How Do the Richest 1% Owns 50% of Wealth in a Small-Open Growth Model with Endogenous Wealth and Human Capital

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Abstract

This paper extends the growth model for a closed national economy by Zhang (2015) to a small-open economy. We attempt to explain some economic mechanisms of how the richest one per cent of the population own 50% of national wealth. We consider endogenous wealth and human capital accumulation by heterogeneous households with different preferences and learning abilities as the main determinants of growth and inequality. We describe the production technologies and economic structure on the basis of the Uzawa two-sector model. By applying Zhang's concept of disposable income and approach to household behavior, we describe consumers' wealth accumulation and consumption behavior. We model human capital accumulation on the basis of Arrow's learning by doing and Zhang's creativity with leisure. We simulate the model with three groups of the population, the rich 1 %, the middle 69%, and the poor 20%. We demonstrate the existence of an equilibrium point at which the rich 1% own more than half of the national wealth and the poor 20% less than 10% of the national wealth. We show how the system moves to the equilibrium from an initial state and confirm that the equilibrium point is stable. We also conduct comparative dynamic analysis.

Keywords: inequality and growth; small-open economy; learning by consuming; wealth and income distribution; heterogeneous households

Introduction

It is nowadays often mentioned that the richest 1% of the world population is owning almost half of the world's wealth. In many countries the richest 1% of a national population owns a great share of the national wealth. Forbes (2000) argues for analyzing determinants of growth and distributions as follows: "careful reassessment of the relationship between these two variables (growth rate and income inequality) needs further theoretical and empirical work evaluating the channels through which inequality, growth, and any other variables are related." Through systematically reviewing the literature of economic growth and inequality of income and wealth, Zhang (2006) concludes that modern theoretical economics has failed in providing a proper analytical framework for dealing with relations between growth and inequality. The purpose of this study is to re-address issues related to growth and inequality by following Zhang's analytical framework of economic growth with heterogeneous households. We are especially concerned with issues related to growth and inequalities in a small-open economy. We introduce some endogenous growth mechanisms to show how the richest 1% of the population own more than almost half of wealth.

This study is based on Zhang's integrated Walrasian general equilibrium and neoclassical growth theory (Zhang, 2006, 2015). We base our model on the Walrasian general equilibrium theory in describing production, consumption, and exchange equilibrium with given physical capital (Walras, 1874, Arrow and Debreu, 1954; Gale, 1955; Nikaido, 1956, 1968; Debreu, 1959; McKenzie, 1959; Arrow and Hahn, 1971; Arrow, 1974; and Mas-Colell *et al.*, 1995). The theory solves equilibrium of pure economic exchanges with heterogeneous supplies and households. The theory fails to properly include endogenous wealth (and other factors such as environment, resources, human capital and knowledge). The Walrasian general equilibrium theory is not proper for addressing issues related to growth and inequality. The neoclassical growth theory enables us to properly describe capital accumulation, even though the neoclassical growth theory lacks proper mechanisms to deal with issues related to income and wealth distribution. Zhang developed his theory by integrating the two approaches with his alternative way to describe household behavior. This study considers both physical wealth and human capital as the determinants of economic growth and inequality. Wealth accumulation is due to propensities to save and human capital accumulation is due to abilities and preferences for learning. Our model also treats human capital accumulation as an endogenous process of economic growth. In economic theory there are only a few theoretical models which study inequality and growth with both endogenous wealth and human capital accumulation. Our approach to human capital accumulation is influenced by Arrow's learning by doing and Zhang's learning through consuming (leisure creativity).

This study deals with the issue similar to Zhang (2015). The main difference between this study and Zhang's model is that this study is concerned with a small open economy, while Zhang's model is for a closed national economy. There is a large number of the literature on economic growth of open economies (e.g., Obstfeld and Rogoff, 1996; Lane, 2001; Kollmann, 2001, 2002; Benigno and Benigno, 2003; Galí and Monacelli, 2005; Uya, *et al*. 2013; and Ilzetzki, *et al*. 2013). We follow this tradition to show how income and wealth distributions change in a small-open economy. We organize the rest of the paper as follows. Section 2 defines the small-open growth model of heterogeneous households with endogenous capital accumulation and human capital accumulation. Section 3 shows that the dynamics of the economy with J types of households can be described by $2J$-dimensional differential equations. As mathematical analysis of the system is too complicated, we demonstrate some of the dynamic properties by simulation when the economy consists of three types of households. Section 4 carries out comparative dynamic analysis with regard to some parameters.

The basic model

The economy consists of one capital good and one consumer good sectors. Most aspects of the production sectors are similar to the standard two-sector growth model by Uzawa (Uzawa, 1965; Burmeister and Dobell 1970; Azariadis, 1993; and Barro and Sala-i-Martin, 1995). Households own assets of the economy and distribute their incomes to consume and to save. Firms use labor and physical capital inputs to supply goods and services. Exchanges take place in perfectly competitive markets. Factor markets work well and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. The population is classified into J groups. Each group has a fixed population, $\overline{N}_j$, ($j = 1, ..., J$). Let prices be measured in terms of the commodity and the price of the commodity be unit. Let $p(t)$ denote the price of consumer good at time t. We denote wage and interest rates by $w_j(t)$ and r^*, respectively. The rate of interest is fixed in international markets. We use $H_j(t)$ to stand for group j's level of human capital. It should be noted that although we call it human capital, the variable $H_j(t)$ may consist of not only human capital such as skills and knowledge but also intangible assets such as social status, reputation, and social relations.

We use subscript index i and s to respectively stand for capital good and consumer good. We use $N_m(t)$ and $K_m(t)$ to stand for the labor force and capital stocks employed by sector m. Let $T_j(t)$ stand for the work time of a typical worker in group j. The variable $N(t)$ represents the total qualified labor force. A worker's labor force is $T_j(t)H_j^{m_j}(t)$, where m_j is a parameter measuring utilization efficiency of human capital by group j. The labor input is the work time by the effective human capital. As the total qualified labor force is the sum of all the groups' labor forces, we have $N(t)$ as follows

$$N(t) = \sum_{j=1}^{J} T_j(t) H_j^{m_j}(t) \overline{N}_j, \qquad j = 1, ..., J. \tag{1}$$

Full employment of labor and capital

The total labor force is employed by the two sectors. The condition of full employment of labor force implies

$$N_i(t) + N_s(t) = N(t). \tag{2}$$

The total capital stock $K(t)$ is allocated between the two sectors. As full employment of capital is assumed, we have

$$K_i(t) + K_s(t) = K(t). \tag{3}$$

Let $\bar{k}_j(t)$ denote per capita wealth of group j at t. Group j's wealth is $\bar{k}_j(t)\bar{N}_j$. As wealth is held by the households, we have

$$K(t) = \sum_{j=1}^{J} \bar{k}_j(t)\bar{N}_j . \tag{4}$$

The capital good sector

Let $F_m(t)$ stand for the production function of sector m, $m = i, s$. The production function of the capital good sector is specified as follows

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \alpha_i, \beta_i > 0, \ \alpha_i + \beta_i = 1, \tag{5}$$

where A_i, α_i, and β_i are positive parameters. The capital good sector employs two input factors, capital and labor force. We assume that all the markets are perfectly competitive. The marginal conditions for the capital good sector are

$$\delta^* = \frac{\alpha_i F_i(t)}{K_i(t)}, \ w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \tag{6}$$

where $\delta^* \equiv r^* + \delta_k$.

The consumer goods sector

The production function of the consumer good sector is specified as follows

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \ \alpha_s + \beta_s = 1, \ \alpha_s, \beta_s > 0, \tag{7}$$

where A_s, α_s, and β_s are technological parameters. The marginal conditions are

$$\delta^* = \frac{\alpha_s p(t) F_s(t)}{K_s(t)}, \ w(t) = \frac{\beta_s p(t) F_s(t)}{N_s(t)}. \tag{8}$$

Consumer behaviors and wealth dynamics

Consumers make decisions on choice of leisure time, consumption levels of services and commodities as well as on how much to save. We note that the wage rate of group j is

$$w_j(t) = w(t)H_j^{m_j}(t), \quad j = 1, \cdots, J. \tag{9}$$

Per capita current income from the interest payment $r^* \bar{k}_j(t)$ and the wage payment $T_j(t)w_j(t)$ is

$$y_j(t) = r^* \bar{k}_j(t) + T_j(t)w_j(t),$$

where $y_j(t)$ is the current income. The total value of wealth that consumers can use is $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t) = \tilde{r}\,\bar{k}_j(t) + W_j(t). \tag{10}$$

where $W_j(t) \equiv T_j(t)w_j(t)$ is the wage income and $\tilde{r} \equiv 1 + r^*$. The disposable income is used for saving, consumption, and education. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in (10) is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute between saving and consumption. The typical consumer distributes the total available budget between saving $s_j(t)$, consumption of consumer good $c_j(t)$. The budget constraint is

$$p(t)c_j(t) + s_j(t) = \hat{y}_j(t) = \tilde{r}\,\bar{k}_j(t) + w_j(t)T_j(t), \tag{11}$$

The time constraint for everyone

$$T_j(t) + \bar{T}_j(t) = T_0, \tag{12}$$

where $\bar{T}_j(t)$ is the leisure time of the representative household and T_0 is the total available time. Substituting (12) into (11) yields

$$w_j(t)\bar{T}_j(t) + p(t)c_j(t) + s_j(t) = \bar{y}_j(t) \equiv \tilde{r}\,\bar{k}_j(t) + T_0\, w_j(t). \tag{13}$$

The variable $\bar{y}_j(t)$ is the disposable income when the household spends all the available time on work. We assume that the consumer's utility function is dependent on $\bar{T}_j(t)$, $c_j(t)$, and $s_j(t)$ as follows

$$U(t) = \bar{T}_j^{\sigma_{j0}}(t)c^{\xi_{j0}}(t)s^{\lambda_{j0}}(t), \ \ \sigma_{j0}, \xi_{j0}, \lambda_{j0} > 0, \tag{14}$$

where σ_{j0} is the propensity to use leisure time, ξ_{j0} is the propensity to consume, and λ_{j0} the propensity to own wealth. This utility function proposed by Zhang (1993) is applied to different economic problems. Maximizing $U_j(t)$ subject to (13) yields

$$\bar{T}_j(t) = \frac{\sigma_j\,\bar{y}_j(t)}{w_j(t)}, \ \ c_j(t) = \frac{\xi_j\,\bar{y}_j(t)}{p(t)}, \ \ s_j(t) = \lambda_j\,\bar{y}_j(t), \tag{15}$$

where

$$\sigma_{j0} \equiv \rho_j\,\sigma_{j0}, \ \ \xi_{j0} \equiv \rho_j\,\xi_{j0}, \ \ \lambda_{j0} \equiv \rho_j\,\lambda_{j0}, \ \ \rho_j \equiv \frac{1}{\sigma_{j0} + \xi_{j0} + \lambda_{j0}}.$$

Change in the household wealth

According to the definitions of $s_j(t)$, the wealth accumulation of the representative household in group j is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{16}$$

This equation simply states that the change in wealth is equal to saving minus dissaving.

Dynamics of human capital

In economic theory there are three sources of improving human capital, through education (Uzawa, 1965), "learning by producing" (Arrow, 1962), and "learning by leisure" (Zhang, 2007). We propose that the human capital accumulation is described as follows

$$\dot{H}_j(t) = \frac{\tilde{\upsilon}_j\, c_j^{a_j}(t)\bar{k}_j^{\upsilon_j}(t)T_j^{\theta_j}(t)}{H_j^{\pi_j}(t)} - \delta_{jh}\, H_j(t), \tag{17}$$

where δ_{hj} is the depreciation rates of human capital, $0 < \delta_{hj} < 1$. In (17), $\tilde{\upsilon}_j$, a_j, υ_j, and θ_j are non-negative parameters, and π_j is a parameter. In our approach different groups may have different depreciation rates of human capital. We now interpret the items in $\tilde{\upsilon}_j c_j^{a_j} \bar{k}_j^{\upsilon_j} T_j^{\theta_j} / H_j^{\pi_j}$. The item $c_j^{a_j}$ which implies a positive relation between human capital accumulation and consumption is influenced by Uzawa's learning through education and Zhang's learning through consumption. As education is classified as the consumption of services, a higher level of consumption may imply a higher investment in education. On the other hand, a higher consumption also implies that the household may accumulate more through other consumption activities. The item $\bar{k}_j^{\upsilon_j}$ which implies a positive relation between wealth and human capital accumulation can be interpreted that more wealth means, for instance, a higher social status. More wealth may also help one to maintain professional reputation. The specification of $T_j^{\theta_j}$ is influenced by Arrow's learning by doing. More work accumulates more human capital. The term $H_j^{\pi_j}$ implies that more human capital makes it easier (more difficult) to accumulate knowledge in the case of $\pi_j < 0$ ($\pi_j > 0$).

Demand of and supply for consumer good

The output of the consumer good sector is consumed only by the households. The demand for consumer good from a group is $c_j(t)\bar{N}_j$. The condition that the total demand is equal to the total supply implies

$$\sum_{j=1}^{J} c_j(t)\bar{N}_j = F_s(t). \tag{18}$$

We completed the model. The model is structurally general in the sense that some well-known models in theoretical economics can be considered as its special cases. For instance, if we fix wealth and human capital and allow the number of types of households equal the population, then the model is a Walrasian general equilibrium model. If the population is homogeneous, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). It is structurally similar to the multi-class models by Pasinetti and Samuelson (e.g., Samuelson, 1959; Pasinetti, 1960, 1974). We now examine dynamics of the model.

The dynamics and its properties

The system consists of any number of types of households and each type of households save. As households have different propensities to save and different attitudes and abilities to accumulate human capital, the dimension of economic system should be twice as the number of types of households. The following lemma shows that the economic dynamics is represented by $2J$ dimensional differential equations.

Lemma

The dynamics of the economy is governed by the following $2J$ dimensional differential equations system with $(\bar{k}_j(t))$ and $(H_j(t))$, where $(\bar{k}_j(t)) \equiv (\bar{k}_1(t), \cdots, \bar{k}_J(t))$ and $(H_j(t)) \equiv (H_1(t), \cdots, H_J(t))$, as the variables

$$\dot{\bar{k}}_j(t) = \Lambda_j((H_j(t)), (\bar{k}_j(t))), \quad j = 1, ..., J,$$

$$\dot{H}_j(t) = \Omega_j((H_j(t)), (\bar{k}_j(t))), \quad j = 1, ..., J,$$

in which Λ_j and Ω_j are unique functions of $(\bar{k}_j(t))$ and $(H_j(t))$ at any point in time, defined in the appendix. For given $(\bar{k}_j(t))$ and $(H_j(t))$, the other variables are uniquely determined at any point in time by the following procedure: w by (A2) → $w_j(t)$ by (A4) → p by (A5) → $N_i(t)$ by (A12) → $N(t)$ by (A11) → $N_s(t)$ by (A8) → $\bar{y}_j(t)$ by (A6) → $K_i(t)$ and $K_s(t)$ by (A1) → $F_i(t)$ and $F_s(t)$ by the definitions → $\bar{T}_j(t)$, $c_j(t)$, and $s_j(t)$ by (15) → $K(t)$ by (4).

We can follow the procedure given in the lemma to follow the motion of the dynamic economic system. This implies that we can simulate the dynamic equations with any number of types of households. As this is a highly dimensional nonlinear dynamic system, it is difficult to get analytical properties of the system. For illustration, we simulate the model by specifying the parameters as follows:

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 69 \\ 20 \end{pmatrix}, \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.15 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.18 \\ 0.2 \end{pmatrix}, \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.94 \\ 0.65 \\ 0.6 \end{pmatrix}, \begin{pmatrix} \sigma_{10} \\ \sigma_{20} \\ \sigma_{30} \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.2 \\ 0.2 \end{pmatrix},$$

$$\begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.4 \\ 0.1 \end{pmatrix}, \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.1 \end{pmatrix}, \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.1 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.1 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.4 \end{pmatrix},$$

$$r^* = 0.024, \; A_i = 1, \; A_s = 0.9, \; \alpha_i = 0.32, \; \alpha_s = 0.34, \; T_0 = 24, \; \delta_{h1} = 0.04, \; \delta_{h2} = 0.06, \; \delta_{h3} = 0.08, \; \delta_k = 0.05. \quad (20)$$

Group $1, 2$ and $3's$ populations are respectively $1, 69$ and $20.$ Group 2 has the largest population. The capital good sector and consumer good sector's total productivities are respectively 1 and $0.9.$ Group $1, 2$ and $3's$ utilization efficiency parameters, m_j, are respectively $0.7,\ 0.15$ and $0.1.$ Group 1 utilizes human capital mostly effectively; group 2 next and group 3 lest effectively. We call groups $1, 2$ and 3 respectively the rich, the middle, and the poor. The population is specified in such a way that the rich is only one percent of the population. We specify the values of the parameters, α_j, in the Cobb-Douglas productions approximately equal to $0.3.$ The rich's learning by doing parameter, $\bar{v}_1$, is the highest. The returns to scale parameters, π_j, are all positive, which implies that human capital accumulation exhibits decreasing returns to scale in human capital. The depreciation rates of human capital are specified in such a way that the rich has lowest rate. The rich's propensity to save is $0.6.$ and the poor's propensity to save is $0.6.$ It is assumed that the rich is most effective in learning through consuming and working. The value of the middle's propensity is between the rich and the poor. In Figure 1, we plot the motion of the system with the following initial conditions

$$\bar{k}_{16}(0) = 8900,\ \bar{k}_2(0) = 90,\ \bar{k}_3(0) = 52,\ H_1(0) = 600,\ H_2(0) = 16,\ H_3(0) = 2. \quad (21)$$

In Figure 1, the national output Y, the share of each group's wealth in the national wealth θ_{jw}, and the ratio between group 1's and another group's wealth φ_j, are respectively defined as

$$Y(t) = F_i(t) + p(t)F_s(t),\ \ \theta_{jw}(t) \equiv \frac{\bar{K}_j(t)}{K(t)},\ \ \varphi_j(t) \equiv \frac{\bar{k}_1(t)}{\bar{k}_j(t)},\ \ j = 2, 3.$$

The wage rate and price of services which are independent of are respectively $w = 1.36$ and $p = 1.06.$ The national output and wealth experience slight changes during the study period. The rich's and the middle's human capital fall and the poor's human capital rises slightly. The ratio between the rich household and the middle household falls. The ratio between the rich household and the poor household falls. The rich own more than half of the national wealth with 1 percent of the population and the poor own about 7 per cent of the national wealth with the 20 percent of the national population. The rich household owns about 160 times wealth than the poor household.

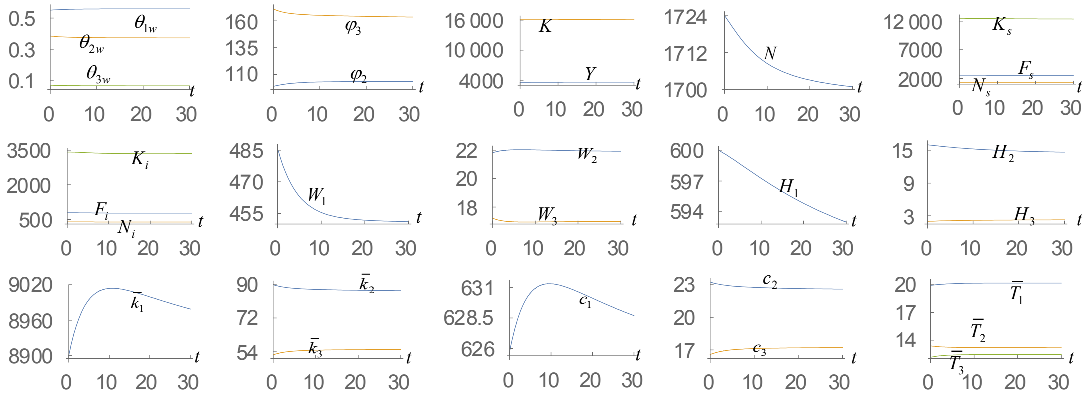

Figure 1. The Motion of the Economic System

We start with different initial states not far away from the equilibrium point and find that the system approaches to an equilibrium point. The equilibrium values of the variables are listed in (22). The rich has highest human capital and highest wage income. The rich household spends the least time on work and the poor household spends the longest hours. The rich household's consumption level and wealth are also much higher than the households from the two other groups.

$$\theta_{1w} = 0.559,\ \theta_{2w} = 0.373,\ \theta_{3w} = 0.069,\ \varphi_2 = 103.4,\ \varphi_3 = 162.7,\ Y = 3503.5,\ K = 16092.3,$$
$$N = 1698.4,\ F_i = 778,\ F_s = 2524,\ K_i = 3364,\ K_s = 12332,\ N_i = 391,\ N_s = 1307.9,$$
$$w_1 = 1617.9,\ w_2 = 2.02,\ w_3 = 1.47,\ W_1 = 450.4,\ W_2 = 22,\ W_3 = 17,\ H_1 = 588,\ H_2 = 14.5,$$
$$H_3 = 2.28,\ \bar{k}_1 = 8919,\ \bar{k}_2 = 87,\ \bar{k}_3 = 55,\ c_1 = 625,\ c_2 = 22.5,\ c_3 = 17.2,\ \bar{T}_1 = 20.2,$$

$$\bar{T}_2 = 13.2,\ \bar{T}_3 = 12.5. \tag{22}$$

It is straightforward to calculate the six eigenvalues as follows

$$-0.387,\ -0.356,\ -0.224,\ -0.11,\ -0.07,\ -0.032.$$

As all the eigenvalues are negative, we see that the equilibrium point is locally stable.

Comparative Dynamic Analysis

We simulated the motion of the dynamic system. We now study how exogenous changes, such as the rich's preference, affect the economic growth, inequality and each class's wealth and consumption. Before carrying out comparative dynamic analysis, we introduce a variable $\overline{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter.

The rich's efficiency of applying human capital being enhanced

Some people may accumulate much human capital but may not effectively apply human capital. Some people may very effectively what they learn. It may be argued that the rich has more opportunities to utilize and tends to be more capable of applying human capital in globalized economies. If the society is developed toward such a direction that enables the rich (and successful ones) to apply their human capital more effectively, gaps between the rich and the poor may be enlarged. We now increase the rich's human capital utilization efficiency as follows: $m_1 : 0.7 \Rightarrow 0.71$. The effects on the dynamic variables are plotted in Figure 2. The wage rate and price of services are not affected. The improved efficiency by the rich benefits the growth of the national wealth, GDP and total labor supply. The output levels and two input factors of the two sectors are augmented. The macroeconomic variables are improved. The rich's share of national wealth is augmented and the shares of the other two groups are reduced. The gaps between the rich household's wealth and the other two groups are enlarged. Although it makes the rich richer, to accumulate more human capital and wealth, to earn more, to work more, and to consume more, the parameter change has almost no impact on the other two groups. This is contrast to the closed economy. As shown in Zhang (2015), in the closed economy the poor and the middle benefit from the improvement in the rich's efficiency of applying human capital. The human capital, wealth, consumption level of services and wage incomes of the poor and the middle are all enhanced. In contrast to the closed economy, the change in the rich's human capital application efficiency benefits the rich, has makes no impact on the other groups, and worsens equality.

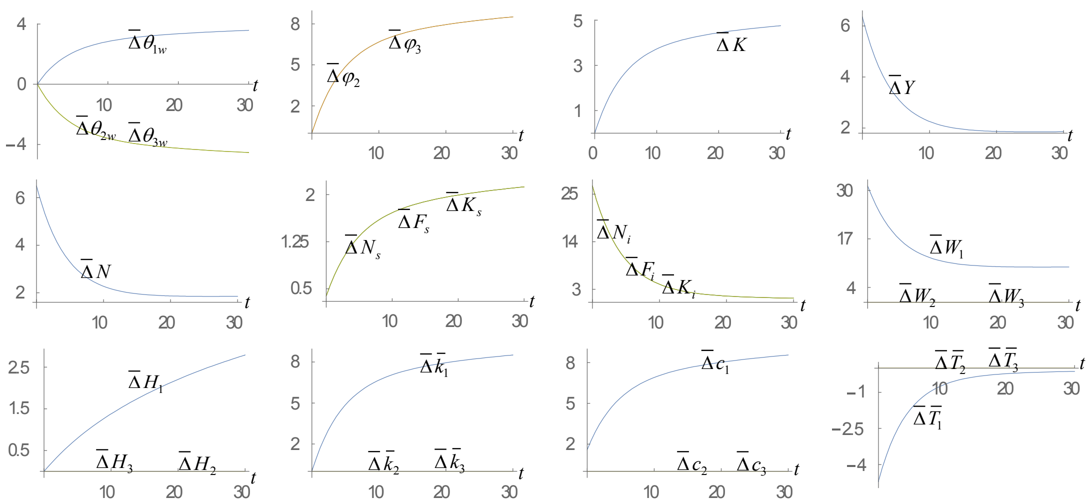

Figure 2. The Rich's Efficiency of Applying Human Capital Being Enhanced

The rich's propensity to save being augmented

If the rich has become too rich to spend their income, their propensity to save may be increased. We now increase the rich's propensity to save in the following way: $\lambda_{10} : 0.94 \Rightarrow 0.95$. The simulation results are plotted in Figure 3. The preference change has no impact on the wage rate and price of service. In contrast to the case for the closed economy (Zhang, 2015), the rise in the rich's propensity to save has no effect on the micro variables related to the poor and the middle. As the country is open with capital flows freely in global markets where the price and cost of capital good are fixed for the economy under consideration, the change in the propensity of the rich affects the macroeconomic variables and the group itself. It should be noted that as the rich changes its preference, the rich works more and consumes more which leads to higher human capital. The enhancements of the rich's human capital increase the national economic performance, but do not benefit the poor and the middle. The gaps between the rich and the poor are enlarged.

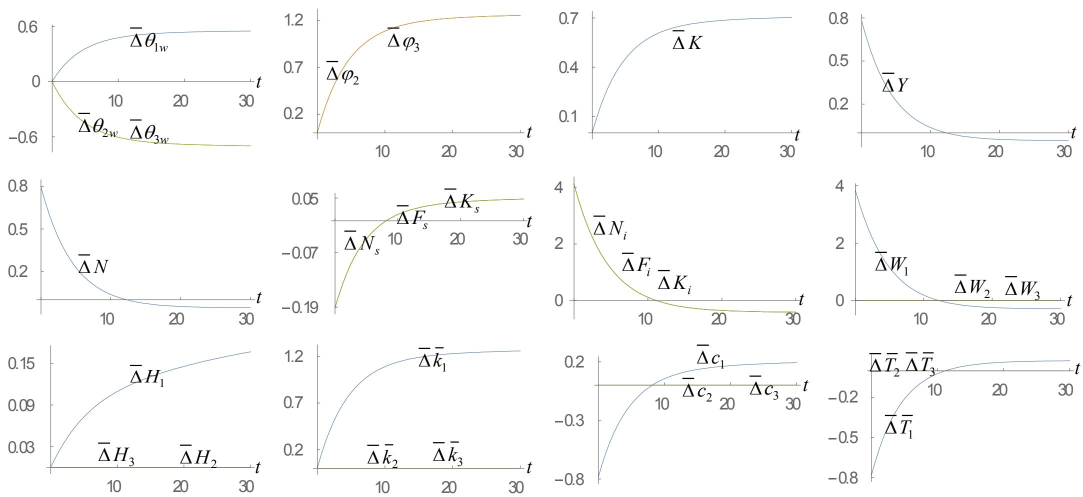

Figure 3. The Rich's Propensity to Save Being Augmented

The rich increasing the propensity to consume

We now consider a case that the rich increases the propensity to consume in the following way: $\xi_{10} : 0.07 \Rightarrow 0.075$. We plot the simulation results in Figure 4. The rate of interest and wage rate are not affected. The micro variables of the poor and middle are not affected. In the short term the inequalities are "improved" as the share of the rich in the national wealth falls and the ratios of the per household's wealth levels are shrunk. In the long term the inequalities are "deteriorated" as the share of the rich in the national wealth rises and the ratios of the per household's wealth levels are enhanced. As the rich works more hours and accumulates more human capital in the long term, the gaps are enlarged. It should remarked that different from the case that the propensity to save is increased, the increase in the propensity to consume results in that the rich work more hours even in the long term.

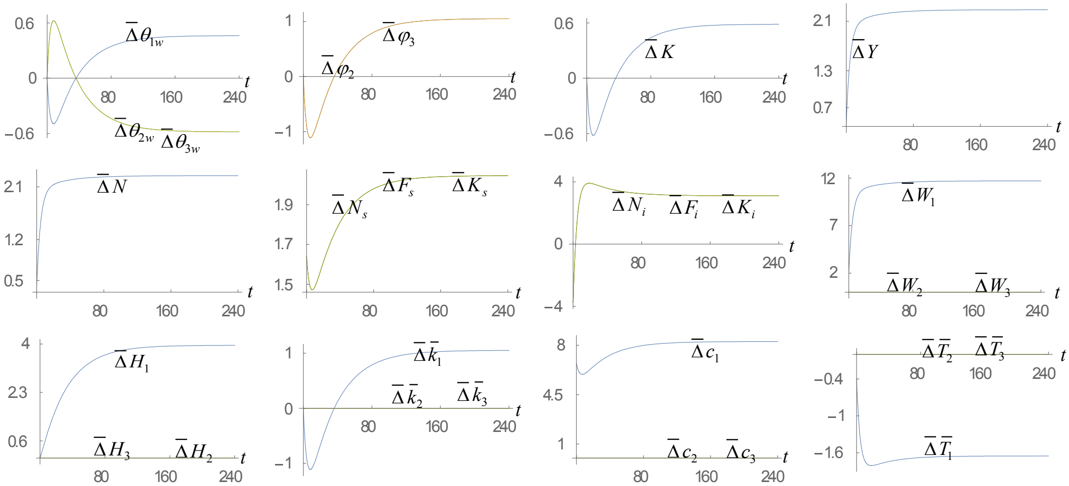

Figure 4. The Rich Increasing the Propensity to Consume

The rich accumulating human capital more effectively

We now strengthen the impact of the rich's learning through consuming on human capital accumulation as follows: $a_1 : 0.3 \Rightarrow 0.31$. We see that the rich's human capital accumulation is more strongly affected by consumption, the wealth gap between the rich and the poor is enlarged. The rich gets higher share of the national wealth and the ratios of per household wealth between the poor and rich and between the middle and rich are enhanced. The rich's human capital is augmented, which also increases the growth of the national wealth, GDP and total labor supply. The output levels and two input factors of the two sectors are augmented. The rate of interest and the price of service are not affected.

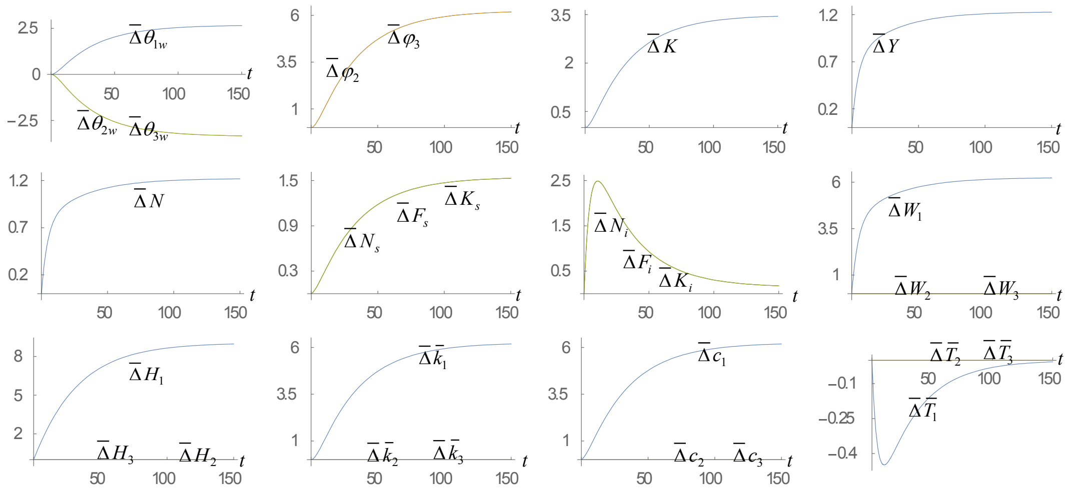

Figure 5. The Rich's Learning through Consuming Being Strengthened

The total factor productivity of the capital good sector being enhanced

Another important question is what will happen to different people and the national economy if the total factor productivity of the capital good sector is increased. We increase the productivity in the following way: $A_i : 1 \Rightarrow 1.05$. The simulation results are plotted in Figure 6. We have following change in the wage rate and price of service

$$\overline{\Delta}\, w = 7.4, \ \ \overline{\Delta}\, p = 4.9.$$

The rise in the productivity increases human capital and wage incomes of all the groups. The price of consumer good rises. The two sectors expand the output levels in the long term. The wealth and consumption levels of all the groups are increased in the long term. The national wealth, GDP and total labor supply are all increased. The inequality between the rich and the poor is enlarged in the long term. The rich get higher share of the national wealth and the ratios of per household wealth between the poor and rich and between the middle and rich are increased in the long term. It should be noted that Kaldor (1956) argues that as income inequality is enlarged, growth should be encouraged as savings are promoted. This positive relation between income inequality and growth is also observed in studies by Bourguignon (1981) and Frank (2009). There are other studies which find negative relations between income inequality and economic growth. Some mathematical models which predicate negative relations are referred to, for instance, Galor and Zeira (1993) and Galor and Moav (2004), and Benabou (2002). The empirical study by Persson and Tabellini (1994) also confirm negative relations. From our simulation, we see that relations between inequality and economic growth are complicated in the sense that these relations are determined by many factors. The relation are expectably ambiguous or development-dependent in the sense that one may observe positive or negative relations according the parameter values combinations and state of economic development.

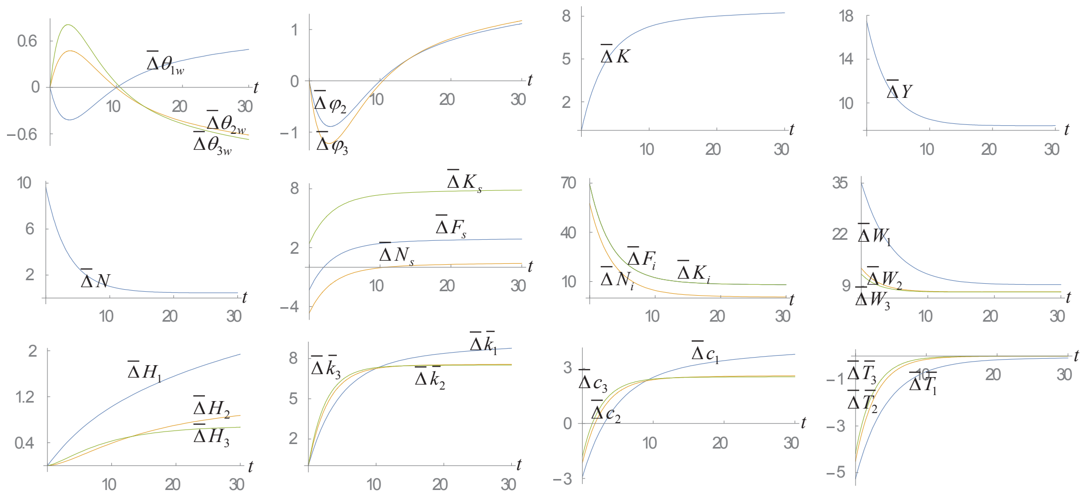

Figure 6. The Total Factor Productivity of the Capital Good Sector Being Enhanced

The total factor productivity of the service sector being enhanced

We now allow the total productivity in the following way: $A_s : 0.9 \Rightarrow 0.95$. The simulation results are plotted in Figure 7. We have following change in the wage rate and price of service

$$\overline{\Delta} w = 0, \ \overline{\Delta} p = -5.3.$$

The rise in the productivity increases human capital and wage incomes of all the groups. The price of consumer good rises. The two sectors expand the output levels in the long term. The wealth and consumption levels of all the groups are increased in the long term. The national wealth, GDP and total labor supply are all increased.

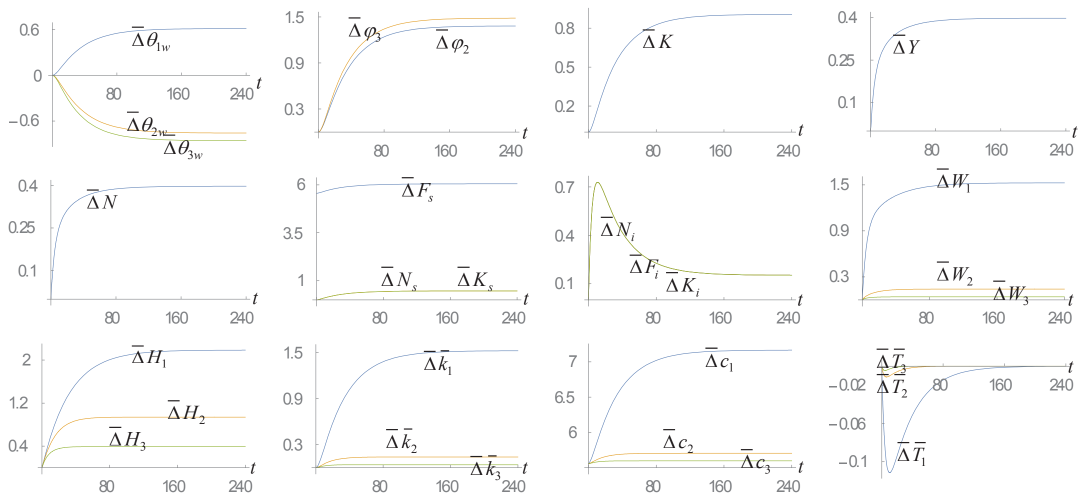

Figure 7. The Total Factor Productivity of the Service Sector Being Enhanced

The rate of interest being increased in global markets

We now allow the rate of interest to be exogenously changed as follows: $r^* : 0.0242 \Rightarrow 0.025$. We plot the simulation results in Figure 8. The wage rate of the price of service

$$\bar{\Delta}w = -0.62, \;\; \bar{\Delta}p = -0.04.$$

The rise in the cost of capital lowers the output levels of the two sectors. The wealth and consumption levels of all the groups are reduced in the long term. The national wealth, GDP and total labor supply are all decreased. The ratio between the rich's wealth and the poor's wealth falls in the long term.

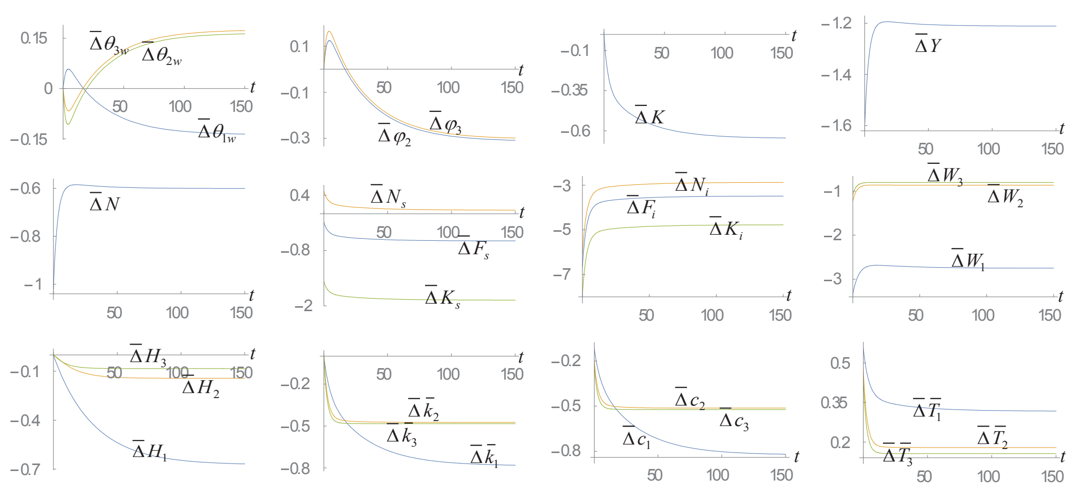

Figure 8. The Rate of Interest Being Increased in Global Markets

Concluding Remarks

This paper extended the growth model for a closed national economy by Zhang (2015) to a small-open economy. The dynamic economic model of heterogeneous households attempted to explain some economic mechanisms of how the richest one per cent of the population own 50% of national wealth. We consider endogenous wealth and human capital accumulation by heterogeneous households with different preferences and learning abilities as the main determinants of growth and inequality. The model is developed for perfectly competitive economies. We described the production technologies and economic structure on the basis of the Uzawa two-sector model. By applying Zhang's concept of disposable income and approach to household behavior, we describe consumers' wealth accumulation and consumption behavior. We modelled human capital accumulation on the basis of Arrow's learning by doing and Zhang's creativity with leisure. We described how wealth accumulation, human capital accumulation, and division of labor, and time distribution interact with each other under perfect competition. We simulated the model with three groups of the population, the rich 1 %, the middle 69%, and the poor 20%. We demonstrated the existence of an equilibrium point at which the rich 1% do own more than half of the national wealth and the poor 20% less than 10% of the national wealth. We showed how the system moves to the equilibrium from an initial state and confirm that the equilibrium point is stable. We also demonstrated how changes in the total factor productivities of the two sectors, the rich's human capital utilization efficiency, the rich's efficiency of learning through consuming, and the rich's propensities to save, to consume, and to enjoy leisure, affect growth and inequality. Our comparative dynamic analysis shows that changes in the rich's propensities and human capital accumulation abilities have slight effects on the economic conditions of the poor and the middle in the open-small economy. This is contrast to what have been demonstrated in Zhang's closed national growth model where, for instance, the poor and the middle benefit from the rich's rise in the propensity to save. The study has many obvious limitations. For instance, we assume that there is no social mobility in the economic system. This study does not consider the role of the government in redistributing wealth and income. It is important to see how the government can affect distribution with various policies. We carried out comparative dynamic analysis each time only with respect change in a single parameter. It is more insightful to allow multiple parameters to be changed simultaneously. Another important issue is how to introduce endogenous change in preferences of different people. We may extend the model in some other directions. We may introduce education and allow some kind of government intervention in education.

Appendix: Proving Lemma

By (6) and (8) we obtain

$$z \equiv \frac{\delta^*}{w} = \frac{N_q}{\bar{\beta}_q K_q}, \ q = i, s, \tag{A1}$$

where $\bar{\beta}_q \equiv \beta_q / \alpha_q$. From (A1) in (6), we have

$$z = \left(\frac{\delta^*}{\alpha_r}\right)^{1/\beta_i}, \ w(z) = \alpha z^{-\alpha_i}, \tag{A2}$$

where

$$\alpha_r = \alpha_i A_i \bar{\beta}_i^{\beta_i}, \ \alpha = \beta_i A_i \bar{\beta}_i^{-\alpha_i}.$$

Insert (A1) in (3)

$$\frac{N_i}{\bar{\beta}_i} + \frac{N_s}{\bar{\beta}_s} = z \sum_{j=1}^{J} \bar{k}_j \bar{N}_j, \tag{A3}$$

where we also use (4). We have

$$w_j(H_j) = H_j^{m_j} w. \tag{A4}$$

Hence, we determine the rate of interest and the wage rates as functions of (H_j). From (7) and (8), we have

$$p = \frac{\bar{\beta}_s^{\alpha_s} z^{\alpha_s} w}{\beta_s A_s}. \tag{A5}$$

From (A4) and the definitions of $\bar{y}_j$, we have

$$\bar{y}_j = (1 + r^*)\bar{k}_j + T_0 w_j. \tag{A6}$$

Insert $p c_j = \xi_j \bar{y}_j$ in (18)

$$\sum_{j=1}^{J} \xi_j \bar{N}_j \bar{y}_j = p F_s. \tag{A7}$$

Substituting (A6) in (A7) yields

$$N_s = \sum_{j=1}^{J} \tilde{g}_j \bar{k}_j + \tilde{g}, \tag{A8}$$

where we use $p F_s = w N_s / \beta_s$ and

$$\tilde{g}_j \equiv \bar{r}\, \beta_s\, \xi_j\, \bar{N}_j, \quad \bar{r} \equiv \frac{\tilde{r}}{w}, \quad \tilde{g}\big((H_j)\big) \equiv \beta_s T_0 \sum_{j=1}^{J} H_j^{m_j}\, \xi_j\, \bar{N}_j .$$

Insert $w_j \bar{T}_j = \sigma_j \bar{y}_j$ in (12)

$$T_j = T_0 - \frac{\sigma_j \bar{y}_j}{w_j}. \tag{A9}$$

Insert (A6) in (A9)

$$T_j = \left(1 - \sigma_j\right) T_0 - \frac{\tilde{r}\, \sigma_j \bar{k}_j}{w_j}. \tag{A10}$$

Insert (A10) in (1)

$$N = n_0 - \sum_{j=1}^{J} n_j \bar{k}_j , \tag{A11}$$

where

$$n_0\big((H_j)\big) \equiv T_0 \sum_{j=1}^{J} \left(1 - \sigma_j\right) \bar{N}_j H_j^{m_j} , \quad n_j \equiv \bar{r}\, \sigma_j\, \bar{N}_j .$$

Substituting (A8) and (A11) into yields

$$N_i\big((H_j), (\bar{k}_j)\big) = n_0 - \tilde{g} - \sum_{j=1}^{J} \left(n_j + \tilde{g}_j\right) \bar{k}_j . \tag{A12}$$

It is straightforward to confirm that all the variables can be expressed as functions of z, $(\bar{k}_j)$ and (H_j) by the following procedure: w by (A2) → w_j by (A4) → N_i by (A5) → N_i by (A12) → N by (A11) → N_s by (A8) → $\bar{y}_j$ by (A6) → K_i and K_s by (A1) → F_i and F_s, by the definitions → $\bar{T}_j$, c_j, and s_j by (15) → by (4). From this procedure, (A13), (16), and (17), we have

$$\dot{\bar{k}}_j = \Lambda_j\left(\left(\bar{k}_j\right), \left(H_j\right)\right) \equiv \lambda_j \bar{y}_j - \bar{k}_j, \quad j = 1, ..., J, \tag{A14}$$

$$\dot{H}_j = \Omega_j\left(\left(\bar{k}_j\right), \left(H_j\right)\right), \quad j = 1, ..., J, \tag{A15}$$

In summary, we proved the lemma.

Bibliography

1. Arrow, K.J. (1962) The Economic Implications of Learning by Doing. Review of Economic Studies 29, 155-73.
2. Arrow, K. J. (1974) General Economic Equilibrium: Purpose, Analytic Techniques, Collective Choice. American Economic Review 64, 253-72.
3. Arrow K.J. and Debreu, G. (1954) Existence of an Equilibrium for a Competitive Economy. Econometrica 22, 265–90.
4. Arrow, K.J. and Hahn, F.H. (1971) General Competitive Analysis. San Francisco: Holden-Day, Inc.
5. Azariadis, C. (1993) Intertemporal Macroeconomics. Oxford: Blackwell.
6. Barro, R.J. and X. Sala-i-Martin (1995) Economic Growth. New York: McGraw-Hill, Inc.
7. Benabou, R. (2002) Tax and Education Policy in a Heterogeneous-Agent Economy: What levels of Redistrbution Maximize Growth and Efficiency? Econometrica 70, 481-517.
8. Benigno, G. and Benigno, P. (2003) Price Stability in Open Economies. Review of Economic Studies 70, 743-64.
9. Bourguignon, F. (1981) Parento-superiority of Unegalitarian Equilibria in Stiglitz's Model of Wealth Distribution with Convex Saving Function. Econometrica 49, 1469-75.
10. Burmeister, E. and Dobell, A.R. (1970) Mathematical Theories of Economic Growth. London: Collier Macmillan Publishers.

11. Debreu, G. (1959) Theory of Value: An Axiomatic Analysis of Equilibrium. Yale University Press, London.
12. Forbes, K. (2000) A Reassessment of the Relationship Between Inequality and Growth. American Economic Review 90, 869-70.
13. Frank, M.W. (2009) Inequality and Growth in the United States: Evidence from a New State-level Panel of Income Inequality Measures. Economic Inquiry 47, 55-68.
14. Gale, D. (1955) The Law of Supply and Demand. Mathematica Scandinavica 3, 33–44
15. Gali, J. and Monacelli, T. (2005) Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Review of Economic Studies 72, 707-34.
16. Galor, O. and Moav, O. (2004) From Physical to Human Capital Accumulation: Inequality in the Process of Development. Review of Economic Studies 71, 1001-26.
17. Galor, O. and Zeira, J. (1993) Income Distribution and Macroeconomics. Review of Economic Studies 60, 35-52.
18. Ilzetzkia, E., Mendozab, E.G., and Véghc, G.A. (2013) How Big (Small?) Are Fiscal Multipliers? Journal of Monetary Economics 60 (2), pp. 239–54.
19. Kaldor, N. (1956) Alternative Theories of Distribution. Review of Economic Studies 23, 83–100.
20. Kollmann, R. (2001) The Exchange Rate in a Dynamic-Optimizing Business Cycle Model with Nominal Rigidities: A Quantitative Investigation. Journal of International Economics 55, 243-62.
21. Kollmann, R. (2002) Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles. Journal of Monetary Economics 49, 899-1015.
22. Lane, P.R. (2001) The New Open Economy Macroeconomics: A Survey. Journal of International Economics 54, 235-66.
23. Mas-Colell, A., Whinston, M.D. and Green, J.R. (1995) Microeconomic Theory. New York: Oxford University Press.
24. McKenzie, L.W. (1959) On the Existence of General Equilibrium for a Competitive Market. Econometrica 27, 54–71.
25. Nikaido, H. (1956) On the Classical Multilateral Exchange Problem. Metroeconomica 8, 135–45.
26. Nikaido, H. (1968) Convex Structures and Economic Theory. New York: Academic Press.
27. Pasinetti, L.L. (1960) A Mathematical Formulation of the Ricardian System. Review of Economic Studies 27, 78-98.

28. Obstfeld, M. and Rogoff, K. (1996) Foundations of International Macroeconomics. Mass., Cambridge: MIT Press.
29. Pasinetti, L.L. (1974) Growth and Income Distribution - Essays in Economic Theory. Cambridge: Cambridge University Press.
30. Persson, T. and Tabellini, G.E. (1994) Is Inequality Harmful for Growth? Theory and Evidence. American Economic Review 84, 600–21.
31. Samuelson, P.A. (1959) A Modern Treatment of the Ricardian Economy: I. The Pricing of Goods and Labor and Land Services. The Quarterly Journal of Economics 73, 1-35.
32. Solow, R. (1956) A Contribution to the Theory of Growth. Quarterly Journal of Economics 70, 65-94.
33. Uzawa, H. (1961) On a Two-Sector Model of Economic Growth. Review of Economic Studies 29, 47-70.
34. Uya, T., Yi, K.M., and Zhang, J. (2013) Structural Change in an Open Economy. Journal of Monetary Economics 60(September), pp. 667–82.
35. Uzawa, H. (1965) Optimal Technical Change in an Aggregative Model of Economic Growth. International Economic Review 6, 18-31.
36. Walras, L. (1874) Elements of Pure Economics, translated from the French by W. Jaffé, 1954. London: Allen and Unwin.
37. Zhang, W.B. (1993) Woman's Labor Participation and Economic Growth - Creativity, Knowledge Utilization and Family Preference. Economics Letters 42, 105-10.
38. Zhang, W. B. (2015). Oscillations in a Growth Model with Capital, Technology and Environment with Exogenous Shocks. *Academicus International Scientific Journal*, (12), 73-93.
39. Zhang, W.B. (2006) Growth with Income and Wealth Distribution. London: Macmillan.
40. Zhang, W.B. (2007) Economic Growth with Learning by Producing, Learning by Education, and Learning by Consuming. Interdisciplinary Description of Complex Systems 5, 21-38.
41. Zhang, W.B. (2015) How Do the Richest 1% Own 50% of Wealth in an Integrated Walrasian Equilibrium and Neoclassical Growth Model. The International Journal of Economic Behavior 5 (1), 59-80.