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Probabilistic Modeling of Structural Forces

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ABSTRACT

Since forces acting on structures fluctuate widely with time and space during the lifetime of a structure, variations of the forces should be considered by probability distributions. Probabilistic definition of forces is expressed by random field variables including stochastic parameters. Structural forces are simulated by adopting Normal and Gamma probability distribution functions. The basic model given by JCSS (Joint Committee on Structural Safety) code principles is used as model to take into account the variations. In the simulation of the live loads comprised of sustained and intermittent loads, time intervals are assumed to follow a Poisson process and their distributions are defined by exponential distributions. The simulated loads are evaluated in terms of percentiles, correlation effects, reduction factors and extreme values. Results are compared with those of deterministic model as well. It has been observed that probabilistic model is more realistic and the results can be used in the calculation of specific fractiles like load and resistance factor design.

Keywords: *Field variable, sustained, intermittent, reduction factor, extreme values.*

1. INTRODUCTION

Loads acting on structures vary with time during the life-time of a structure as well as structural properties (rigidity, strength and etc.). The main purpose of a structural design is to provide serviceability during its lifetime for considered performance level under effects of probable forces. Structural loads and resistances are non-deterministic variables and they vary with time and space. The uncertainties in the structural loads such as live loads, earthquakes and wind effects display larger variations than those of the structural characteristics such as strength, stiffness and dimensions of sections. In traditional designs, these loads are taken into consideration as deterministic parameters by safety factors. However, the uncertainties obstruct to determine exact response of a structural system. In fact, the variability in the load effects requires probabilistic models to reveal more realistic behavior and therefore the safety of a structure can be provided by probabilistic definitions depending on degree of the uncertainty. In reliability analysis of a nuclear reactor, seismic and climatic effects were modeled by uniform Poisson process and load intensities were evaluated by extreme value distributions (Schueller, 1973). Ditlevsen (1988) modeled the unit weight of the material and boundary of the body by random field variables. He suggested a stochastic model for self-weight in terms of variances and means by linearization technique. For modeling of the uncertain boundary, random vector-field was used in the stochastic model. In the determining of the probability distribution function of the extreme values, time-invariant loads were idealized by various Poisson process for the combination of dead and live loads for floors. Load intensities were modeled by Gamma distributions and new combination rule was suggested (Floris, 1998). Researches of probabilistic load models were initially realized over field and load surveys by Ellingwood and Culver (1977), McGuire and Cornell (1974), Peir and Cornell (1973), Hasofer (1968). Later, the studies were performed by many other researchers such as Pearce and Wen (1984), Corotis and Sheehan (1986), Ditlevsen and Madsen(2007).

In this study, probabilistic procedures are used to obtain random intensities of the dead and live loads. For this purpose, probabilistic modeling of variations in the loads is defined by stochastic field parameters. The probabilistic loads are simulated by developed computer algorithms in MATLAB complier. Simulations of probabilistic loads are realized automatically in accordance with proper probability distribution of considered stochastic model. All structural loads are modeled depending on "Joint Committee on Structural Safety" (JCSS) probabilistic model code principles.

2. PROBABILISTIC MODELS FOR STRUCTURAL LOADS

Characteristics of actions like intensity and duration generally vary with time and space during the life-time of a system. In a specific time interval, large changes in the actions occur with short returning periods. In terms of the rate of variation, structural loads may be simply categorized by two types of action: permanent and variable actions. Dead loads and deformations are permanent actions that contain small variations with low rates of variation. Impact effects, wind velocities, ocean waves and earthquake motions are variable actions that have much more variability than the permanent actions. Large variations can not be accurately determined by considering deterministic models at the beginning of design; and they need to be evaluated by probabilistic approximations. Typical load models to be modeled as stochastic process are given in Figure 1. Time intervals of the load processes, should be handled with the nature of load occurring by a proper distributions.

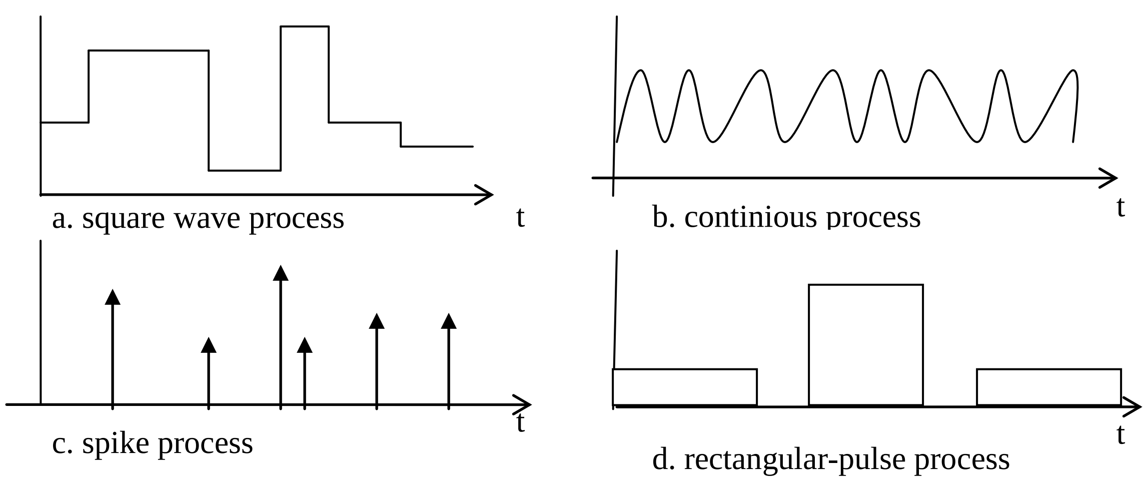


Figure 1. Load process varying with time

A general type of load is defined by various parameters such as duration, location and direction variables etc. For probabilistic analyses, all variables related to actions are adopted as random process. The intensity function of a structural action can be evaluated by basic action variables and field variables. An action variable denotes either a pure effect or combination forces acting on the structure. The model of the action may include many variables with their characteristic models. An action model can de idealized by the variables in two categories (JCSS),

$$P=\xi(P_b, W) \tag{1}$$

where ξ is generally a simple product function, P_b is time dependent variable related to external action and W is time-invariant variable stating transition factor between P_b and P variables. The variables in an action model basically can be defined by stochastic fields / variables and deterministic functions.

3. DEAD LOAD MODEL

Dead loads are mainly related the self weight of structural and other elements. Variations in time and space are very low and therefore, their probability of occurrence is close to 1 for any point and time-instant. Variations with time in these parameters, of course, can be neglected. Most of dead loads have much less variability (variation coefficient %1-10) during the lifetime in comparison with others such as occupancy loads or external actions. They are known as permanent actions and the main reason of the uncertainty arises from variability in unit weight values and sizes of dimension. Stochastic definition of dead load can be expressed by a field random variable including spatial variability and weight density parameters. Ditlevsen (1987) presented a relation integrated with influence function for considering the self-weight effect. A random field model is defined as a random function of the spatial coordinates at a certain instant. The basic model given by JCSS code for the self weight may be used as model to take into account the gravity effects over a volume V. The self weight effect is,

$$G = \int_V \rho dV \tag{2}$$

where ρ is the unit volume weight. The random intensities of dead load are usually modeled with Gaussian distribution. The variation parameters of the self-weight are used of high strength concrete in this study given by Table 1 for the some concrete material. Load combination model for dead and live load is shown in Figure 5.

Table 1. Variation parameters for self-weight

Concrete type	**Mean value (kN/m^3)**	**Coefficient of variation**
Ordinary*	24	0.04
High strength	24-26	0.03

Variatons in dimensions are considered as time-independent variables by statistical parameters. For a dimension h, deviations from actual size are defined by:

$$D=h-h_n \tag{3}$$

h_n is the nominal value. When the nomimal value is lower than 1000 mm, then the first two moments of the variations are expected as,

$$\mu_s = 0.003\ h_n \leq 3\ \text{mm} \qquad (4)$$

$$\sigma_s = 4\ \text{mm} + 0.006\ h_n \leq 10\ \text{mm} \qquad (5)$$

For a beam element (with T section), dead loads are simulated depending on presented parameters of variation. Dimensions are assumed to follow lognormal distribution. The exceedance probability functions of a beam element (on the 2 nd floor) are given in Figure 2 in terms of daily and annual extreme values. From figures it can be understand that with decreasing exceedance probabilities, the increasing in expected load intensities becomes faster for annual extreme model. In Table 2, variances and some fractile values of the expected values are compared for beam elements of the example frame. There aren't significant differences among the loads of beam elements in terms of statistic values. Thus, the small variations in dead loads that can be neglected are proved by simulated random load values and calculated statistical values.

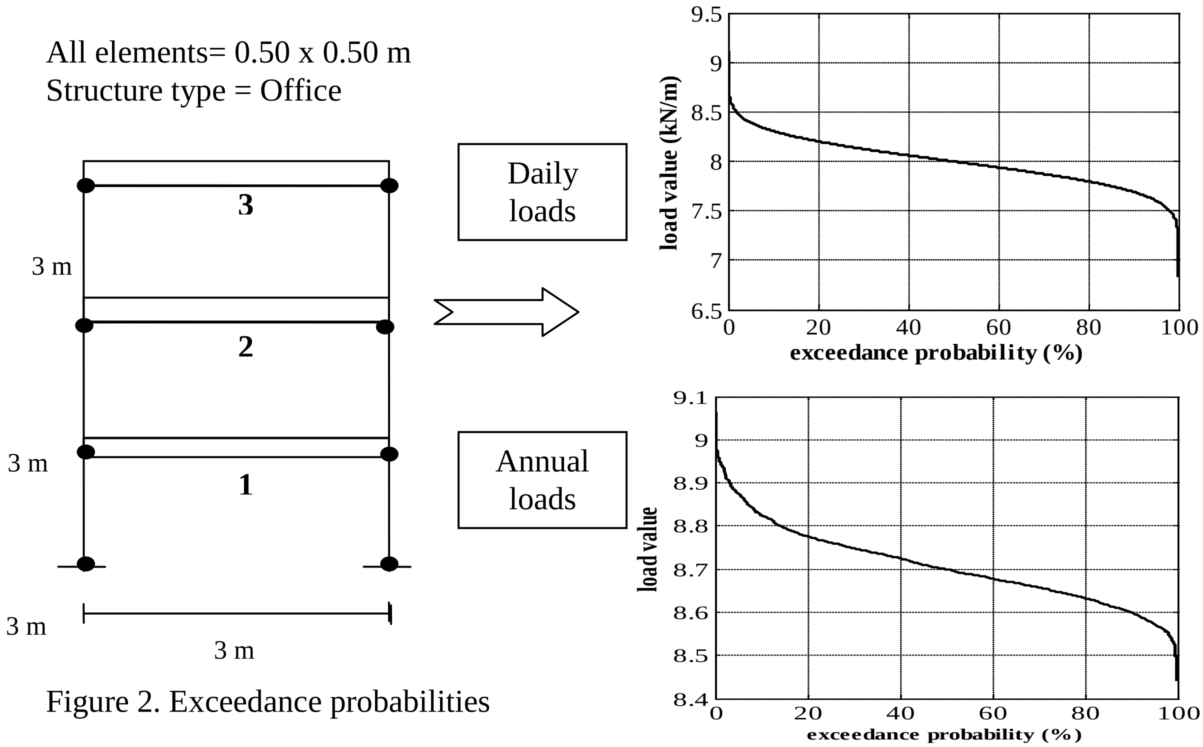


Figure 2. Exceedance probabilities

. Table 2. Statistical parameters for daily and annual extreme self-weight loads

	Variance	Fractiles -daily		Variance	Fractiles-annual	
Element	σ_x^2	%50	%98	σ_x^2	%50	%98
1	0.0577	7.206	7.441	0.0078	8.523	8.531
2	0.0575	7.383	7.440	0.0080	8.515	8.533
3	0.0577	7.379	7.438	0.0086	8.522	8.534

Variances: (kN/m)2, Fractile values: kN/m

4. LIVE LOAD MODEL

The live loads consist of self-weight of objects and all living creatures on floors and they have more randomness than dead loads. Live loads usually have various values depending on building types (hospital, school and etc.) and they can be categorized with respect to their returning periods, magnitudes or probability of occurrence as well. A general live load process is comprised of sustained and intermittent load process (see Fig. 3). The sustained loads consist of weight massive household goods (wardrobe, refrigerator etc.) with long returning periods. The duration of this load type vary depending on the occupancy of a tenant. The intermittent loads include special load cases with short returning periods and their uncertainties come from short-term fluctuations, such as replacement of equipments, gathering humans, exceptional cases and etc. They usually happen in the way of concentrated load. Since the duration of sustained loads are quite larger than that of intermittent loads, therefore the intermittent-variations are observed more often than other. Load events occur in the manner of contiguous and discrete pulses and during a pulse, the load magnitude remains virtually constant until the next event. It is assumed that the distribution of live loads matched to the Gamma distribution.

As the simulation of live loads is related to time, it is also required to simulate durations for each load processes. The time changes occur along the consecutive events and they appear randomly in different instants (for instance t_i and t_n) for sustained and intermittent loads. The occurrence rate of the time changes are defined by $1/\lambda$ and $1/\nu$ (Table 4), respectively. In the simulation of live load processes, time intervals (T_{st}, T_{int}) are assumed to follow Poisson process and their variations are modeled by exponential distribution.

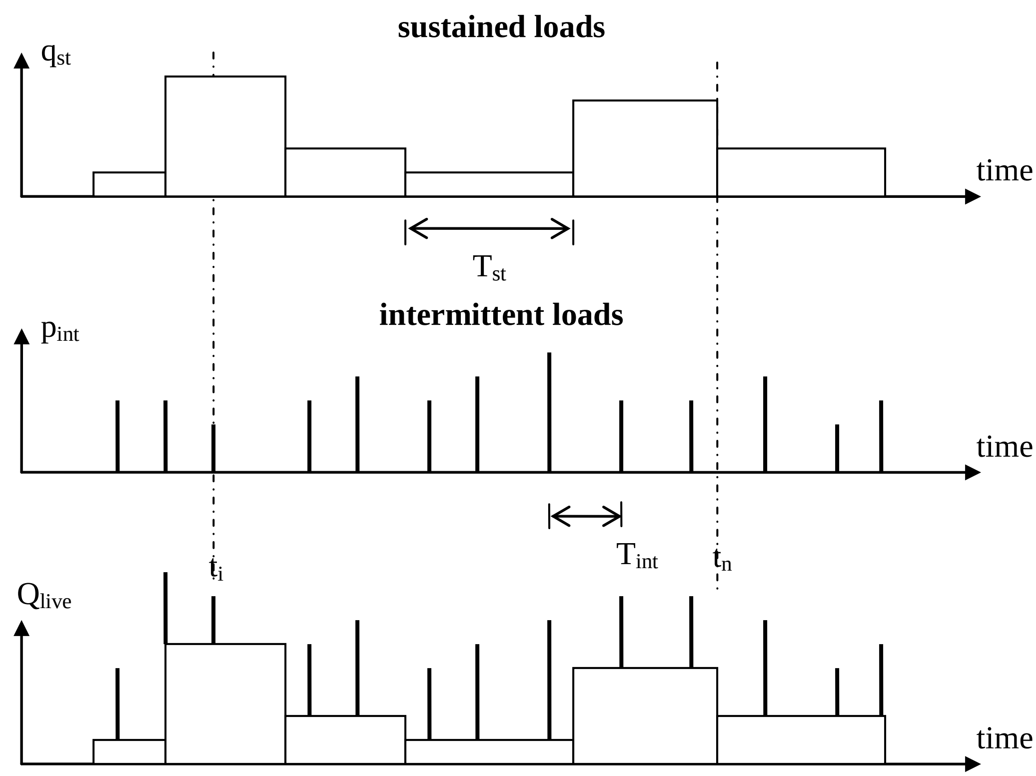


Figure 3. Sustained and intermittent load process with combination model (FBC).

In the combining of load processes, Ferry Borges-Castanheta (FBC) model containing sequence pulses is utilized to idealize time-dependent loads. This model considers occurrence rate of load changes and occupancy durations as well. The live load magnitude is defined by stochastic field variable (F(x, y)) with referenced by JCSS code. Field variable includes ensemble mean (μ), normal distributed variable (C, mean=0) and random field factor (R(x, y)) having also zero mean.

$$F(x, y) = \mu + C + R(x, y) \quad (6)$$

The load effect, S, is related to influence function z(x, y) and considered area A for elastic system:

$$S = \int_A F(x,y) z(x,y) dA \quad (7)$$

For sustained load (q_{st}), equivalent value of uniform distributed load is:

$$q_{st} = \frac{\int_A F(x,y) z(x,y) dA}{\int_A z(x,y) dA} \quad (8)$$

and sustained loads are simulated by Gamma distribution with the expected mean value and variance:

$$E[q_{st}] = \mu \quad (9)$$

$$\sigma_{st}^2 = \sigma_C^2 + \sigma_R^2 \frac{A_o}{A} \beta(A) == \sigma_C^2 + \sigma_R^2 \frac{\int_A i^2(x,y) dA}{[\int_A i(x,y) dA]^2} \quad (10$$

where β is influence factor defining the load shape over random fields and σ_R^2 is variance of stochastic field variable. The reference area A_o is related to the usage type of the floor (see Table 1). The area of the tributary surfaces arising from load effects like moment, axial forces and etc is defined by influence area A. An influence area is related to the floor area designating the load of a given element. The ratio of the A>Ao is an area reduction factor especially in case of large areas. Statistical and deterministic parameters (given in Table 1) vary depend on building type. if $A<A_0$ then, the ratio of A_0/A will be taken as 1.0. Since spatial variabilities for the same floor loads are assumed independent, the $\beta(A)$ can be considered as a constant factor β (e.g. β=2.2 for column loads). The intermittent loads are simulated also by using the $[q_{st}]$ equation depending on their parameters given in Table 3. They are modeled by the exponential distribution with $E[p_{int}]=\mu_{int}$ and its variance (JCSS):

$$\sigma_{p-\text{int}}^2 = \sigma_{R\text{int}}^2 \frac{A_o}{A} \beta(A) \quad (12)$$

Table 3. Live load parameters for some building types

	Sustained loads					Intermittent loads			
Building type	A_0 (m^2)	μ_{qst} (kN/m^2)	σ_C (kN/m^2)	σ_R (kN/m^2)	$1/\lambda$ (1/year)	μ_{int} (kN/m^2)	σ_{Rint} (kN/m^2)	$1/\lambda$ (1/year)	d_p (year)
Residence	20	0.3	0.15	0.3	7	0.3	0.4	1.0	1-3
Office	20	0.5	0.3	0.6	5	0.2	0.4	0.3	1-3
School class.	100	0.6	0.15	0.4	>10	0.5	1.4	0.3	1-5

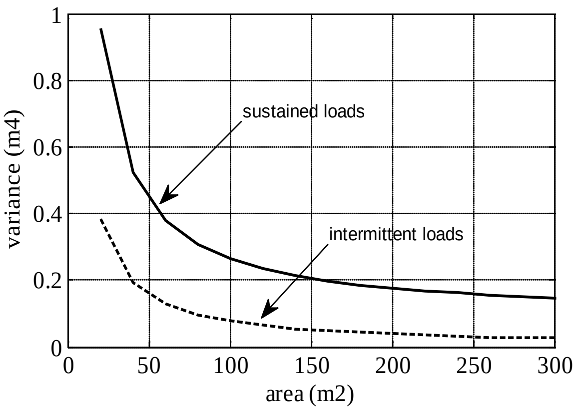
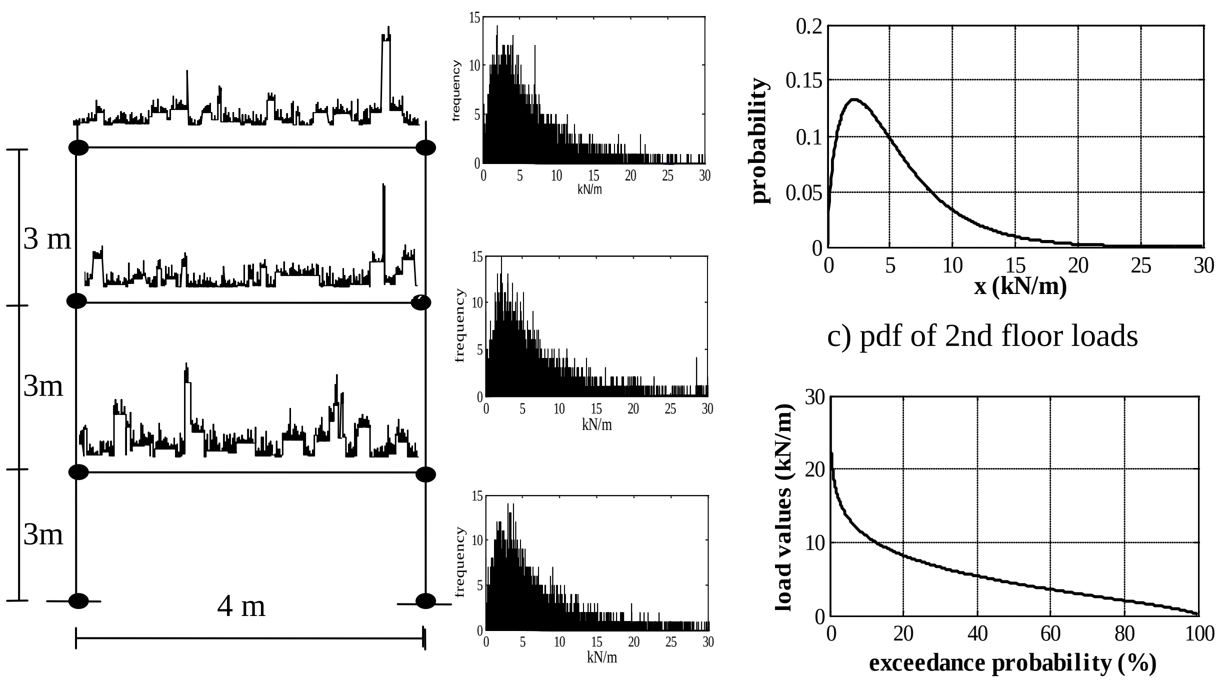


Figure 4. Effect of influence areas

In office usage, variances of floor loads are plotted in Figure 4 for various influence area with some parameters (β=1.0, σ_c=0.30, σ_R=0.60, σ_{int}=0.30). As expected, the variances of sustained loads are larger than those of intermittent. As the influence area increase, the variances decrease rapidly for both load types. The combinations of live loads are obtained by using the simulated loads as seen on the frame system given in Figure 5. Schematic illustrations on beam elements denote simulations of daily live loads for the duration of



a) simulated daily loads

b) frequency distributions for annual extreme floor loads

d) exceedance probabilities of 2nd floor annual extreme loads

Figure 5. Simulated loads and probability functions of annual extreme loads

3650000 days. The annual extreme values are evaluated by peak load intensities for each beam element. Histograms of the extreme values are plotted and it is easily seen that the distributions are compatible with the gamma distribution. Probability density function and exceedance probabilities are evaluated as well for the annual extreme values and their functions are plotted only for the 2nd floor (Figure 6(c,d)). These statistical results may be used in further analyses such as probabilistic design, loads and resistance factor design and reliability analyses. It has been observed that the variances and fractiles of live loads are quite high and this situation leads to large variability in the annual extreme values (Table 4).

Table 4. Statistics for annual extreme loads

	Variance	**Fractiles**	
Element	σ_x^2 (kN/m2)	%50 (kN/m)	%98 (kN/m)
1	21.25	4.07	19.12
2	20.20	4.24	18.63
3	18.19	4.27	17.29

Another goal in this study is to show comparatively the effect of reduction factors in live loads in terms of number of storey for a multi-storey frame. Probabilistic model results are compared by the results of deterministic model (DIN 1055-3) in Figure 6. Probabilistic analyses are implemented for A=20 m^2 and β=1 values. The effects of number of storey are studied by considering correlations (ρ=0.5) between floor loads as well. With increasing of number of storey, the influence areas in a structure would also increase thus the variances decrease. This phenomenon leads to stronger reductions for higher structures. It is easily seen that the probabilistic model provide more capacity by decreasing correlation in multistory structures. The correlation among the floor loadings quite affects the values of the reduction factors.

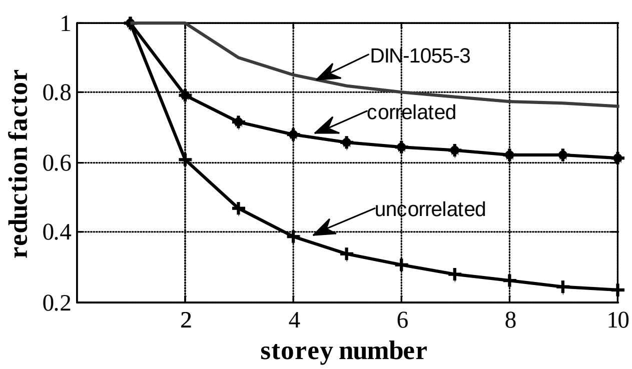


Figure 6. Reduction factors for the live loads

As the number of storey increase, differences between the results of both methods become larger. The duration of stochastic load simulation affects adversely the distributions of annual extreme loads and their fluctuations appear in a wide interval for small simulation durations. Therefore the simulation duration should be suitably selected to obtain accurate probabilistic results.

5. CONCLUSION

- Structural loads are evaluated in terms of probabilistic effects for dead and live loads. The variations on loads are adopted as random variables and they are taken into account by stochastic process with proper distributions. Dead loads are simulated by considering variations in unit weight-volume and sizes of the cross-sections. It has been show that the changes in self-weight loads are small and they can be neglected.
- In the obtaining of the extreme load intensities during life time, time-dependence of the loads is disregarded and the problem is converted to time invariant variables. However, live load intensities are appeared by arbitrarily time intervals, therefore the intensities are assumed as time-dependent by use Poisson process. The effect of influence area is examined. As the influence area increases, considerably decreases are observed in the quantiles over the regions with small areas.
- The reduction factors of deterministic and probabilistic models are evaluated and the differences occur in large scale as the areas increase. In probabilistic model, stronger reduction factors are observed with increasing number of storey. This model provides more capacity by increasing influence area and decreasing correlation for higher structures.
- In the obtaining of probabilistic results, total duration of the simulation affect adversely the accuracy. If the time period defined as short term, the extreme values differ from correspondent distributions. Therefore, the duration of simulation should be properly defined to obtain accurate results.

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