Cournot-Nash Family Decision and Economic Growth in an Extended Solowian Model

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Abstract

The purpose of this paper is to study economic growth with family-based microeconomic foundation. It develops a neoclassical growth model of endogenous wealth accumulation and consumption of two-person families. Growth mechanism and economic structures are based on a generalized Solowian growth model with Zhang's concept of disposable income and utility. Each spouse maximizes his/her utility function which is dependent on his/her egocentric utility function and the spouse's egocentric utility function. One's egocentric utility function is related to one's private consumption of goods, consumption of family goods, and saving made out of one's own disposable income. The couple's decisions are interdependent and are modelled as a Cournot-Nash game. Our model endogenously determines intra-household wealth accumulation and resource allocation on consumption and saving. The paper make an integration of some basic ideas in neoclassical growth theory and family economics. We conduct comparative dynamic analyses to show how the movement of the economy is affected by different exogenous changes in gender relations, preferences, and technologies.

Keywords: family decision; Cournot game; Solow model.

Introduction

The purpose of this study is to propose a simple economic growth model with family. In his AEA Presidential Address, Becker (1988) addressed the significance of family economics for macroeconomics. Since then, there is a sizeable amount of publications on the topic. Many important decisions, for instance, on consumption, labor supply, savings, number of children, and education, are made within families. These decisions are essential determinants of the creation and allocation of human as well as physical capital, labor force, and economic structure. In fact, Becker (1965) published his seminal work on economic rationality in the allocation of time and gender issues in a formal and rigorous theory. Since then, there are many studies on various issues of family and gender (e.g., Chiappori, 1992; Gomme, Kydland, and Rupert, 2001; Campbell and Ludvigson, 2001; and Vendrik, 2003). On the other hand, most of these studies are concentrated on microeconomics. It has become evident that macroeconomic theories without taking account of family decisions ignore a basic determinant for explaining economic growth and income and wealth distribution. There are many studies on family related to macroeconomics (for recent literature, e.g., Blundell, Dias, and Shaw, 2016; Doepke and Tertilt, 2016; and Chiappori, et al., 2018). This study deviates from the traditional literature in applying the concept of disposable income and utility function proposed by Zhang (1993, 2005) to examine growth with family decisions.

This study models behavior of a two-person family in a non-cooperative or Cournot-Nash framework (e.g., Chen and Woolley, 2001). Cournot game is a mathematically well-examined in game theory and economically widely applied in microeconomics. Cournot (1801-1877) developed Cournot competition theory in his 1838 volume *Recherches sur less Principles Mathematiques de la Theorie des Richesses* when he studied the competition with a market dominated a duopoly. The best response function is defined for each firm for given exogenous output level of the other firm. An equilibrium point is determined at intersection point of the best response functions. This equilibrium is now called a Nash equilibrium in game theory. This study applies the concept to study behavior of the couple. Neoclassical growth theory is a main approach to economic growth with endogenous wealth as the main machine of growth. The theory is built on microeconomics. But It is mainly concerned with firms and economic structure rather than families. The key model, Solow model, in neoclassical growth theory treats capital accumulation by industry as the main economic growth mechanism (e.g., Solow, 1956; Uzawa, 1961; Burmeister and Dobell 1970; Azariadis, 1993; Jensen and Larsen, 2005; and Ben-David and Loewy, 2003). As emphasized by Doepke and Tertilt (2016: 1791), "typical macroeconomic models ignore the family and instead build on representative agent modelling that abstracts from the presence of multiple family members, who may have conflicting interests, who might make separate decisions, and may split up and form new households." As discussed generally in Zhang (2005, 2008), neoclassical economic theory has not been properly integrated with different microeconomic theories partly due to analytical difficulties in association with the Ramsey approach to human behavior. Zhang constructs a new approach to modelling household behavior. This study applies this approach to introduce family decisions to neoclassical growth theory. It should be noted that Zhang (2012, 2016) introduces gender to neoclassical growth theory. This study applies family decision with Cournot-Nash equilibrium approach, rather than a representative household decision. The paper is constructed as follows. Section 2 defines a neoclassical one-sector growth model with family decisions. Section 3 simulates movement of the economy and identifies equilibrium of the national economy. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study.

The one-sector growth model with family decisions

This section introduces family decisions into the neoclassical growth model with the concept of disposable income and utility function proposed by Zhang. Most aspects of the model with regards to production and market structures are similar to are the Solow growth model, except modelling behavior of family. The economy is composed of two homogenous populations, man and woman, with the same number $\overline{N}$. Man and woman form a family. All the markets are perfectly competitive. We use subscript $j=1$ and $j=2$ to describe, man and woman, respectively. There is one commodity which is produced by a single sector and is used for investment and consumption. Capital depreciates at a rate δ_k. The family members own all assets. Input factors are in full employment. All prices are measured in terms of commodity with unity price. The wage rate of spouse j $w_j(t)$ and rate of interest $r(t)$ are determined by free markets. The economy has the total capital stock. We use h_j to stand for a constant level of gender $j's$ human capital· As there is no discrimination, we have $w_j(t)=h_j w(t)$, where $w(t)$ is the wage rate. The total labor supply N is given by:

$N=h_1\overline{N}+h_2\overline{N}.$ (1)

Current income and disposable income

We model decisions of man and woman who live together and form a family. Each maximizes his or her utility. As a member of the union, they will care each other. They have a family (public) goods, their own consumption goods and wealth. They make decisions interdependently. Each spouse's utility is dependent not only his/her own consumption and wealth, but also the partner's and utility. Cournot-Nash approach implies that each player maximizes his/her utility with the other's behavior as given. We use $\overline{k}_j(t)$ to stand for the wealth held by spouse j. The family members have the following current incomes:

$y_j(t)=r(t)\overline{k}_j(t)+h_j w(t),\ j=1,2,$ (2)

where $r(t)\overline{k}_j(t)$ and $h_j w(t)$ are spouse $j's$ incomes from interest payment and wage.

The spouse's $j's$ disposable income is the sum of the current income and value of wealth:

$\widehat{y}_j(t)=y_j(t)+\overline{k}_j(t)=R(t)\overline{k}_j(t)+h_j w(t),$ (3)

where $R(t)\iint 1+r(t)$. This study neglects possible transfers of disposable income between husband and wife.

Utility functions and budgets

Gender $j's$ well-being is given by an egocentric utility function $U_j(t)$ which is dependent on gender $j's$ private consumption $c_j(t)$, contribution to the family saving $s_j(t)$, and family good $\overline{c}(t)$ as follows:

$$U_j(t) = c_j^{\xi_{j0}}(t)\, \bar{c}^{-\gamma_{j0}}(t)\, s_j^{\lambda_{j0}}(t), \xi_{j0}, \gamma_{j0}, \lambda_{j0} > 0, \text{ (4)}$$

where ξ_{j0} is gender j's propensity to consume private goods, γ_{j0} is propensity to consume family goods, and λ_{j0} is propensity to make contribution to the family's savings. Household good is characterized by being non-rival. Husband and wife's egocentric utility functions are different. We consider each spouse cares the other's well-being. Spouse $j's$ utility function is thus specified by

$$\widetilde{U}(t) = U_j(t) U_i^{\in_j}(t), \in_j >= 0, i \neq j, \text{ (5)}$$

where $\in_j$ is a parameter measuring how strongly spouse j cares about the other. When each spouse cares the other's utility rather than consumption levels, the preference is termed caring preferences (1988; see also Bourguignon and Chiappori, 1992).

Family goods is bought by the couple:

$$\bar{c}(t) = \bar{c}_1(t) + \bar{c}_2(t). \text{ (6)}$$

where $\bar{c}_j(t)$ is spouse $j's$ contribution to family goods. Spouse j spends the disposable income on consuming private goods, paying family goods, and making savings. The budget is formed as:

$$c_j(t) + \bar{c}_j(t) + s_j(t) = \widehat{y}_j(t). \text{ (7)}$$

Spouse j maximizes the utility function under (7). As shown in Appendix A-1, we solve the optimal problem as follows:

$$\bar{c} = \overline{\gamma \widehat{y}}, c_j = \frac{\xi_{j0} \overline{\gamma \widehat{y}}}{\bar{\gamma}_j}, s_j = \frac{\lambda_{j0} \overline{\gamma \widehat{y}}}{\bar{\gamma}_j}, \bar{c}_j = \widehat{y}_j - (\xi_{j0} + \lambda_{j0}) \frac{\overline{\gamma \widehat{y}}}{\bar{\gamma}_j}, \text{ (8)}$$

where we omit time index and

$$\widehat{y}(t) = \widehat{y}_1(t) + \widehat{y}_2(t), \bar{\gamma}_j \equiv \gamma_{j0} + \in_j \gamma_{i0}, i \neq j, \bar{\gamma} \equiv \left(\frac{\xi_{10} + \lambda_{10}}{\bar{\gamma}_1} + \frac{\xi_{20} + \lambda_{20}}{\bar{\gamma}_2} + 1 \right)^{-1}.$$

From (8), we see that for $\bar{c}_2(t)$ and $\bar{c}_2(t)$ to be positive, we should require:

$$\left(\frac{\xi_{10} + \lambda_{10}}{\bar{\gamma}_1} + 1 \right) \left(\frac{\xi_{20} + \lambda_{20}}{\bar{\gamma}_2} \right)^{-1} > \frac{\widehat{y}_1(t)}{\widehat{y}_2(t)} > \left(\frac{\xi_{10} + \lambda_{10}}{\bar{\gamma}_1} \right) \left(\frac{\xi_{20} + \lambda_{20}}{\bar{\gamma}_2} + 1 \right)^{-1}.$$

This implies that if the disposable gap between the couple is too large, then one spouse will not spend his/her disposable income on family goods. In our simulation we are concerned with situations that both of the spouses purchase family goods.

Wealth accumulation

According to the definition of $s_j(t)$, the change in spouse $j's$ wealth is given by:

$\bar{k}_j(t) = s_j(t) - \bar{k}_j(t)$. (9)

This equation states that the change in wealth is saving minus dissaving.

Production sector

The production function $F(t)$ is taken on the Cobb-Douglas form:

$F(t) = AK^{\alpha}(t)N^{\beta}, \alpha, \beta > 0, \alpha + \beta = 1,$ (10)

where A, α and β are positive parameters. The marginal conditions are:

$r(t) + \delta_k = \dfrac{\alpha F(t)}{K(t)}, w(t) = \dfrac{\beta F(t)}{N(t)}.$ (11)

Demand and supply of goods

The equilibrium condition that the output of the production sector is equal to the depreciation of capital stock and the net savings is expressed as:

$C(t) + S(t) - K(t) + \delta_k K(t) = F(t),$

where

$S(t) = s_1(t)\bar{N} + s_2(t)\bar{N}, C(t) = c_1(t)\bar{N} + c_2(t)\bar{N}.$

The family wealth is equal to national wealth

$K(t) = \bar{k}_1(t)\bar{N} + \bar{k}_2(t)\bar{N}.$ (12)

The model is completed. It is an extension of the core model in neoclassical growth theory and is based on some ideas in family economics. We now study behavior of the model.

The movement of the economy

This section simulates the movement of the system. As shown in Appendix A-2, the movement of the economic system is given by two differential equations:

$\bar{k}_j(t) = s_j(\bar{k}_j(t)) - \bar{k}_j(t), j = 1,2.$

By this equation we determine $\bar{k}_j(t)$ over time. Once we solve $\bar{k}_j(t)$, as demonstrated in Appendix A-2 we solve all the variables in the dynamic system. The parameter values are specified as follows:

$A = 1.4, \alpha = 0.35, \overline{N} = 100, h_1 = 2.5, h_2 = 2, \delta_k = 0.05, \lambda_{10} = 0.6, \gamma_{10} = 0.2,$
$\xi_{10} = 0.2, \lambda_{20} = 0.62, \gamma_{20} = 0.15, \xi_{20} = 0.15.$ (18)

The national population is 200. The choice of population sizes is not important as far as our purposes of providing some insights into economic mechanisms of the system and comparative dynamic analysis. The total factor productivity is 1.4. The parameter α in the Cobb-Douglas production is taken on 0.35. In empirical studies the value is often taken on 1/3 (for instance, Miles and Scott, 2005; Abel, Bernanke, and Croushore, 2007). The depreciation rate of physical capital is fixed at 0.05. Under (18) and the initial conditions:

$k_1(0) = 9, h_2 = 2,$

we plot the movement of the economy as in Figure 1.

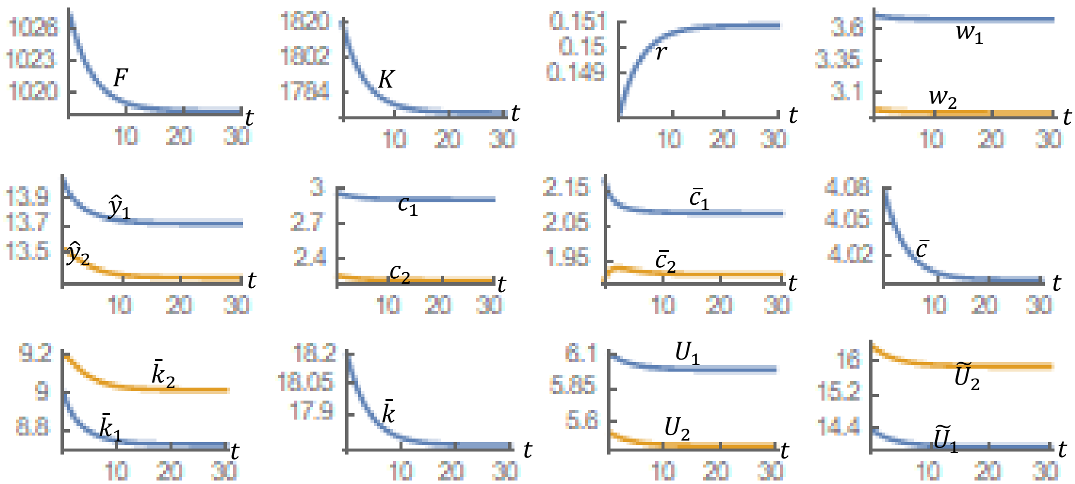

Figure 1. The Motion of the Economic System

The system has an equilibrium point given as follows:

$$F = 1018.3, K = 1774, r = 0.151, w_1 = 3.68, w_2 = 2.94, \hat{y}_1 = 13.7,$$

$$\hat{y}_2 = 13.3, c_1 = 2.91, c_2 = 2.22, \bar{c}_1 = 2.09, \bar{c}_2 = 1.91, \bar{c} = 4, \bar{k}_1 = 8.73,$$

$$\bar{k}_2 = 9.02, \bar{k} = 17.74, U_1 = 5.99, U_2 = 5.43, \widetilde{U}_1 = 14, \widetilde{U}_2 = 15.9. \quad (19)$$

At equilibrium the husband has higher income than the wife; he consumes more consumption goods than she. The husband makes more contribution to family goods than the wife. The wife has more wealth than the husband. The husband has higher egocentric utility than the wife; but the wife has higher well-being than the husband as she derives much more pleasure from her husband than he from his wife. The two eigenvalues are -1 and -0.245. The equilibrium point is stable. We can thus effectively conduct comparative dynamic analysis.

Comparative statics analysis

We just gave the movement of the economy system, found the existence of an equilibrium point and guaranteed the stability of the equilibrium point. This section is concerned with how the dynamics is affected when some parameters are exogenously changed. For instance, one may ask about whether the national economic growth is encouraged or discouraged, if one spouse derives less well-being from his/her spouse. Let $\bar{\Delta} x(t)$ represent the change rate of variable $x(t)$ in percentage caused by an exogenous change in a parameter.

The husband derives less well-being from his wife's well-being

We analyze how the economy is affected if the husband derives less well-being from his wife's well-bring in the following way: $\in_1$: $0.5 \Rightarrow 0.45$. Figure 2 provides the simulation result. As the husband derives less well-being from his wife's well-being, he spends less on family goods. The wife spends more on family goods. The family holds less family goods initially but has more in the long term. Each spouse has more wealth. The national output and capital are enhanced. The rate of interest is reduced. The wage incomes are increased. Each spouse has more disposable income and consumes more goods. Each spouse has higher egocentric utility level. The wife has higher utility, while the husband has lower utility.

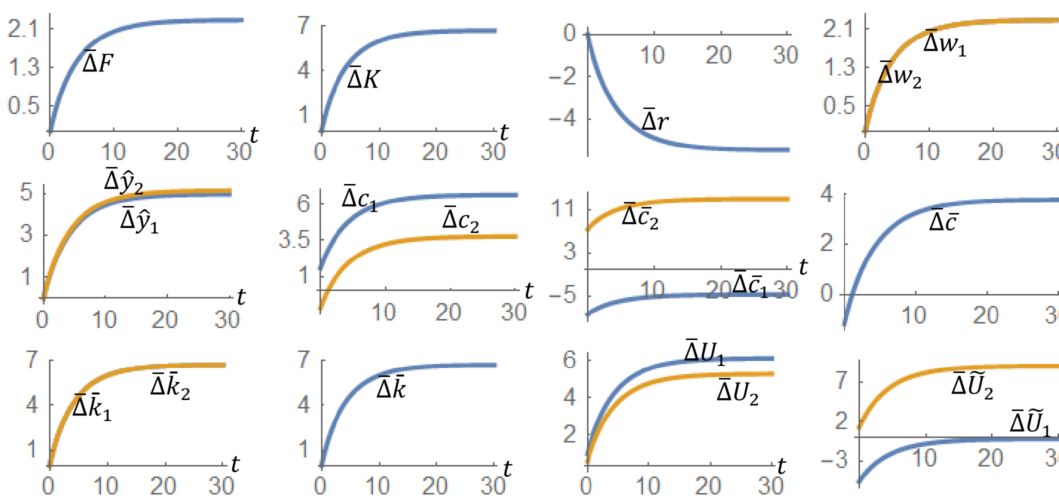

Figure 2. Man Derives Less Well-being from his Wife's Well-being

The wife's human capital is enhanced

We analyze how the economy is affected if the wife's human capital is enhanced in the following way: h_2: $2 \Rightarrow 2.1$. Figure 3 provides the simulation result. The wife's wage is enhanced, while the husband's wage income is slightly affected. The wife spends more on family goods, while the husband spends less on family goods. The total consumption of family goods is enhanced. The rate of interest rises initially and changes slightly in the long term. The national output and national capital are increased. Each spouse spends on consumption goods and has more wealth. All the utility levels are enhanced.

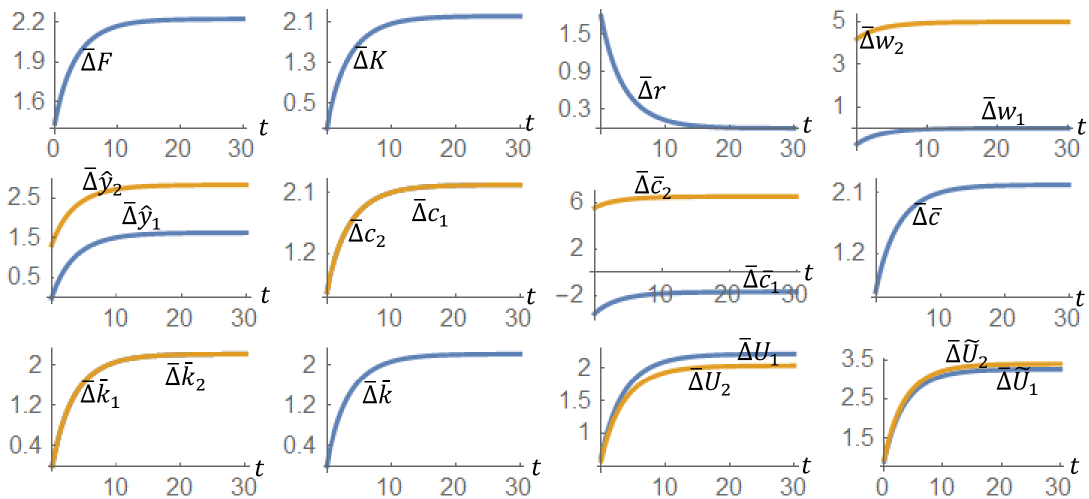

Figure 3. The Wife's Human Capital is Enhanced

The wife increases her propensity to save

We analyze how the economy is affected if the wife's human capital is enhanced in the following way: λ_{20}: $0.62 \Rightarrow 0.64$. Figure 3 provides the simulation result. The wife accumulates more wealth. The husband has less wealth initially but more in the long term. The rate of interest falls. The wife spends less on family goods initially but more in the long term. The husband spends more on family goods initially but less in the long term. The total consumption of family goods falls initially but rises in the long term. The utility levels are enhanced.

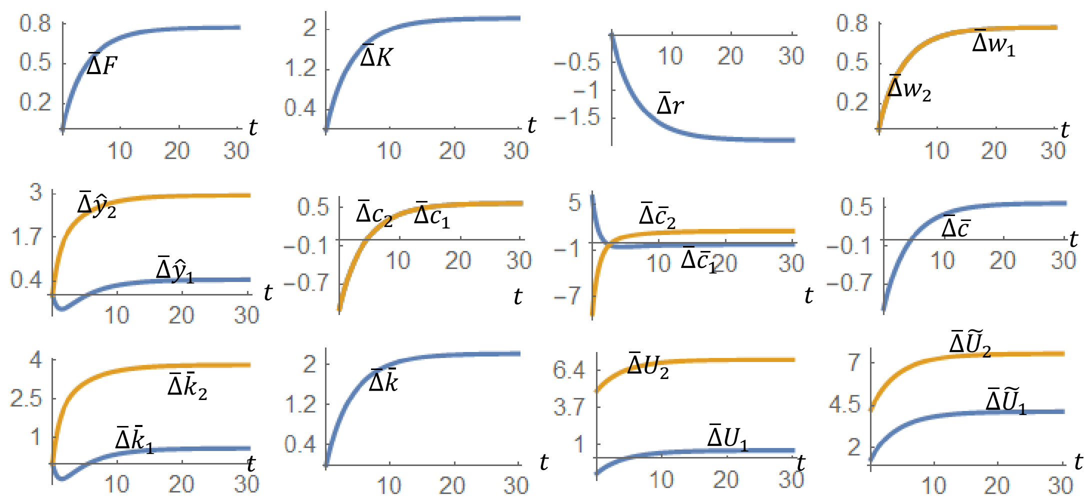

Figure 4. The Wife Increases Her Propensity to Save

The wife increases her propensity to consume consumption goods

We analyze how the economy is affected if the wife increases her propensity to consume consumption goods in the following way: ξ_{20}: $0.15 \Rightarrow 0.17$. Figure 5 provides the simulation result. The wife spends more on consumption goods and less on family goods. Her disposable income and wealth are reduced. The husband spends less on consumption goods and more on family goods. His disposable income and wealth are reduced. The economy produces less and has less capital stocks. In the long term all the utility levels are reduced.

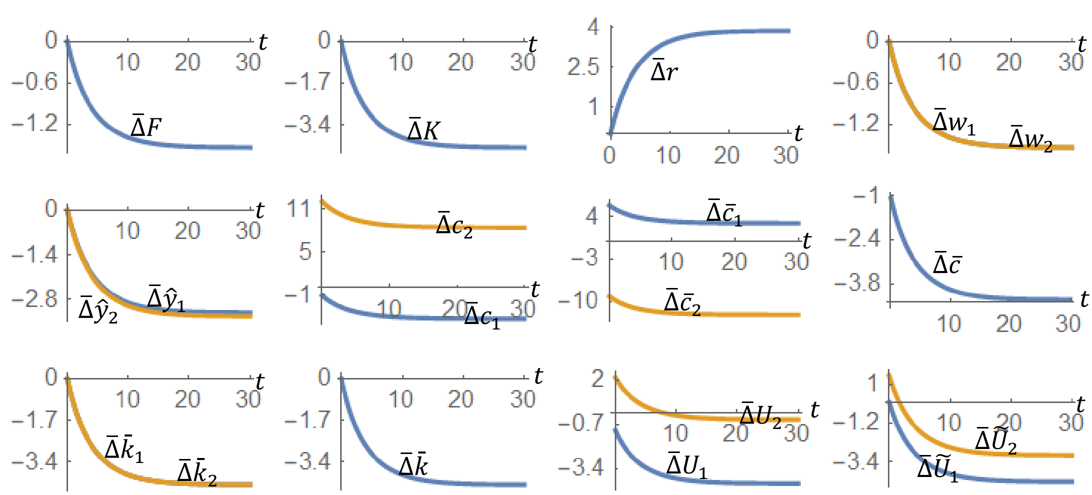

Figure 5. The Wife Increases Her Propensity to Consume Consumption Goods

The wife increases her propensity to consume family goods

We analyze how the economy is affected if the wife increases her propensity to consume family goods in the following way: γ_{20}: $0.15 \Rightarrow 0.17$. Figure 6 provides the simulation result. The wife purchases more family goods, while the husband purchases less. The total consumption of family goods is enhanced. The wife spends less consumption goods initially but slightly increases in the long term. The husband spends more on consumption goods. The national wealth and national capital stocks are enhanced. The rate of interest falls. The wage incomes are increased. The wealth levels of spouses are increased. All the utility levels are reduced.

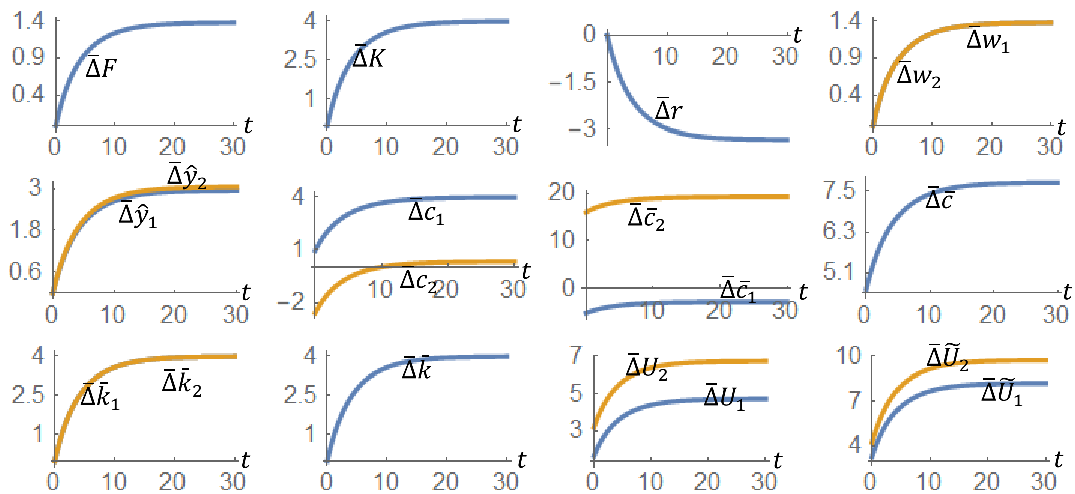

Figure 6. The Wife Increases Her Propensity to Consume Family Goods

The total factor productivity is enhanced

We analyze how the economy is affected if the total factor productivity is enhanced in the following way: A: $1.4 \Rightarrow 1.42$. Figure 7 provides the simulation result. The national output and national capital stocks are enhanced. The couple's wage incomes are enhanced. The rate of interest rises initially and changes slightly in the long term. Each spouse has more wealth, more disposable income, spends more on consumption goods and family goods. All the utility levels are enhanced.

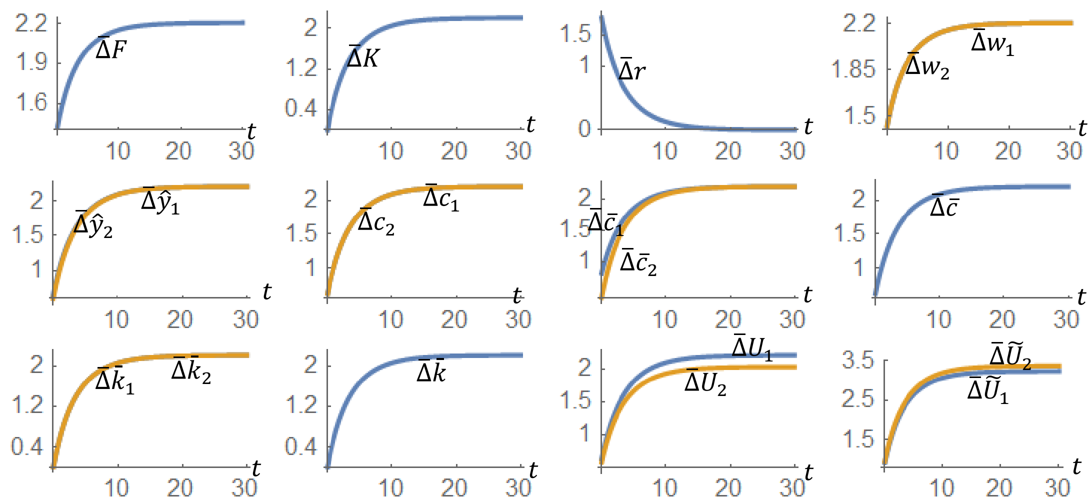

Figure 7. The Total Factor Productivity is Enhanced

Conclusions

This paper develops a neoclassical growth model with homogenous two-person families. Growth mechanism and economic structures are based on a generalized Solowian growth model with Zhang's concept of disposable income and utility. Each spouse maximizes the spouse's utility which is related to the spouse's own consumption and saving and the other spouse's well-being. The couple's decisions are interdependent and are modelled as a Cournot-Nash game. The model endogenously determines intra-household wealth accumulation and resource allocation. Saving is endogenous due to interdependent decisions of family members. The paper introduced an alternative microeconomic mechanism in neoclassical economic growth theory. The paper made an integration of some basic ideas in neoclassical growth theory and family economics. It provided some insights into dynamics of gender game and factor distribution in a perfectly competitive economy with capital accumulation. We conducted comparative dynamic analyses with regards to some parameters. This paper can be generalized by many ways according to the literature of family economics. Many models for different issues of families provide basic ideas for further research within the framework proposed in this study. For instance, the family is a driving force for institutional change. There are issues related to modelling interregional resource transmission (Becker, et al., 2018). Marriage and divorce are interdependent with growth, preference and education (Chiappori, Dias, and Meghir, 2018). It is also important to introduce heterogeneous households and use more general functional forms of utility.

Appendix A-1: Optimal behavior

The Lagrangian expression is given by:

$$L_j = c_j^{\xi_{j0}} \bar{c}^{\bar{\gamma}_j} s_j^{\lambda_{j0}} s_i^{\in_j \lambda_{i0}} c_i^{\in_j \xi_{i0}} + \lambda_j \left(\hat{y}_j - c_j - \bar{c}_j - s_j\right), \text{ (A1)}$$

where

$$\bar{\gamma}_j \equiv \gamma_{j0} + \in_j \gamma_{i0}.$$

The first-order conditions imply:

$$\frac{\partial L_j}{\partial c_j} = \frac{\xi_{j0} \tilde{U}_j}{c_j} - \lambda_j = 0, \text{ (A2)}$$

$$\frac{\partial L_j}{\partial \bar{c}_j} = \frac{\bar{\gamma}_j \tilde{U}_j}{\bar{c}} - \lambda_j = 0, \text{ (A3)}$$

$$\frac{\partial L_j}{\partial s_j} = \frac{\lambda_{j0} \tilde{U}_j}{s_j} - \lambda_j = 0, \text{ (A4)}$$

$$\frac{\partial L_j}{\partial \lambda_j} = \hat{y}_j - c_j - \bar{c}_j - s_j = 0. \text{ (A5)}$$

From (A5) and (6), we have:

$$\hat{y}_1 + \hat{y}_2 - c_1 - c_2 - \bar{c} - s = 0. \text{ (A6)}$$

From (A2)-(A4), we solve:

$$c_j = \frac{\xi_{j0} \bar{c}}{\bar{\gamma}_j}, s_j = \frac{\lambda_{j0} \bar{c}}{\bar{\gamma}_j}, j = 1, 2. \text{ (A7)}$$

From (A6) and (A7), we get:

$$\bar{c} = \overline{\gamma} \hat{y}, \text{ (A8)}$$

From (A5) and (A7), we thus have:

$$c_j = \frac{\xi_{j0} \overline{\gamma} \hat{y}}{\bar{\gamma}_j}, s_j = \frac{\lambda_{j0} \overline{\gamma} \hat{y}}{\bar{\gamma}_j}, \bar{c}_j = \hat{y}_j - (\xi_{j0} + \lambda_{j0}) \frac{\overline{\gamma} \hat{y}}{\bar{\gamma}_j}. \text{ (A9)}$$

By (A8) and (A9), we determine behavior of the family as functions of $\hat{y}_j(t)$.

Appendix A-2: Proving the Lemma

By (11) we get:

$$r(\bar{k}) = \frac{\alpha A N^{\beta}}{\bar{N}^{\beta} \bar{k}^{\beta}} - \delta_k, w(\bar{k}) = \frac{\beta A \bar{N}^{\alpha} \bar{k}^{\alpha}}{N^{\alpha}}. \text{ (A10)}$$

where we use (12). By (3) and (A10) we have:

$R(\bar{k}) = r(\bar{k}) + 1, \ \widehat{y}_j(\bar{k}) = R\bar{k}_j + h_j w.$ (A11)

By (8) we have

$\bar{c} = \overline{\gamma\widehat{y}}, c_j = \frac{\xi_{j0}\overline{\gamma\widehat{y}}}{\bar{\gamma}_j}, s_j = \frac{\lambda_{j0}\overline{\gamma\widehat{y}}}{\bar{\gamma}_j}.$ (A12)

It is straightforward to check that all the variables in the system can be expressed as functions of $\bar{k}_j(t)$. By (9) we have:

$\dot{\bar{k}}_j(t) = s(\bar{k}_j(t)) - \bar{k}_j(t).$ (A13)

We see that once we determine $\bar{k}_1(t)$ and $\bar{k}_2(t)$ we get the values of all the other variables at any point of time.

Bibliography

1. Abel, A., Bernanke, B.S., & Croushore, D. (2007). *Macroeconomics*. New Jersey: Prentice Hall.
2. Azariadis, C. (1993). *Intertemporal macroeconomics*. Oxford: Blackwell.
3. Becker, G.S. (1965). A theory of the allocation of time. *Economic Journal*, 75(299), 493-517.
4. Becker, G.S. (1988). Family economics and macro behavior. *American Economic Review*, 78(1), 1-13.
5. Becker, G. S., Kominers, S.D., Murphy, K.M., & Spenkuch, J.L. (2018). A theory of intergenerational mobility. *Journal of Political Economy*, 126(S1), 7-25.
6. Ben-David, D. & Loewy, M.B. (2003). Trade and neoclassical growth model. *Journal of Economic Integration*, 18(1), 1-16.
7. Blundell, R., Dias, M.C., & Shaw, C.M.J. (2016). Female labor supply, human capital and welfare reform. *Econometrica*, 84(5), 1705-1753.
8. Bourguignon, F. & Chiappori, P.A. (1992). Collective models of household behavior: An introduction. *European Economic Review*, 36(2-3), 355-364.
9. Burmeister, E. & Dobell, A.R. (1970). *Mathematical theories of economic growth*. London: Collier Macmillan Publishers.
10. Campbell, J.Y. & Ludvigson, S. (2001). Elasticities of substitution in real business cycle models with home production. *Journal of Money, Credit and Bankings*, 33(4), 847-875.
11. Chen, Z.Q. & Woolley, F. (2001). A Cournot-Nash model of family decision making. *The Economic Journal*, 111 (October), 722-748.

12. Chiappori, P.A. (1992). Introducing household production in collective models of labor supply. *Journal of Political Economy*, 105(1), 191-209.
13. Chiappori, P.A., Dias, M.C., & Meghir, C. (2018). The marriage market, labor supply, and education choice. *Journal of Political Economy*, 126(S1), 26-72.
14. Doepke, M. & Tertilt, M. (2016). Families in macroeconomics, Chapter 23. In J.B. Taylor & H. Uhlig (Eds.), *Handbook of macroeconomics* (Vol. 2B, pp. 1789-891). Amsterdam: Elsevier B.V.
15. Gomme, P., Kydland, F.E., & Rupert, P. (2001). Home production meets time to build. *The Journal of Political Economy*, 109(5), 1115-1131.
16. Jensen, B.S. and Larsen, M.E. (2005). General equilibrium dynamics of multi-sector growth models. *Journal of Economics*, 85(S1), 17-56.
17. Miles, D. & Scott, A. (2005). *Macroeconomics – Understanding the wealth of nations*. Chichester: John Wiley & Sons, Ltd.
18. Vendrik, M.C.M. (2003). Dynamics of a household norm in female labour supply. *Journal of Economic Dynamics & Control*, 27(5), 823-841.
19. Zhang, W.B. (1993). Woman's labor participation and economic growth - Creativity, knowledge utilization and family preference. *Economics Letters*, 42(1), 105-110.
20. Zhang, W.B. (2005). *Economic growth theory*. London: Ashgate.
21. Zhang, W.B. (2008). *International trade theory: Capital, knowledge, economic structure, money and prices over time and space*. Berlin: Springer.
22. Zhang, W.B. (2012). Economic growth with heterogeneous households, gender division of labor, and elastic labor supply. *Journal of Income Distribution*, 21(2), 15-37.
23. Zhang, W.B. (2016). Population growth and preference change in a generalized solow growth model with gender time distributions. *Oradea Journal of Business and Economics*, 1(2), 7-30.
24. Zhang, W. B. (2015). Oscillations in a Growth Model with Capital, Technology and Environment with Exogenous Shocks. Academicus International Scientific Journal, 6(12), 73-93.
25. Zhang, W. B. (2018). Dynamic Interactions Between Health, Human Capital and Wealth. Academicus International Scientific Journal, 9(17), 122-145.