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Comparison of Probability Based Design and Eurocode 7 in Slope Stability Analysis

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ABSTRACT

Since it is recently noticed that probability methods are being a vital part of the engineering design, in this article are taken into account two different methods such as Probability Based Design (PBD) and Eurocode 7. Both of these methods are based upon reliability design but each of them expresses this concept in different ways.

In Probability Based Design, reliability (R) is strictly related to probability of failure (P_f). So, to define the value of reliability using Probability Based Design, it is initially needed to assess the value of probability of failure through statistical calculations. Reliability calculations provide a means of evaluating the combined effects of uncertainties and a means of distinguishing between conditions where uncertainties are particularly high or low.

Eurocode 7 is a commonly used method which introduces the reliability design concept in calculations through the usage of the partial coefficients (γ) using three design methods.

To compare these methods, in this article is represented the detailed calculation of two case studies related to slope stability analysis using each of the methods separately. The results drawn by these methods are then combined together and interpreted in order to retrieve the conclusions. Both of these methods are found to be able to give satisfying results.

Keywords: *partial factors; reliability; probability of failure; coefficients of variation; reliability index;*

INTRODUCTION

Frequent incidences of slope movement are observed throughout the world. The occurrence of landslides in the area depends on a complex interaction of natural as well as human induced factors. In recent years much progress has been made in the development of a satisfactory understanding of the behaviour of soils, but many features are still only partially understood.

This paper presents a review of two methods which have been proposed for analyzing the stability of slopes, with comparisons of the results they furnish; Probability Based Design (PBD) and Eurocode 7. These methods which appear to give consistent and reliable results have been studied in considerable detail, and complete mathematical solutions have been obtained.

The examples which are considered in the third part of the paper are only hypothetical case – studies.

RELIABILITY AND FACTORS OF SAFETY

General

This method introduces the concept of reliability "R" in slope stability analysis. The reliability of a slope (R) can be defined as a way of measuring the stability that takes into account all the uncertainties involved in the process. The reliability of a slope is equal to the probability that a slope will not fail and it can be calculated as 1.0 minus the probability of failure:

$$R = 1 - P_f \tag{1}$$

Although R and P_f are of the same importance as the factor of safety (FS) in characterizing slope stability, the latter is more widely used from the designers. In this paper is presented the simplest way of reliability analysis which does not require special knowledge or efforts to be applied.

Standard deviation and coefficients of variation

Let us consider a test which is done to measure one of the soil properties. If we take a close look to the values obtained from the test, can be easily distinguished that they are characterized by a scatter, but there is no any systematic variation of the values. [1] This change between values is due to natural variation, amount of sample disturbance etc. In order to define the scatter of a variable, a quantitative measure is introduced such as the standard deviation which can be calculated using the formula [1]

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{1}^{N}\left(x - x_{av}\right)^2} \tag{2}$$

where σ is the standard deviation, N the number of measurements, x the measured variable and x_{av} the average value of x.

The coefficient of variation is a very important and suitable dimension of uncertainty in the value of the variable. [1] It can be calculated as the ratio of the standard deviation and the most expected value of the variable, which, for practical cases is accepted to be the average value.

$$COV = \frac{\sigma}{\text{average value}} \tag{3}$$

COV of factor of safety and the reliability index

In order to calculate the reliability (R) and probability of failure (P_f) it is required to estimate firstly the factor of safety and the coefficient of variation of factor of safety (COV_F). The factor of safety can be calculated as usual, using a computer or by hand calculations. In this paper the factors of safety are retrieved from the commercial software GEOSLOPE SLOPE/W. After this, it can be estimated the value of the coefficient of variation of factor of safety using Taylor series method. To apply this method, first of all we need to estimate the standard deviations of all the quantities included in the slope stability process and then to use these formula

$$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \ldots + \left(\frac{\Delta F_n}{2}\right)^2} \quad (5)$$

$$COV_F = \frac{\sigma_F}{F_{MLV}} \quad (6)$$

where $\Delta F_n = \left(F_n^+ - F_n^-\right)$, F_n^+ is the value of the factor of safety calculated with the value of the n –th parameter plus one standard deviation from its most likely value, and F_n^- is the value of the factor of safety calculated with the value of the n – th parameter minus one standard deviation. [1] Meanwhile calculating F_n^+ or F_n^- the other variables are kept at their most likely value. After these assessments the value of σ_F and COV_F can be easily retrieved. Using these two values the value of the probability of failure can be determined according to the graph represented in Figure 1.

On the other hand the probability of failure P_f is strongly related to the *lognormal reliability index* (β) which is determined using the formula

$$\beta_{LN} = \frac{\ln\left(F_{MLV} / \sqrt{1 + COV_F^2}\right)}{\sqrt{\ln\left(1 + COV_F^2\right)}} \quad (7)$$

Values of P_f that corresponds to the calculated β_{LN} values can be estimated using the "NORMSDIST" function in Microsoft Excel. The function returns the value of R, but using equation (1) we can obtain P_f.

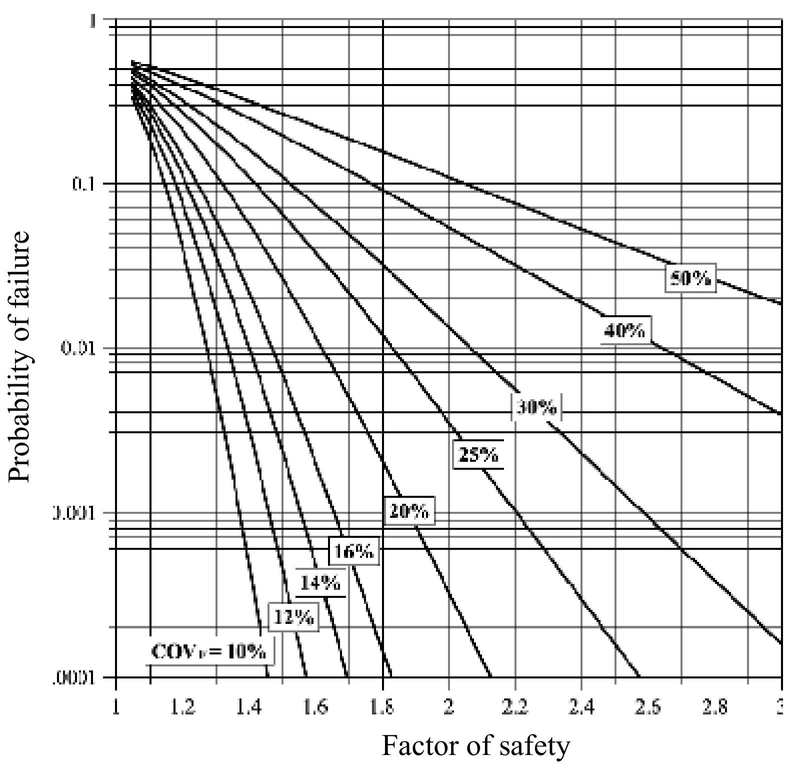


Figure 1 Relation between FS and P_f

SLOPE STABILITY ANALYSIS ACCORDING TO EUROCODE 7

The design of slopes using Eurocode 7 is done considering two limit states: (1) Ultimate limit states (ULS) and (1) Serviceability limit states (SLS). These limit states may involve loss of overall stability of the ground and associated structures, excessive movements and loss of serviceability.

The ultimate limit state can be further divided into two types of limit states:

a) GEO – type limit states, where failure occurs in the ground only [2]
b) STR – type limit states, with combined failure or large movement in ground and in any structural element [2]

Serviceability limit state includes excessive movements in the ground, vibration, or heave in the supported structures. In case of *natural slopes* it is recommended to avoid the occurrence of SLS. The serviceability limit state design is ignored because it goes beyond the scope of this paper.

Ultimate limit state design

The stability analysis of a slope should be performed using the factored values of material properties and resistances. The design approaches and the partial factors for this purpose are stated in Eurocode 7.

The case studies presented in this paper need to be checked only for the GEO – limit state and in such case, it is recommended to use only Design Approach – 1(DA – 1) combination 2. Even it is difficult to be applied, Design Approach – 2 (DA – 2) is also used for stability analysis selecting the partial factors as suggested in "Designers guide to Eurocode 1997 – 1" (R. Frank, C. Bauduin and others, 2004).

Since Design Approach – 1 is not relevant in our cases and Design Approach – 3 gives the same result as Design Approach – 1, they are not taken into account. The recommended partial factors for DA – 1 combination 2 and DA – 2 are shown respectively in Table 1 and Table 2.

Table 1 Partial factors used for Design Approach 1 Combination 2

Actions and effects		Material properties		Resistance	
γ_G	1.00	$\gamma_{\varphi'}$	1.25	γ_{Re}	1.00
γ_Q	1.30	$\gamma_{c'}$	1.25	-	-
-	-	γ_γ	1.00	-	-

Table 2 Partial factors used for Design Approach 2

Actions and effects		Material properties		Resistance	
γ_G	1.00	$\gamma_{\varphi'}$	1.00	γ_{Re}	1.10
γ_Q	1.00	$\gamma_{c'}$	1.00	-	-
-	-	γ_γ	1.00	-	-

In the calculations is also included the concept of the over – design factor (ODF) which can be estimated with the formula

$$ODF = \frac{F}{\gamma_G \gamma_{\mathrm{Re}}} \tag{8}$$

where

- F – factor of safety
- γ_G – partial factor for permanent unfavorable actions. The recommended value for DA – 1 combination 2 is $\gamma_G = 1.00$ and for DA – 2 is $\gamma_G = 1.35$ [2]
- γ_{Re} – partial resistance factor

Based on the values of ODF we can judge if the considered slope is over designed or not.

CASE STUDY

Using reliability analysis

In Figure 2 is presented the geometry and the soil profile of the considered slope, and also the underground water level. Meanwhile in Table 3 are presented the effective values of cohesion (c'), friction angle (φ') and unit weight of soil from different measurements carried out in the site.

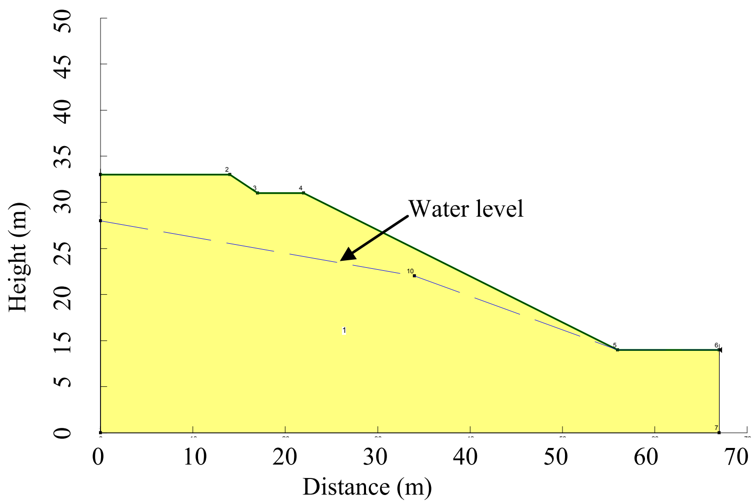


Figure 2 Geometry and soil profile of slope 1

Table 3 Soil properties

φ' (°)	c' (kPa)	γ (kN/m³)
24.0	6.9	16.0
24.9	7.8	16.0
26.3	9.8	16.0
26.6	11.8	16.0
27.2	12.8	16.0
27.5	13.7	16.0
27.7	-	-
28.1	-	-
29.2	-	-
29.4	-	-

The first step of the calculation process is estimating the standard deviations of the soil properties using the formula number (2) stated above in the text. Doing so, we obtain $\sigma_{c'} = 2.75$ and $\sigma_{\varphi'} = 1.72$. After this, as it is mentioned earlier in the paper we input these values into the SLOPE/W software in order to obtain the factors of safety (F). The procedure is simple and clear, we add one standard deviation to the average value of one parameter while all the other parameters are kept at their average value (e. g. we input to the software $c' = c'_{avg} + \sigma_{c'}$, $\varphi' = \varphi'_{avg}$ and $\gamma = \gamma_{avg}$) and we get the factor of safety (F^+) through the software analysis. We follow the same steps but now we subtract the value of the standard deviation from the average value of the parameter and so it is obtained the other value of the factor of safety (F^-). The calculations and results are presented further in Table 4.

Table 4 Values of factors of safety and ΔF_n using the procedure above

Soil parameter	Value	Factor of safety (F)		$\Delta F = \left(F^+ - F^-\right)$
φ' (°)	$\varphi' + \sigma_{\varphi'} = 28.81$	F^+	1.165	0.116
	$\varphi' - \sigma_{\varphi'} = 25.37$	F^-	1.049	
c' (kPa)	$c' + \sigma_{c'} = 13.22$	F^+	1.186	0.166
	$c' - \sigma_{c'} = 7.711$	F^-	1.020	

Pursuant to the calculations using the formulas (5), (6) and (7) we derive $\sigma_F = 0.101$, $COV_F = 0.091$ and $\beta_{LN} = 1.068$. Via the Excel function "NORMSDIST", from the value of β_{LN} we get $R = 0.85724$ and substituting R in formula (1) the result is $P_f = 0.14276$.

Finally refereeing to the plot shown in Figure 1 and with the known values of P_f and COV_F is retrieved the factor of safety F = 1.11

For the next case study we will introduce only the slope geometry, soil properties and the results in tabular form.

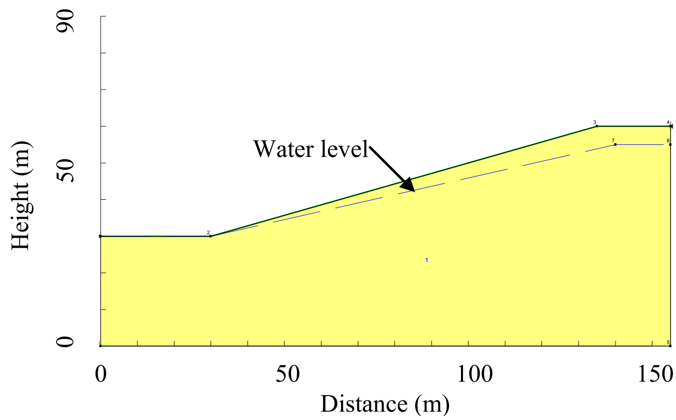


Figure 3 Geometry and soil profile of slope 2

Table 4 Soil properties

φ' (°)	c' (kPa)	γ (kN/m^3)
21.99	34.3	16.3
23.84	32.4	16.3
28.01	27.5	16.6

Table 5 Calculations results for slope number 2

σ_F	COV_F	β_{LN}	R	P_f	F
0.416	0.269	1.515	0.935	0.065	1.51

Using Eurocode 7

The examples demonstrated above, will be solved in this section of the paper according to the recommendations of Eurocode 7. The procedure consists of two steps.

The first one is factoring all the soil parameters, actions or effects of actions and resistances that will affect the analysis process using the partial factors introduced in Table 1 and Table 2, respectively for DA – 1 combination 2 and for DA – 2.

The second step concerns of inputting these values in the software and getting the factor of safety after running the analysis.

The calculations and results for both considered slopes will be presented below in tabular form, separately for DA – 1 combination 2 and for DA – 2.

Table 6 Input parameters and results for slope 1 according to DA – 1 combination 2

Input parameters			Analysis results	
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right)$	$c_d^{'} = \frac{c_k^{'}}{\gamma_{c'}}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma}$	F	$ODF = \frac{F}{\gamma_G \gamma_{\mathrm{Re}}}$
22.25^{o}	8.372 kPa	16 kN/m^3	0.885	0.885

Table 7 Input parameters and results for slope 1 according to DA – 2

Input parameters			Analysis results	
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right)$	$c_d^{'} = \frac{c_k^{'}}{\gamma_{c'}}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma}$	F	$ODF = \frac{F}{\gamma_G \gamma_{\mathrm{Re}}}$
27.09^{o}	10.466 kPa	16 kN/m^3	1.106	0.744

Table 8 Input parameters and results for slope 2 according to DA – 1 combination 2

Input parameters			Analysis results	
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right)$	$c_d^{'} = \frac{c_k^{'}}{\gamma_{c'}}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma}$	F	$ODF = \frac{F}{\gamma_G \gamma_{\mathrm{Re}}}$
20.16^{o}	25.12 kPa	16.34 kN/m^3	1.233	1.233

Table 9 Input parameters and results for slope 2 according to DA – 2

Input parameters			Analysis results	
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right)$	$c_d^{'} = \frac{c_k^{'}}{\gamma_{c'}}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma}$	F	$ODF = \frac{F}{\gamma_G \gamma_{\mathrm{Re}}}$
24.65^{o}	31.4 kPa	16.34 kN/m^3	1.546	1.041

RESULTS AND DISCUSSIONS

In this part of the paper are introduced the results obtained from the above calculations for slope 1 and slope 2 plotted in Figure 4 and Figure 5 respectively.

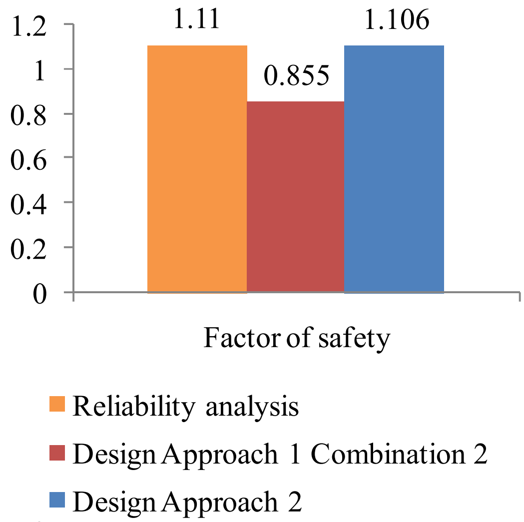
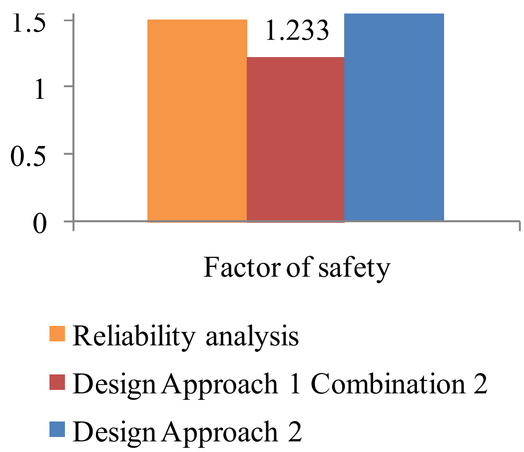


Figure 4 Factors of safety for slope 1

Figure 5 Factors of safety for slope 2



If we take a look to Figure 4 and 5 it is very clear that Reliability analysis and Design approach 2 provide us almost the same values of factor of safety (*F*), on the other hand Design approach 1 Combination 2 gives always lower values of *F*. The latter happens because DA – 1 Combination 2 considers a higher level of uncertainty related to the design values of soil parameters, it means that bigger partial factors are used in this method.

CONCLUSION

1. Both methods are strongly affected by the designer's experience.
2. The best way to judge on the stability of a slope is by using both methods because they seem to complement each – other.
3. In cases that are available several data regarding the soil parameters, reliability analysis is more appropriate for usage in order to obtain more realistic results.
4. Since it is very difficult to distinguish between the favorable and unfavorable actions affecting slope stability it becomes difficult to apply the partial factors appropriately so, it is evident that the presented reliability analysis is more suitable for usage.

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REFERENCES

[1] J. Michael Duncan and Stephen G. Wright (2005) Soil strength and slope stability. *John Wiley & Sons, Inc.,*USA.

[2] R. Frank, C. Bauduin and others (2004) Designer's guide to EN 1997 - 1. *Thomas Telford Publishing*., London, Great Britain.

[3] Eurocode 7: Geotechnical design (2004). *European Committee for Standardization.,* Brussels, Belgium.

[4] A. Bond, A. Harris (2008) Decoding Eurocode 7. *Taylor & Francis e – Library*., Great Britain.