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Finite Element Analysis of Laminated Composite Beams under Moving Loads

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ABSTRACT

In this study, a multilayered shear deformable finite element for dynamic analysis of laminated composite beams subjected to moving loads is presented. The beam model includes separate rotational degrees of freedom for each lamina but does not require additional axial and transverse degrees of freedom beyond those necessary for a single lamina. The shape functions ensure compatibility between the laminae. Making use of Timoshenko beam theory and Galerkin method, stiffness and mass matrices for the multilayered beam element are derived. In solution, interfacial slip or delamination between laminae is not allowed. Comparisons between the results obtained by proposed element with available results in the literature are given to show its validity.

INTRODUCTION

Increase in use of fiber-reinforced composite laminates as a structural member in place of conventional structural materials such as concrete and steel makes important to understand mechanical behavior of them under static and dynamic loads. As well as other construction areas, laminated composites are preferred in bridge design due to their merits such as low density, high strength, long-term durability, and resistance to corrosion and fatigue. Despite these advantageous properties, there are some technical issues need to be addressed before engineers can develop confidence in bridge design with laminated composites. It is well known bridges are subject to moving traffic loads. Due to their dynamic nature, moving loads cause greater deflections and stresses than those of static loads. Thus, their dynamic nature must be considered in the analysis and design of bridges. There are many studies on the response of bridges to moving loads in the literature. However, in all of these studies, the bridge is idealized by an isotropic beam or plate structure [1, 2, 17, 18 and references therein].

Although vibration and stability of laminated composite beams are well-studied [3-7], there is a few studies on dynamic analysis of these type of structures under moving loads [8-14]. Chonan [8] considered the steady-state response of a thick sandwich strip plate to a moving line load of constant magnitude. Based on higher-order shear deformation theory, Kadivar and Mohebpour [9] derived a laminated beam element including Poisson effect and bend-stretch, shear-stretch, and bend-twist couplings to analyze moving load-induced vibrations of unsymmetric laminated composite beams. They also considered other laminated beam theories such as classical lamination and first-order shear deformation theories by using appropriate modifications on their beam element. Zibdeh and Abu-Hilal [10] investigated stochastic vibrations of laminated composite coated beams traversed by a moving random load. Kiral *et al*. [11] investigated dynamic behavior of laminated composite beams subjected to moving loads using a three-dimensional finite element model based on classical lamination theory. Kavipurapu [12] investigated the dynamic response of simply supported glass/epoxy composite beams subjected to moving loads in a hygrothermal environment using the general purpose finite element program ANSYS. Kahya and Mosallam [13] studied the moving mass problem of composite sandwich beams using modal superposition. They investigated effects of vehicle mass and speed, fiber orientation, and lamina thickness on the beam response and the contact force between the mass and the beam. Mohebpour *et al*. [14] developed an algorithm based on the finite element method to study the dynamic response of laminated composite beams subjected to moving oscillators. In derivation of the finite element, they used first-order shear deformation theory.

In this study, a multilayered beam element is presented for dynamic analysis of laminated composite beams under moving loads. This element includes separate rotational degrees of freedom for each lamina while it does not require any additional axial and transversal degrees of freedom beyond those necessary for a single lamina. To include shear deformation, Timoshenko beam theory is used for transverse vibrations. The shape functions ensure compatibility between the laminae. Interfacial slip and delamination are not allowed.

FORMULATION

The element will be described here was previously used to investigate static response and natural frequencies of laminated composite beams [3, 4]. Here, we will give formulation of the method briefly and apply it to moving load analysis of laminated composite beams.

Governing Equations

Free axial vibrations of a linear elastic bar are governed by the following differential equation.

$$m\ddot{u} - EAu'' = 0 \tag{1}$$

where $u(x,t)$ is axial displacement, *m* is mass per unit length, *E* is Young's modulus, and *A* is cross-sectional area of the beam.

In order to include shear deformation, Timoshenko beam theory is used. According to this theory, transverse vibrations of a beam is governed by

$$\begin{aligned} &m\ddot{v} - KGA\left(v'' - \phi'\right) = p(x,t) \\ &EI\phi' + KGA\left(v' - \phi\right) - \rho I\ddot{\phi} = 0 \end{aligned} \tag{2}$$

where $v(x,t)$ is transverse displacement, $\phi(x,t)$ is rotation of the cross-section. *G*, *I* and *ρ* are shear modulus, second moment of area and mass density of the beam, respectively. *K* is shear correction factor which is taken as 5/6 for rectangular cross-sections. In Eqs. (1) and (2), over dot and prime denote derivatives with respect to time (*t*) and spatial coordinate (*x*), respectively. For a concentrated load moving on the beam, $p(x,t)$ can be defined as

$$p(x,t) = P\delta(x - ct) \tag{3}$$

where *P* and *c* denote magnitude and speed of the moving load, respectively. $\delta(\cdot)$ is Dirac delta function.

Finite Element Formulation

Multilayered beam element consists of N stacked layers. As shown in Figure 1, single lamina element has five nodes consisting of four equally spaced nodes and a node at the middle. The nodal displacements measured at each node are:

- at the exterior nodes (nodes 1 and 5), axial displacement u, transverse displacement v, and cross-sectional rotation ϕ,
- at two equally spaced interior nodes (nodes 2 and 4), transverse displacement v,
- at the middle node in the interior (node 3), axial displacement u, and cross-sectional rotation ϕ.

All nodal displacements are measured at midplane of the beam and are expressed as

$$\mathbf{u} = \{u_1 \quad u_2 \quad u_3 \quad v_1 \quad v_2 \quad v_3 \quad v_4 \quad \phi_1 \quad \phi_2 \quad \phi_3\}^T \tag{4}$$

Multiple laminae are accommodated merely by adding only rotational degrees of freedom. No additional axial and transversal degrees of freedom are necessary.

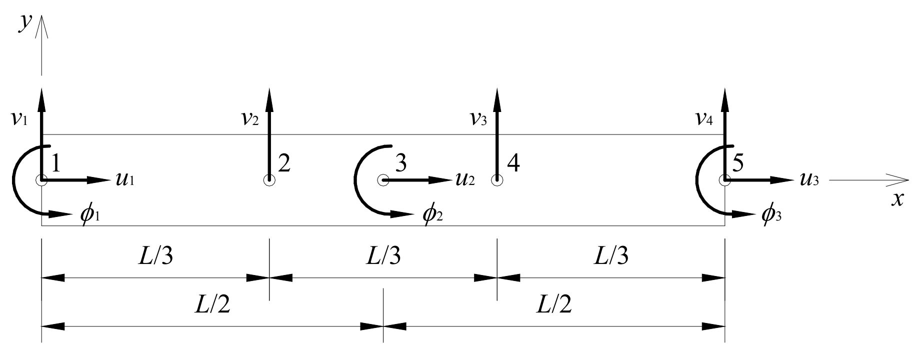

Figure 1 10-degree of freedom single lamina element

According to first-order shear deformation theory, *i.e.*, Timoshenko beam theory for beams [15], axial and transversal displacements at any point in the lamina can expressed as

$$U(x,t) = u(x,t) - y\phi(x,t) \qquad V(x,t) = v(x,t) \tag{5}$$

Since the axial displacement of a point not on the midplane of the beam is a linear function of ϕ as well as u as seen in Eq. (5), the degree of polynomial for u and ϕ must be the same order, and they are chosen to obey a quadratic polynomial. Because the shear strain is a linear function of both ϕ and $\partial v / \partial x$, the degree of polynomial for v must be one order higher than those used for u and ϕ in order to ensure compatibility. Therefore, a cubic polynomial for v is chosen [3]. Deflection behavior of a single lamina element according to Timoshenko beam theory can, thus, be described as follows.

$$u(x,t) = \sum_{i=1}^{3} \varphi_i(x) u_i(t) \qquad v(x,t) = \sum_{i=1}^{4} \psi_i(x) v_i(t) \qquad \phi(x,t) = \sum_{i=1}^{3} \varphi_i(x) \phi_i(t) \tag{6}$$

where $\varphi_i(x)$ and $\psi_i(x)$ are Lagrange interpolation functions, $u_i(t)$, $v_i(t)$ and $\phi_i(t)$ are the generalized nodal displacements. Following the usual finite element procedure, equation of motion of a single lamina can be written as

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{f} \tag{7}$$

where **m** is mass matrix, **k** is stiffness matrix of a single lamina, and **f** is load vector, which are defined as

$$\mathbf{m} = \begin{bmatrix} \mathbf{m}^{11}_{3\times3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{m}^{22}_{4\times4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{m}^{33}_{3\times3} \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} \mathbf{k}^{11}_{3\times3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{k}^{22}_{4\times4} & \mathbf{k}^{23}_{4\times3} \\ \mathbf{0} & \mathbf{k}^{23\,T}_{3\times4} & \mathbf{k}^{33}_{3\times3} \end{bmatrix} \qquad \mathbf{f} = \left\{ \mathbf{0} \quad \mathbf{f}_{1\times4} \quad \mathbf{0} \right\}^{\mathrm{T}} \tag{8}$$

where

$$\begin{aligned} & m^{11}_{ij} = \int_0^{L_e} m\,\varphi_i\,\varphi_j dx \quad (i,j=1-3) \qquad m^{22}_{ij} = \int_0^{L_e} m\,\psi_i \psi_j dx \quad (i,j=1-4) \\ & m^{33}_{ij} = \int_0^{L_e} \rho I \varphi_i \varphi_j dx \quad (i,j=1-3) \qquad k^{11}_{ij} = \int_0^{L_e} EA\,\varphi_i'\,\varphi_j' dx \quad (i,j=1-3) \\ & k^{22}_{ij} = \int_0^{L_e} KGA\,\psi_i'\,\psi_j' dx \quad (i,j=1-4) \quad k^{23}_{ij} = -\int_0^{L_e} KGA\,\psi_i'\,\varphi_j dx \quad (i=1-4, j=1-3) \\ & k^{33}_{ij} = \int_0^{L_e} (EI\,\varphi_i'\,\varphi_j' + KGA\,\varphi_i\,\varphi_j) dx \quad (i,j=1-3) \\ & f_i = P\psi_i(ct) \quad (i=1-4) \end{aligned} \tag{9}$$

where L_e represents element length. Elements of **m** and **k** matrices cannot be included here due to space limitations.

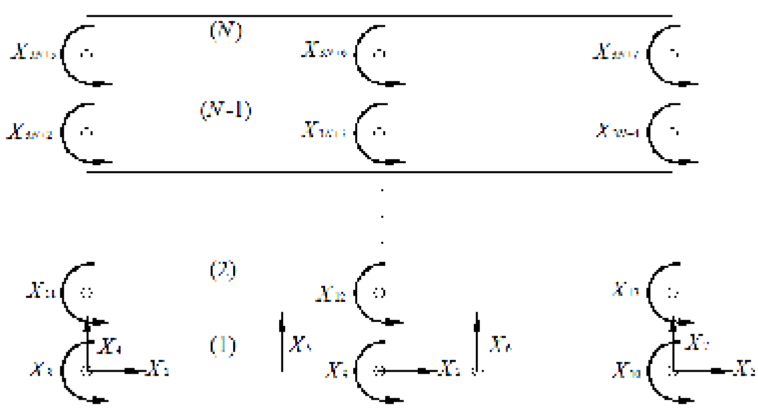

Figure 2 Multilayered beam element

For N-layer beam element as shown in Figure 2, the total number of degrees of freedom is (3N+7). According to Eq. (7), the load-displacement relations for each lamina are

$$\begin{aligned} & \mathbf{f}^{(N)} = \mathbf{m}^{(N)}\ddot{\mathbf{X}}^{(N)} + \mathbf{k}^{(N)}\mathbf{X}^{(N)} \\ & \mathbf{f}^{(N-1)} = \mathbf{m}^{(N-1)}\ddot{\mathbf{X}}^{(N-1)} + \mathbf{k}^{(N-1)}\mathbf{X}^{(N-1)} \\ & \vdots \\ & \mathbf{f}^{(1)} = \mathbf{m}^{(1)}\ddot{\mathbf{X}}^{(1)} + \mathbf{k}^{(1)}\mathbf{X}^{(1)} \end{aligned} \tag{10}$$

where $\mathbf{f}^{(i)}$ denotes the nodal force vector, and $\mathbf{X}^{(i)}$ is a column vector including the local variables of ith lamina as well as the rotational variables of the other laminae between i and N. The following relations are employed to convert the local displacement vector $\mathbf{u}^{(i)}$ for each lamina to $\mathbf{X}^{(i)}$.

$$
\begin{aligned}
&\mathbf{u}^{(N)} = \mathbf{R}^{(N)}\mathbf{X}^{(N)} \\
&\mathbf{u}^{(N-1)} = \mathbf{R}^{(N-1)}\mathbf{X}^{(N-1)} \\
&\vdots \\
&\mathbf{u}^{(1)} = \mathbf{R}^{(1)}\mathbf{X}^{(1)}
\end{aligned}
\tag{11}
$$

where $\mathbf{R}^{(i)}$ has dimension $10\times(10+3N-3i)$ and all $R_{jk}^{(i)}=0$ except $R_{jj}^{(i)}=1$ for j=1-10. The column vector $\mathbf{X}^{(i)}$ has dimension $(10+3N-3i)\times 1$. To convert $\mathbf{X}^{(i)}$ to $\mathbf{X}^{(i-1)}$, the following relations are used.

$$
\begin{aligned}
&\mathbf{X}^{(N)} = \mathbf{T}^{(N-1)}\mathbf{X}^{(N-1)} \\
&\mathbf{X}^{(N-1)} = \mathbf{T}^{(N-2)}\mathbf{X}^{(N-2)} \\
&\vdots \\
&\mathbf{X}^{(2)} = \mathbf{T}^{(1)}\mathbf{X}^{(1)}
\end{aligned}
\tag{12}
$$

where $\mathbf{T}^{(i)}$ has dimension $(10+3N-3i-3)\times(10+3N-3i)$. Elements of $\mathbf{T}^{(i)}$ matrix are given as follows.

$$
\begin{aligned}
&T_{jj}^{(i)}=1 \quad (j=1-7) \qquad T_{j(j+7)}^{(i)}=-h^{(i)}/2 \qquad T_{j(j+10)}^{(i)}=-h^{(i+1)}/2 \quad (j=1-3) \\
&T_{j(j+3)}^{(i)}=1 \quad (j=8-(10+3N-3i-3)) \qquad \text{All others are zero}
\end{aligned}
\tag{13}
$$

The local load vectors in Eq. (10) can be transformed the global ones by using the following relations.

$$
\begin{aligned}
&\mathbf{F}^{(1)} = \mathbf{R}^{(1)^{\mathrm{T}}}\mathbf{f}^{(1)} \\
&\mathbf{F}^{(2)} = \mathbf{T}^{(1)^{\mathrm{T}}}\mathbf{R}^{(2)^{\mathrm{T}}}\mathbf{f}^{(2)} \\
&\vdots \\
&\mathbf{F}^{(N)} = \mathbf{T}^{(1)^{\mathrm{T}}}\mathbf{T}^{(2)^{\mathrm{T}}}\cdots\mathbf{T}^{(N-1)^{\mathrm{T}}}\mathbf{R}^{(N)^{\mathrm{T}}}\mathbf{f}^{(N)}
\end{aligned}
\tag{14}
$$

Combining Eqs. (10), (11), (12) and (14) together gives the final expressions for the stiffness and mass matrices of a multilayered beam element as in the following.

$$
\begin{aligned}
\mathbf{K}^{e} = {}&\mathbf{R}^{(1)^{\mathrm{T}}}\mathbf{k}^{(1)}\mathbf{R}^{(1)} + \mathbf{T}^{(1)^{\mathrm{T}}}(\mathbf{R}^{(2)^{\mathrm{T}}}\mathbf{k}^{(2)}\mathbf{R}^{(2)} + \mathbf{T}^{(2)^{\mathrm{T}}}(\mathbf{R}^{(3)^{\mathrm{T}}}\mathbf{k}^{(3)}\mathbf{R}^{(3)} + \ldots \\
&+ \mathbf{T}^{(N-2)^{\mathrm{T}}}(\mathbf{R}^{(N-1)^{\mathrm{T}}}\mathbf{k}^{(N-1)}\mathbf{R}^{(N-1)} + \mathbf{T}^{(N-1)^{\mathrm{T}}}\mathbf{k}^{(N)}\mathbf{T}^{(N-1)})\mathbf{T}^{(N-2)})\ldots)\mathbf{T}^{(2)})\mathbf{T}^{(1)}
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
\mathbf{M}^{e} = {}&\mathbf{R}^{(1)^{\mathrm{T}}}\mathbf{m}^{(1)}\mathbf{R}^{(1)} + \mathbf{T}^{(1)^{\mathrm{T}}}(\mathbf{R}^{(2)^{\mathrm{T}}}\mathbf{m}^{(2)}\mathbf{R}^{(2)} + \mathbf{T}^{(2)^{\mathrm{T}}}(\mathbf{R}^{(3)^{\mathrm{T}}}\mathbf{m}^{(3)}\mathbf{R}^{(3)} + \ldots \\
&+ \mathbf{T}^{(N-2)^{\mathrm{T}}}(\mathbf{R}^{(N-1)^{\mathrm{T}}}\mathbf{m}^{(N-1)}\mathbf{R}^{(N-1)} + \mathbf{T}^{(N-1)^{\mathrm{T}}}\mathbf{m}^{(N)}\mathbf{T}^{(N-1)})\mathbf{T}^{(N-2)})\ldots)\mathbf{T}^{(2)})\mathbf{T}^{(1)}
\end{aligned}
\tag{16}
$$

The multilayered beam considered here is discretized along its length by using the multilayered beam element described above. The equation of motion of the entire system is

$$
\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \tag{17}
$$

where **F** is nodal force vector of the entire system. Eq. (17) can be solved numerically in an incremental sense by using Newmark's method. It should be noted that force vector contains zeroes at all nodes of the beam except those of the element on which the moving load acts.

NUMERICAL RESULTS

In this section, convergence behavior and accuracy of the present multilayered beam element is investigated by comparing our results with available ones in the literature. In all examples given below, the beam is traversed by a concentrated load with magnitude P = 4.45 N along its length.

As a first example, a simply supported single layer isotropic beam is selected. The beam has length L = 10.16 cm, width b = 0.635 cm and thickness h = 0.635 cm, Young's modulus E = 206.8 GPa, Poisson ratio ν = 0.3, and mass density ρ = 10686.9 kg/m^3 [9]. Table 1 gives first three nondimensional natural frequencies of the selected beam. According to Table 1, results obtained from the present element are in good agreement with the exact solutions. Since the element converges quickly, it is concluded that N_e = 16 elements along the beam length is enough for dynamic analyses.

Table 2 presents dynamic magnification factors for midspan deflections, which can be defined as the ratio of maximum dynamic deflection to static one. Here, the critical speed is defined as $c_{cr} = \omega_1 L / \pi$. Results seen in the last two columns are obtained by using analytical method described in Refs. [17] for Bernoulli-Euler beam theory and [18] for Timoshenko beam theory. According to Table 2, the present element gives satisfactory results compared to the previous works.

Table 1 Convergence and accuracy of nondimensional natural frequencies ($\sqrt{12}\hat{\omega}_n$) of a single layer isotropic beam

Number of elements	1st mode	2nd mode	3rd mode
4	9.819	38.798	86.282
8	9.816	38.600	84.931
12	9.816	38.652	84.849
16	9.816	38.651	84.835
20	9.816	38.651	84.831
Exact [16]	9.870	39.478	88.826

$\hat{\omega}_n = \omega_n (L^2 / h)\sqrt{\rho / E}$

Table 2 Dynamic magnification factors for midspan deflections of a single layer isotropic beam

$\alpha = c / c_{cr}$	v (m/s)	Present	Ref. [14]	Ref. [9]	Ref. [17]	Ref. [18]
0.0625	15.6	1.0568	1.053	1.063	1.0597	1.0470
0.125	31.2	1.1150	1.139	1.151	1.2110	1.1095
0.25	62.4	1.2476	1.267	1.281	1.2580	1.2507
0.375	93.6	1.5827	1.569	1.586	1.5732	1.5614
0.5	124.8	1.7205	1.687	1.704	1.7057	1.7072
0.625	156.0	1.7579	1.711	1.727	1.7312	1.7427
0.75	187.2	1.7199	1.681	-	1.7017	1.7100
1	250.0	1.5519	1.528	1.542	1.5481	1.5461

As a second example, nondimensional fundamental frequencies of laminated beams with various lamina lay-ups are compared to those of Reddy's exact solution according to Timoshenko beam theory [15]. Here, E_1/E_2 = 25, G_{12} = 0.5 E_2, and ν_{12} = 0.3 are selected, and beams have simple end conditions. For multilayered beams, all laminae have the same thickness. As seen in Table 3, results are in good agreement with those of the exact solutions especially for thin beams. For angle-ply laminates, our results show some difference in comparison to the exact ones because the present element does not consider the characteristic couplings between in-plane and out-of-plane responses of anisotropic laminates.

Table 3 Nondimensional fundamental frequencies ($\hat{\omega}_n$) of laminated composite beams with various lamina lay-up

L/h	[0°]		[90°]		[0°/90°/90°/0°]		[45°/-45°/-45°/45°]	
	Present	Ref. [15]	Present	Ref. [15]	Present	Ref. [15]	Present	Ref. [15]
10	11.635	11.635	2.810	2.771	10.929	10.488	3.196	3.663
20	13.430	13.430	2.839	2.829	12.595	12.434	3.258	3.739
100	14.210	14.210	2.849	2.848	13.330	13.334	3.278	3.765

$\hat{\omega}_n = \omega_n (L^2 / h)\sqrt{\rho / E_2}$

Table 4 Dynamic magnification factors for midspan deflections of [0°/90°/90°/0°] symmetric cross-ply elastic beam with simple supports

$\alpha = v / v_{cr}$	*v* (m/s)	Present	Ref. [12]	Ref. [9]
0.0625	15.6	1.0401	1.063	1.063
0.125	31.2	1.0888	1.149	1.151
0.25	62.4	1.2062	1.280	1.281
0.375	93.6	1.5730	1.586	1.586
0.5	124.8	1.6925	1.710	1.704
0.625	156.0	1.6964	1.726	1.727
0.75	187.2	1.6522	-	-
1	250.0	1.4902	1.542	1.542

For further validation purpose, moving load-induced deflection behavior of a four-layer [0°/90°/90°/0°] symmetric cross-ply laminated composite beam with simple ends is considered as a third example. All laminae have the same thickness. Material properties are E_1 = 144.8 GPa, E_2 = 9.65 GPa, G_{12} = 4.136 GPa, ν_{12} = 0.25, and ρ = 1389.297 kg/m^3. The beam length L = 10.16 cm, width b = 0.635 cm and thickness h = 0.745 cm are considered [9]. Table 4 gives dynamic magnification factors for midspan deflections of the considered beam. As seen in the table, the present element gives acceptable results compared to the previous works.

CONCLUSION

A multilayered finite element for dynamic analysis of laminated composite beams is presented. This element allows separate rotational degrees of freedom for each lamina but does not require any additional axial and transversal degrees of freedom beyond those necessary for a single lamina. Comparisons between the results obtained by proposed element with available results in the literature show good agreement. This element can be further applied to buckling problems of laminated composite beams.

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