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The Assessment Of Slopes Stability With Probability Methods

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ABSTRACT

This paper aims to deal with the assessment of the stability of slopes, taking in consideration several methods for analysis:

- the limit equilibrium methods such as the Infinitive Slope method, Bishop's simplified method etc. upon which assumptions are made on the considered failure surface and the internal forces interacting between the blocks, the equilibrium equations to be satisfied, the unknown variables to be determined (such as the safety factor and normal force) and the areas within which these methods could apply.

- the probability methods in which the coefficient of variation of the factor of safety (COV_F) is determined, (as the ratio of standard deviation to the mean coefficient of the safety factor, previously calculated), reliability R (as the probability of a certain case occurring in a certain way, at a certain time, in given conditions), the probability of failure P_f(as 1-R).

Following the theoretical analysis, a number of numerical examples are included in order to illustrate in a more explicit manner the methodology used for the application of the assessment methods mentioned above, as well as to highlight the particularities and differences of each method.

Keywords: *safety factor, resistance parameters, reliability, coefficient of variation of the factor of safety, probability of destruction.*

INTRODUCTION

The field of slope stability encompasses several analyses. The analysis and calculation can be computed via the methods of limiting equilibrium or probability methods. The analysis of slope stability using the methods of limiting equilibrium requires the laboratory parameters in order to calculate the safety factor Fsig (the only factor that can evaluate the slope stability). Probability analysis and reliability take into consideration the change in the physical and mechanical parameters of the soil during the control or design phase. This article aims to deal with 3 topics: the methods of limiting equilibrium, probability methods in slope stability and solved examples using the classic method and the probability method. The scope of this paper is the evaluation of the applicability of the probability methods in the determination of the stability of slopes.

THE LIMIT STATE EQUILIBRUM METHODS (BISHOP'S SIMPLIFIED METHOD, JANBU'S SIMPLIFIED METHOD (GPS) AND MORGENSTERN & PRICES METHOD)

In this article we will present the methods of limiting equilibrium for calculating and analyzing the stability of a given slope. For each method we see fit to highlight some the key points:

a) The safety factor F which can be defined as the ratio of shear strength of the soil to the shear stresses.

$$F = \frac{S}{\tau} \qquad (1)$$

Where:

S = shear strength of the soil

τ = shear stress of the soil.

According to these methods it is calculated by the formula given below:

$$F = \frac{\sum \left[\dfrac{c' \cdot \Delta l \cdot \cos\alpha + (W - u \cdot \Delta l \cdot \cos\alpha) \cdot tg\varphi'}{\cos\alpha + \left(\dfrac{\sin\alpha \cdot tg\varphi'}{F} \right)} \right]}{\sum W \cdot \sin\alpha} \qquad (2)$$

Where:

F = safety factor

c' =internal effective cohesion of the soil

ϕ' = internal effective friction angle of the soil

α = the inclination angle

W = self weight of the block

Δl = the length of the calculated element

u = water pore pressure

b) The assumptions regarding the failure surface (the failure surface is considered the surface with the minimal safety factor) and the internal forces interacting between the blocks: Bishop's simplified method assumes a circular failure surface and the horizontal forces between the slices are taken into account. Janbu's method uses the positioning line. It is the imaginary line drawn between the points where the internal forces act E (or Z). The Morgenstern & Price's method considers the existence of shear forces between the slices and the horizontal forces relate to them as follows:

$$X = \lambda \cdot f(x) \cdot E \qquad (4)$$

Ku:

X and E = vertical and horizontal forces between the blocks
λ = unknown factor
$f(x)$ = the function that represents the value of every limit block

c) The equilibrium equations to be satisfied: It is satisfied the equilibrium of the moments related to the circle's center for each block foe each method.

d) The applicability of each method: These methods are used for inhomogeneous slopes where the resistant parameters are: $c, \varphi \neq 0$ and the failure surface can be circular and also for every kind of slope in different types of soils.

PROBABILITY METHODS AND RELIABILITY

Reliability R is an alternative parameter that measures stability in regard to the uncertainty of the analysis for determining the stability of slopes. Hence, the calculated probability to avoid failure of the slope is:

$$R = 1\text{-}Pf \qquad (5)$$

Where:
P_f = probability of destruction
R = reliability.

Reliability of a slope is the probability of the slope to be in equilibrium under the influence of different factors. In order to calculate the reliability we have to initially determine: the factor of safety and the coefficient of safety variation which can be determined by calculating the standard deviations of the parameters that influence the stability of the slope. Furthermore, using Taylor's Numerical Method, the standard deviations and coefficient of variation can be determined as follows:

$$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \ldots + \left(\frac{\Delta F_n}{2}\right)^2} \qquad (6)$$

$$COV_F = \frac{\sigma_F}{F_{MLV}} \qquad (7)$$

Where:
$\Delta F_1 = F_1^+ - F_1^-$
$F_1^+ \rightarrow$ The safety factor calculated with the value of the first parameter increased with a standard deviation.

$F_1^- \rightarrow$ The safety factor calculated with the value of the first parameter decreased with a standard deviation.
F_{MLV} = approximate value of the safety factor using the approximate values of all the parameters.
σ_F = standard deviation of the safety factor.
COV_F = coefficient of variation of the factor of safety.
If we calculate F_{MLV} and COV_F we can define the probability of destruction P_f using table 1.

Table 1 Probability of destruction

COV_F coefficient of variation of the factor of safety									
F_{MLV} [a]	10%	12%	14%	16%	20%	25%	30%	40%	50%
1.05	33.02	36.38	38.95	41.01	44.14	47.01	49.23	52.63	55.29
1.10	18.26	23.05	26.95	30.15	35.11	39.59	42.94	47.82	51.37
1.15	8.83	13.37	17.53	21.20	27.2	32.83	37.10	43.24	47.62
1.20	3.77	7.15	10.77	14.29	20.57	26.85	31.76	38.95	44.05
1.25	1.44	3.54	6.28	9.27	15.20	21.68	26.98	34.95	40.66
1.30	0.49	1.64	3.49	5.81	11.01	17.3	22.75	31.26	37.48
1.35	0.15	0.71	1.86	3.53	7.83	13.66	19.06	27.88	34.49
1.40	0.04	0.29	0.95	2.08	5.48	10.69	15.88	24.8	31.7
1.50	0	0.04	0.23	0.67	2.57	6.38	10.85	19.49	26.69
1.60	0	0.01	0.05	0.2	1.15	3.71	7.29	15.21	22.4
1.70	0	0	0.01	0.06	0.49	2.11	4.84	11.81	18.75
1.80	0	0	0	0.01	0.21	1.18	3.18	9.13	15.67
1.90	0	0	0	0	0.08	0.65	2.07	7.03	13.08
2.00	0	0	0	0	0.03	0.36	1.34	5.41	10.91
2.20	0	0	0	0	0.01	0.1	0.56	1.19	7.59
2.40	0	0	0	0	0	0.03	0.23	1.88	5.29
2.60	0	0	0	0	0	0.01	0.09	1.11	3.7
2.80	0	0	0	0	0	0	0.04	0.66	2.6
3.00	0	0	0	0	0	0	0.02	0.39	1.83

The index of reliability is denoted with the symbol β and it is a varying parameter of safety (reliability) that is related to the probability of failure and indicates the number of standard deviations F = 1 and F_{MLV} as shown below.

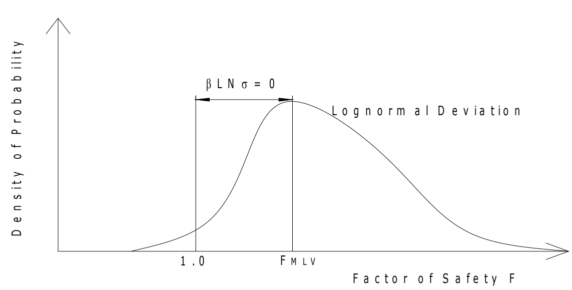


Fig 3

Parameter β is also related to the probability of destruction and the reliability. The lognormal reliability index β_{LN} is defined by F_{MLV} and COV_F using the following equation:

$$\beta_{LN} = \frac{\ln\left(\frac{FMLV}{\sqrt{1 + COV_F^2}}\right)}{\sqrt{\ln\left(1 + COV_F^2\right)}} \quad (8)$$

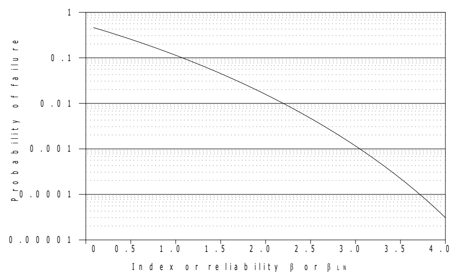


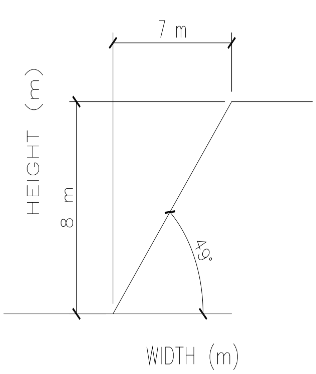
Fig.4

The relation between β and P_f shown in the fig.4 is the lognormal distribution function. The corresponding value of P_f for a given value of β can be calculated using Excel. Below the "Insert Function" option,"Statistical" we can choose NORMSDIST and the value of β_{LN}. The result obtained is the value for reliability R.

APPLICATIONS

A) Limiting equilibrium methods (Bishop, Janbu, Morgenster & Price)

For the slope in fig.5 the factor of safety is calculated using GEO STUDIO 2007, SLOPE/W programme, which gives these values of the safety factor according to the methods above:



Morgenstern&Price F = 1.413
Bishop F = 1.414
Janbu F = 1.390

Fig.5

B) Probability method

Probability of destruction must be determined for the slope in the first application using the geotechnical parameters which are given in the table 2:

Table 2 The results of the laboratory tests

Depth (m)	φ	c (kPa)	γ
1	21	17	17.1
2	19	20	18.9
3	19	21	17
4	21	19.5	18
5	17	18	19
6	23	19.5	19.5
7	20	18	16.5
8	20	19	18

The standard deviations for the parameters given in the table 2 are:

$\sigma_\phi = 2.75^\circ$
$\sigma_c = 1.44$ KPa
$\sigma_\gamma = 0.9\ kN/m^3$

$F_{MLV} = 1.413$

The coefficient of variation of the factor of safety is the ratio of the standart deviation to the value of a variable, which is taken as the mean value:

COVϕ = 14%
COV c = 8%
COV γ = 6%

Table 3 The calculation of ΔF

Variable	Value	F	ΔF
φ	$\varphi + \sigma_\varphi = 22.75^0$	$F_1^+ = 1.499$	0.169
	$\varphi - \sigma_\varphi = 17.25^0$	$F_1^- = 1.330$	
c	$c + \sigma_c = 20.44 kPa$	$F_2^+ = 1.477$	0.128
	$c - \sigma_c = 17.56 kPa$	$F_2^- = 1.349$	

γ	$\gamma + \sigma_\gamma = 18.9 kN/m^3$	$F_3^+ = 1.373$	0.085
	$\gamma - \sigma_\gamma = 17.1 kN/m^3$	$F_3^- = 1.458$	

The standard deviation of the safety factor is: $\sigma_F = 0.114$
The coefficient of variation of the factor of safety is: $COV_F = 8\%$
Reliability index β_{LN} is: $\beta_{LN} = 4.29$

Excel calculation: $\beta_{LN} = 4.29$, R=0.99999 which correspond to $P_f = 0.00001$. The stability of the slope is guaranteed.

CONCLUSIONS

1. The probability methods represent a modern methodology which can be successfully applied for the assessment of the stability of slopes.

2. The reliability and probability of failure can be easily determined by relating to the factor of safety (F_{MLV}) and the coefficient of variation for the factor of safety (COV_F).

3. The event with a specific probability does not necessarily represent a catastrophic failure, but the probability of failure illustrates more significantly the nature and the aftermath of a failure.

4. The numerical examples demonstrate once more the fact that a failure probability of about 1% (as recommended in books) would guarantee a firm stability of the slope.

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