aggregate size of 31.5 mm. G4 represented the designed grading for a maximum aggregate size of 25 mm.

For the first three grading G1, G2, G3 the particle size distribution was done using standard sieves with mass proportions for each sieve group as suggested by EN 933-2 and calculated for 5500 gr amount of each aggregate mixture sample (table 2).

Table 2 – Mass proportions for each standardized limit grading

Size group (mm)	Mass proportions (g)			
	LOWER LIMIT	**INTERMEDIATE**	**UPPER LIMIT**	
<0.25-0.25	110	467.5	825	**FINE**
0.25-0.5	220	495	770	
0.5-1	110	412.5	715	
1-2	330	467.5	605	
2-4	495	577.5	660	
4-8	825	742.5	660	**Coarse**
8-16	1320	990	660	
16-31.5	2090	1357.5	605	
TOTAL	5500	5500	5500	

After the mass proportions for each sieve group were obtained by sieving the sample and weighting the retained mass on the each sieve group, a graded mixture resulted. The graded mixture was uniformly mixed and the dry rodded unit weight was found for each grading mixture of two aggregates samples using a cubic container following the description in section 2.3.

For the designed grading G4 for the obtaining of mass proportions as this grading suggested an intermediate standardized sieve of 11.2 mm was added to the basic set. For convenience the masses were weighted in standard sieve size groups that were closest to the diameter size of the particles as being determined on the model (table 3).

Table 3– Mass proportions for the designed grading

Estimated particle size (mm)	**Closest size group (mm)**	**Mass proportions (g)**
		DESIGNED GRADING
0.23	<0.25	661
0.23, 0.33	0.25-0.5	320
0.54	0.5-1	132
0.72	0.5-1	196
1.03	1-2	118
1.25	1-4	595
2.7	4-11.2	1677
10.35, 25	11.2-31.5	1802
TOTAL		5500

The resulted mixture was then mixed uniformly throughout and the dry rodded unit weight was obtained using a cubic container following the description in section 2.3. Then, the mixtures for the two types of aggregates were sieved with reference to EN 933-2 and their gradation curve were checked and compared with the standardized gradation limit curves.

Figure 3- Compaction for rodded unit weight determination; sieve shaking for sieve analysis (starting from the left)

RESULTS AND DISCUSSION

4.1 Measured results

The results obtained for diameter sizes for inner spheres close packed on FCC pattern are shown in table 4.

Table 4 - The obtained diameter sizes performing Soddy Circle Theorem

Calculated Radius (mm)		Diameters (mm)
R1	12,500	25,000
R2	12,500	25,000
R3	5,175	10,350
R4	1,352	2,704
R5	0,623	1,246
R6	0,514	1,028
R7	0,358	0,716
R8	0,270	0,540
R9	0,113	0,226

Performing Soddy Circle Theorem, closely packed spheres of different diameters were obtained per ‘unit’ plane. R1, R2, R3 were the initial spheres of packing.

The volume proportions for the maximum packing density of the generated particles closely arranged on FCC structure coming from Fuller Distribution with a grading factor n = 0.45 and Dmax = 25 mm are shown in table 5.

Table 5 – Volume percentages of sphere aggregates

Particle size d (mm)	Volume proportion of particles under D_{max} (%)
25.00	100
10.35	67.24
2.70	36.76
1.25	25.93
1.03	23.79
0.72	20.22
0.54	17.82
0.33	14.33
0.23	12.01

The cumulative contents for each sphere-aggregate particle were calculated according equation [3] and are shown in table 6.

Table 6 - Particle size distribution of the proposed model

Particle size d (mm)	Cumulative contents (%)
25.00	-
10.35	32.76
2.70	30.49
1.25	10.83
1.03	2.15
0.72	3.57
0.54	2.40
0.33	3.48
0.23	2.32
<0.23	12.01

The particle size distribution generated under this designed gradation is plotted above with horizontal axes representing closest sieve sizes on a logarithmic plot.

4.2 The effect of aggregate gradation on the dry rodded unit weight of aggregates samples

The dry rodded unit weights of two types of aggregates were determined experimentally and are presented in table 7. Under same compacted condition it is evident the effect of collection of various sizes in reducing the void spaces in the resulting aggregate mixtures. For the graded mixtures of SAMPLE V the dry rodded unit weight resulted to be 1.81 gr/cm3 for grading G1, 1.95 gr/cm3 for grading G2, 1.98 gr/cm3 for grading G3 and 1.96 gr/cm3 for grading G4. For the graded mixtures of SAMPLE T the dry rodded unit weight were resulted to be 1.76 gr/cm3 for grading G1, 2.05 gr/cm3 for grading G2, 1.97 gr/cm3 for grading G3 and 1.98 gr/cm3 for grading G4.

Table 7 - Measured results for dry rodded unit weight of the graded mixtures of the two types of samples

SAMPLE V MIXTURE GRADING	RODDED UNIT WEIGHT (gr/cm3)	SAMPLE T MIXTURE GRADING	RODDED UNIT WEIGHT (gr/cm^3)
G1	1,81	G1	1,76
G2	1,95	G2	2.05
G3	1,98	G3	1,97
G4	1,96	G4	1,90

Grading G1 designates less void spaces being filled by fine particles as far as it has a mass proportion of coarse particles higher than that of fine particles. This is reflected even in the measurements: for both samples grading G1 gave the less rodded density.

Grading G2 designates higher void space being filled because has a mass proportion of fine particles higher than that of coarse particles and is more dense for both aggregate samples in comparison with graded mixtures obtained for lower limits as suggested by standard.

Intermediate grading G3 for both samples has a wide particle size distribution. The generated dense mixtures as it is noticed have relatively void space decrease in comparison with G1 and G2. Even though the grading is idealized considering the upper and lower limit, proportioning of aggregates samples under adjustment of this grading are acceptable.

It is noticed that the designed grading G4 resulted in mixtures for both aggregates samples with rodded unit weight closer to intermediate grading G4 and higher to them of grading G1 and G2. That matches with the fact that this grading was designed for the maximum possible density of particles arranged on a cubic-closed-pack lattice like FCC and such kind of result is a good indicator of the validation of the proposed grading.

In summary, measurements of rodded unit weight under same compaction efforts had an impact on the decreasing of the void space between graded particles regardless of their particle size distribution. Also, as it may be noticed that the void contents vary inversely with the rodded unit weight.

4.3 Sieve analysis of the designed grading mixtures

Table 8 and 9 represent the sieve analysis data for the designed grading mixtures for both samples.

Table 8 - Sieve analysis for the G4 grading mixture of SAMPLE V

Sieve analysis of designed grading mixture for SAMPLE V				
Sieve size (mm)	**Retained (gr)**	**Retained (%)**	**Cumulative retained (%)**	**Passing (%)**
31.5	0	0	0	100
16	989	17,91	17,91	82,07
8	1007	18,24	36,16	63,85
4	1137	20,59	56,75	43,26
2	381	6,90	63,65	36,36
1	635	11,50	75,15	24,86
0,5	364	6,59	81,74	18,26
0,25	537	9,73	91,47	8,53
<0,25	471	8,53	100	0
	5521			

Table 9 - Sieve analysis for the G4 grading mixture of SAMPLE T

Sieve analysis of designed grading mixture for SAMPLE T				
Sieve size (mm)	**Retained (gr)**	**Retained (%)**	**Cumulative retained (%)**	**Passing (%)**
31.5	0	0	0	100
16	1165,5	21,06	21,06	78,94
8	870	15,72	36,79	63,21
4	1291	23,33	60,12	39,88
2	468	8,46	68,57	31,44
1	431	7,79	76,36	23,64
0,5	284	5,13	81,50	18,51
0,25	468	8,46	89,95	10,05
<0,25	556	10,05	100	0
	5533.5			

Also, the corresponding gradations curves were plotted and compared with the standardized one.

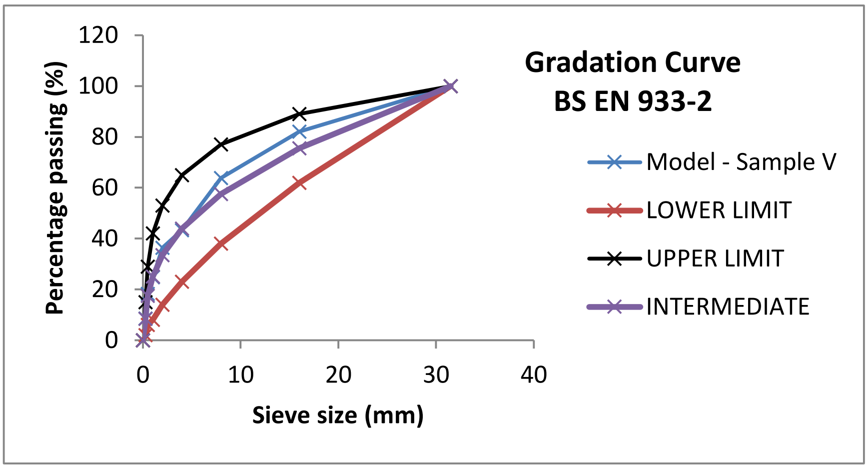

Figure 4 – Gradation Curve Sample V

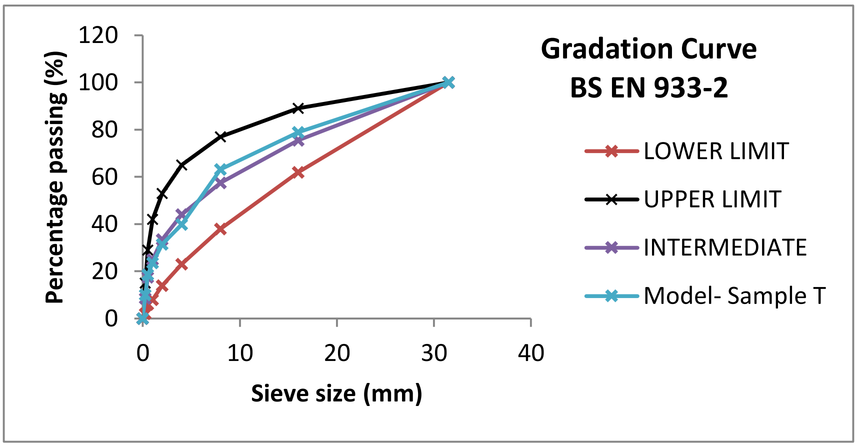

Figure 10 – Gradation Curve Sample T

For the both graphical representations the grading curves fit between two grading limits. This surprising result indicates that the aggregate gradation adjusted under this proposal can be employed for concrete mix design because a well particle size distribution is obtained. This further validates the use of the proposed model for the grading of aggregates.

One of the purposes of the developed model was to reproduce a distribution curve that match those used in concrete technology and the developed one gave acceptable prediction.

CONCLUSION

The designed grading proved to be good on providing a dense aggregate mixture and an acceptable particle size distribution. Even though there is no ideal particle size distribution, the designed one was under what standard restricted. In these restricted bases, the proposed grading is a good target and further results may be obtained if testing and checking this gradation for the impact on concrete mix designs.

An interesting point is that, the designed gradation proved to be essential even considering the particle shape and its impact on rodded unit weight. It is known that rounded particles pack more closely than angular ones. This result in fewer voids between particles and the obtained rodded densities proved this.

For the sample V which consisted on river pebbles the rodded unit weight was defined to be as 1.96 gr/cm3 and for the sample T which consisted on natural crushed stone the roddded unit weight was define to be as 1.90 gr/cm3. Void content varies inversely with rodded unit weight, meaning that the river pebbles of sample V had a void content less than the crushed stone of sample T, so they are more closed packed.

Improvements on the model may be done, because the suggested particle distribution is characterized by a narrow range of fine particles which usually are not commonly preferred in concrete technology.

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