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Safety and Stability of Structures with Semi-Rigid Connections of Members in Nodes

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ABSTRACT

The structural design of a system in which connections of members are absolutely rigid or perfectly pinned has been thoroughly worked out in the existing literature. In real structures in general, and particularly in the precast ones, connection in joints may be partially rigid, which can have a significant impact on the change of stresses and strains in the structure, i.e. on the stability and security of the structure.

Testing of engineering structures, i.e. conditions that must be met so that they remain in stable equilibrium position is of special importance. To determine the limit state of stability in use are experimental and analytical methods based on static, dynamic and energetic principles.

This article presents the methods used to ensure technical safety and security, as well as additional safety conditioned by public and social reasons. The functional damage and fatigue are discussed, as well as the concept of the projected lifespan of the structure. Increased strength provides a higher degree of security, although many factors are variable.

Keywords: *Safety, stability, semi-rigid connections.*

INTRODUCTION

In order to provide safety, the structures should be built according to the design with all the materials of specified strength and correct analyses and assumptions of design. The majority of influential factors is variable, so, in order to safeguard against uncertainty, materials of increased strength are used. The ultimate degree of safety consists of the parts for provision in respect to the technical and social consequences of failure. Structures of various degrees of safety can be constructed. The adopted level largely depends on the amount of available finances, because the cost rises along with the increase of safety. The safety determination methods must give solutions meeting both technical and social aspects. Engineering structures fail due to collapse or due to functional deficiencies. In determining the technical safety there exists a social aspect, too. The solution to this is in introduction of an additional of a degree of safety to that required by technical considerations. This additional degree protects the owner, the user or general public from the consequences of failure. The owner has a financial stake, and possibly others in the structure, the user is at risk of injury of death at failure, while the general public may lose confidence in a certain part of structure. Until recently, according to this the safety was observed from two aspects: in designing the safety of an element or of one part is dominant; while in service the safety of the structure as a whole is of fundamental importance. Nowadays, the endeavors to view the entire structure from the standpoint of safety are worth noticing. The designing includes treatment of structures with a predetermined level of safety and minimum cost. There are the following stages of this process: a)Determination of designing loads, b)Determination of structural strength, c)Determination of the quantitative value of safety.

In order to treat the issue on the statistic basis, these researches require extensive data, which is difficult and costly to provide. After the catastrophic earthquake in Japan of 4^{th} of March 2011, the increasing focus is on the issue of seismic energy absorption and analysis of structural safety via the reliability factor of nuclear power plants which were damaged. For this reason experimental research is required for the equipment belonging to the first class in order to verify mathematical models. The testing based on the contemporary scientific achievements and with provisions guaranteeing as high safety as possible is mandatory. For instance, nuclear gates (providing that the water cooling the radioactive material is sealed and safe from leaking) must retain their function during and after seismic action in order to provide the total safety of the power plant, thus they are designed to a max=3,0g, i.e. to 30 times higher acceleration than the high-rise buildings in the 9^{th} seismic zone. The first step in definition of the reliability index *b* (probabilistic safety coefficient) is normalization of fundamental variables in order to create a mathematical model.

During the aforementioned earthquakes the tsunami waves around 10 m high were formed, thus the pumps used for circulation of nuclear reactor cooling water were damaged, which lead to overheating and explosion of the reactor and contamination of the wider environment. This means that the islands and coastal areas are less safe due to the occurrence of tsunamis which cause more severe damage and casualties than the very earthquake.

METHODS FOR DETERMINATION OF SAFETY

The methods for determination of safety (from the technical aspect) are intended to avoid the loads due to which the bearing capacity of the structures would be exceeded.

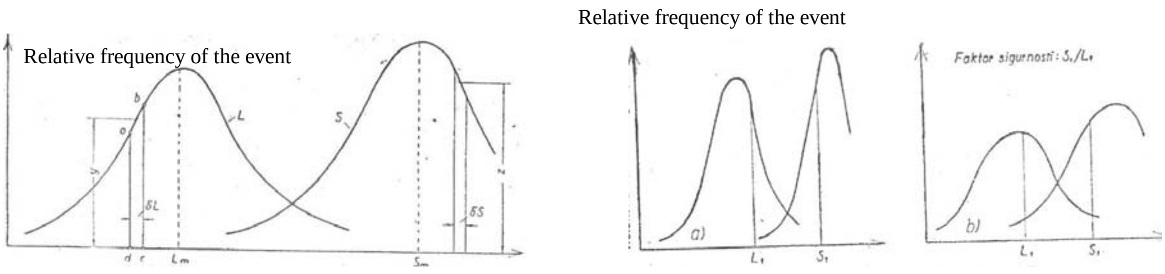


Figure 1. Load and strength values

The curve in Fig. 1, drawn according to the relative frequency of the events, shows different values of load an idealized structure can be exposed to. The S curve in Fig. 1 shows variability of the value of strength of a large number of structures which are designed and constructed according to the same requirements. Damage occurs where these two lines intersect, and were for a specific structure any value of L is higher than S. The attempts to provide the safety of the structure, for such typical curves can be done by adjusting the ratio S_m/L_m, known as the safety factor, to be minimal. The term L_m provides for the maximum load such as can be expected under working conditions, and S_m is the bearing capacity obtained by the choice of the variables in the calculation, so as to minimize the damage, and minimize the safety factor related to the maximum probability of damage. The additional safety, higher than that required by technical reasons, which provides security from social consequences due to functional damage or failure, is determined by varying certain values. The social consequences include financial loss of the proprietor, loss of life, public damage, conflict with the users, repair and reconstruction cost which resulted from the collapse. When making decision about the degree of safety, one should consider the structures which are not constructed, but which could be constructed with the money which would be saved if the degree of safety was lower. These instances are social, administrative and political, so they are taken into consideration in the proposed way, as they affect the cost of the structure. Take for instance the bridge degree of safety. Here the influence of the degree of safety on the number of constructed bridges in a country should be considered. The increase in the degree of safety can be applied to the whole structure or to individual parts, or for only certain failure models. From the engineering standpoint, only one degree of safety should be used for all parts and for all structures. However, due to social reasons, there may be variations in the degree of safety, though it is not logical to have different degrees of safety for failure of steel and concrete in the set concrete. Many of the social consequences of collapse, or risk of collapse can be expressed through the cost of the structure, so the decision about the degree of safety assumes the engineering aspect. By introducing soft steel, the large permanent strain without failure occur, on the basis of the load obtained by the research, the load factor became the ratio of test and design load. The test load is that when there is the onset of permanent strain, or the load at which the structure was tested prior to service. The ratio known as safety factor γ_s is defined as ratio of the bearing capacity of the structure, for instance S_s and the design load L_s. The design load is the highest overall justified predicted load during service.

The previous implies

$$\gamma_s = \frac{S_s}{L_s} > 1 \text{ , } \quad \text{for safety} \qquad (1)$$

Values for S_s and L_s, are obtained by the choice of variables from the design, cannot be complete. If S_1 and L_1, are multiplied by some variable factors, then

$$iL_1 = \frac{S_1}{j} \quad ; \quad \frac{S_1}{L_1} = i, j \quad \text{and} \quad \gamma_1 = i, \quad j \qquad (2)$$

γ_1 is the technical safety factor.

The social safety factor is satisfied by the increase of adopted resistance $S_1/(j\ k)$, then:

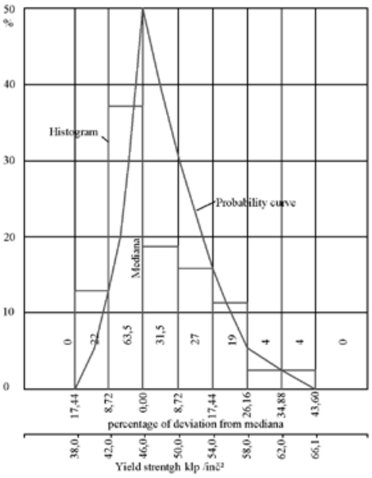
$$iL_1 = \frac{S_1}{jk} \ ; \quad \frac{S_1}{L_1} = i, j, k \quad \text{i} \quad \gamma_2 = i, j, k \qquad (3)$$

γ_2 is the ultimate safety factor (i, j, k>1).

In the equation (1) the values for S_1 and L_1 can vary. The strength of the structure is time dependent. What is certain is that there is static load, and when the structure, according to its purpose, is approved for service, the initial load will also act, only occasionally or permanently. Rüsch proposed that it is permissible for the road loads to adopt the highest load which can be expected, which ensured that the value γ is the same as for the static load. Application of probability method is not easy in the case of the large and massive long lasting and unique structures. The probability of collapse can be determined applying the load or stresses provoked on the load basis. In making decisions about the overall degree of safety it can be observed only in conjunction with the probability of collapse. In application of the probability method in determining the safety, we can decide about the safety by comparing the existing acceptable values. For any acceptable procedure of determining safety, it is necessary to have adequate data on the load and strength of the structure, and treat these data statistically so they could be implemented in a most useful way.

The methods for providing the functional safety have two intertwined areas. First, change of conditions in the mentioned method, and second, determination of limits which must not exceed the design loads. Both for the stress factor and the load factor, the changes of safety factor can be implemented in the relationships, because the goal is to retain the elastic behavior of the structure due to designed loads. In the probability method, such value of probability of functional load is determined, which must not be exceeded in the calculation. The strength related to the functional load is usually a load or a stress when the yield of steel occurs as is the case with reinforced concrete. The second method for providing the degree of safety from the functional load includes determination of certain limits for deflection, vertical load and other characteristics which, if exceeded, could lead to a limited serviceability of the structure.

The effects of fatigue, wear, creep and corrosion impose a limited service life on a structure. In designing, usually it is ensured that the degree of safety is present until the end of service life of a structure. The predicted service life of a structure can be indeterminate. If the design load is increased, it would mean the increase of the cost of the structure and the replacement, for instance, of the existing bridges which would fall below the standard which would necessitate increased maintenance costs, thus it is more sensible to keep the existing design load, and limit the vehicle weight by the law. Such argument can be applied to almost all types of structure. For the bridges, it is accomplished by designing the roads with various load capacities. It should be clearly perceived that the terms, such as the safety factor, without any real sense, as long as they are not related to the adequate probability. The material strength and the maximum influence of the load to which the structures are exposed can vary. Ultimately, the strength of the structure can come close to any positive value lower than the permissible value. The correlation of the safety factor and serviceability factor with the endurance probability for each individual designed structure is impractical. The answer to the question what the term “safety factor” means is not simple. Two definitions are given: “The minimal required safety factor, which provides that some given collapse probability of P_F structure is not exceeded, defined as a quotient (higher than one) of the estimated resistance R_0 to the failure during the predicted service life” , „Minimal factor of serviceability which provides that a given probability, that the structure is not serviceable during the predicted service life, for which it was designed, will not be exceeded; it is defined as a similar quotient, both in respect to serviceability and in respect to failure “. These definitions mostly refer to the structures and not to their individual parts. For example, the structure which has superficial elements or connections will be safer and more serviceable if some of the elements are removed or rendered unserviceable. In order to take into account dynamical and other effects which depend on the characteristics of the structure, most often the term “load effect” is used than the load as well as the ratio of load and time during use.



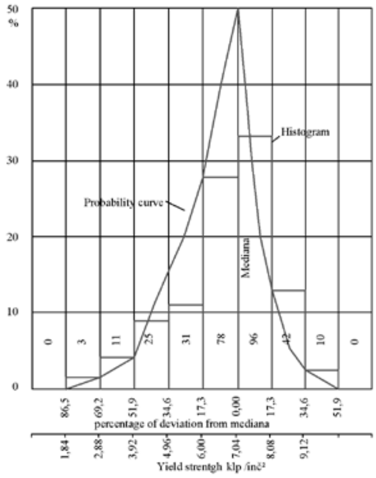


Figure 2. Histogram and probability curve

Safety factor and serviceability factor may be observed as a “load factor” which can be used to equate the median calculation “load effect” with the median calculated strength (resistance) which in some cases can be limited with a relatively small deformation which renders the structure unserviceable, while in other cases, it can be almost as big as the limit strength. In order to evaluate the strength of the structure in the design stage, it is necessary to posses statistical data related to the existing strength, the lower yield point and resistance to fatigue of the material similar to that which will be used.

In Fig.2. is presented the variation of concrete strength which can be expected if the control is poor.

The ordinates of the histogram (Fig. 2.), are drawn as dotted line staircase, present a relative frequency of testing results which occur inside the width of the staircase marked by abscissae on their bases.

STABILITY OF STRUCTURES WITH SEMI-RIGID CONNECTIONS

Basic assumptions of the design

Linear static of the structures, such as is used in everyday engineering practice is based on the following three assumptions:

1) Assumption that the strains of the ε axis of the member and rotation φ of the member cross-section as well as their derivatives, small values whose squares and higher powers can be neglected (assumption of small deformations).
2) Assumption by which the values of displacement of attack points of external forces on the internal forces girder small in respect to the basic dimensions of the girder (assumption of small displacements of attack points of external and internal forces).
3) Assumption of the linear relation between strain and stress, that is, temperature changes (Hooke’s law).

The first assumption provides - geometrical and the second - static linearity, whereas the third assumption provides physical linearity in solving the tasks of structural statics.

The second order theory rejects only the second of the mentioned assumptions, and retains the first and the third assumption.

The assessment of the stability criteria can be conducted on the basis of the experiments, and determine that system possesses a certain coefficient of safety only if during the test the designed load was actually achieved. This means that the actual coefficient of safety is determined on the basis of the level of critical load at which the safety was lost.

This creates an objective difficulty only if the test is performed ”in situ” and the destruction of the system is not allowed. For this reason the wrong conclusion is based on using the results of experimental tests where the limit state of stability was not reached. The concept of stability is a regular physical concept whose interpretation will be observed in the following example.

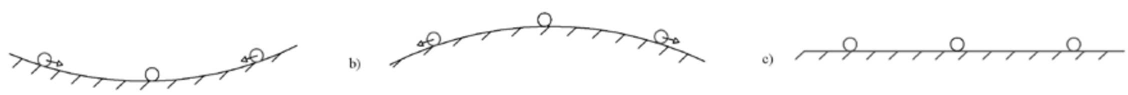


Figure 3.Various cases of equilibrium of a smooth ball on a frictionless surface

In Fig.3. are presented various positions of a smooth on a frictionless surface whose surface has the shapes marked as a), b), c). If the ball, in the case of concave surface is moved directly to the designated position, it will continue movement to the middle position until it returns to it. For this reason the case a) can be characterized as a stable equilibrium.

Simultaneously the ball which taken out of the middle position in the case of convex surface cannot return to that position, thus the case b) is defined as unstable equilibrium. Finally, in the case of flat surface, the ball keeps any position, so we will characterize it a neutral (indifferent) equilibrium.

THE CRITICAL LOAD CONCEPT

In theory of structures, we are testing stability of engineering structures, that is, conditions which must be satisfied so that they would remain in stable equilibrium position. The stable equilibrium position was defined in the previous section, in three ways, applying static, dynamic and energy criterion of stability. We, in fact, are seeking the lowest value of the load, which is called the critical load, at which the originally stable equilibrium position becomes unstable. The static definition of critical load according to Euler is: ”Critical load is the least load of the structure, at which, beside its basic equilibrium position there is at least another equilibrium position”. Dynamic definition of critical load according to M. Đurić [3] is: “Critical load of the structure is the least load of the structure, at which the appropriate disturbances cause movement (vibrations) structures which is not limited to the immediate surrounding of the basic equilibrium position of the structure”. We will use the simpler static criterion of stability (that is static definition of critical load), which is for the elastic systems loaded by conservative forces (and this is the most frequent load in practice) equivalent to dynamic criterion. The analytic formulation of critical load can be expressed in the following way: “The critical load is the least value of the load at which the homogeneous system of linear equations has at least one solution different from the trivial”. According to M. Đurić [3] it is: “The critical load is the least value of load at which the system of equations of linearized theory of the second order has at least one solution except the trivial one”. The solution of the task is still identical to purely mathematical task of determining the eigenvalues of the system of homogeneous differential equations. If we analyze the case when there is no transversal load or temperature differences, at which we will assume that ψ(x) and f(x) are random functions, we will obtain:

$$(\psi v``)`` \pm k^2 (f v`)` = 0 \qquad (4)$$

Where: $I = I_c \psi(x)$; $H = \mp S f(x)$; $k^2 = \dfrac{S}{EI_c}$ (5)

The upper designation is valid for the pressed and the lower for the tensioned member. The differential equation of the constant cross section member (ψ=1) exposed to the action of constant axial force of pressure (f=1) is:

$$v^{IV} + k^2 v^{II} = 0 \qquad (6)$$

whose solution can be obtained by the method of initial parameters in the form:

$$u_h = C_1 + C_2 kx + C_3 \sin kx + C_4 \cos kx \qquad (7)$$

where: C_1, C_2, C_3 are C_4 integration constants, which do not have, but can be assigned a certain physical meaning.

SOLUTION OF THE PROBLEM OF STABILITY OF STRAIGHT MEMBERS BY DEFORMATION METHOD

The critical load has been defined as the least value of load at which the homogeneous system of equations of linearized theory of the second order has at least one solution except the trivial one. Similar to the linear theory, a system of m equations of node rotation and n equation of displacement is given, and it is :

$$A`_{ii}\varphi_i+\sum_k A`_{ik}\varphi_k+\sum_{j=1}^{n} B`_{ij}\Delta_j=0, \qquad (i=1,2,.....,m) \tag{8}$$

$$\sum_{i=1}^{n} B``_{ji}\varphi_i+\sum_{i=1}^{n} C`_{jl}\Delta_l=0, \qquad (j=1,2,.....,n)$$

Or in the matrix form, expressed by the block matrix:

$$\begin{bmatrix} A`B` \\ B``C` \end{bmatrix}\begin{bmatrix} \varphi \\ \Delta \end{bmatrix}=0 \tag{9}$$

The critical load is determined form the condition that the system determinant (9) equals to zero:

$$\det\begin{bmatrix} A`B` \\ B``C` \end{bmatrix}=0 \tag{10}$$

Here the following designations are introduced:

$$A`_{ii}=\sum_k a`_{ik}+\sum_S l`_{iS}, \qquad A`_{ik}=b`_{ik}, \qquad \mathbf{B}`_{ij}=-\sum_k c`_{ik}\psi_{ik,j}=B``_{ji} \tag{11}$$

$$C`_{jl}=\sum_{ik}(c`_{ik}+c`_{ki})\psi_{ik,j}\psi_{ik,l}\mp EI_c\sum_{ab}\frac{\omega_{ab}^2}{L`_{ab}}\psi_{ab,j}\psi_{ab,l},$$ where "-" is for compressed members.

The *k* type and *g* type members are treated as a single type *k*, is semi-rigidly fixed on both ends, and the *s* type member is pinned at both ends.

In the cases when the $A'_0\approx0$ and $C'_0\approx0$ are free terms, we can also apply this procedure of finding the critical load of the system of members, where we must be aware that the obtained critical load has a slightly slower value than the exact value, which results to a smaller dimensions of members (in their dimensioning) than the actually required dimensions, which does not favor safety. However, taking into account that the real connections are elastic - semi-rigid, the reliability of the structure is achieved to a greater extent if the system is calculated in this way.

CONCLUSION

The ultimate degree of safety consists of the parts for provision in respect to the technical and social consequences of failure. Structures of various degrees of safety can be constructed, depending on the available finances. By the methods of determining safety from the technical aspect, we tend to avoid the loads which would exceed the bearing capacity of the structure. Safeguarding against social consequences due to functional damage or failure is determined by varying certain values. The social consequences include financial loss of the proprietor, loss of life, public damage, conflict with the users, repair and reconstruction cost which resulted from the collapse. The increase in the degree of safety can be applied to the whole structure or to individual parts. From the engineering standpoint, only one degree of safety should be used for all parts and for all structures. However, due to social reasons, there may be variations in the degree of safety. The methods for providing the functional safety have two intertwined areas. First, change of conditions in the mentioned method, and second, determination of limits which must not exceeded the design loads. The effects of fatigue, wear, creep and corrosion impose a limited service life on a structure, and the degree of safety must be present until the end of service life of a structure. The material strength and the maximum influence of the load to which the structures are exposed can vary. Safety factor and serviceability factor may be observed as a “load factor” .

Linear static of the structures, such as is used in everyday engineering practice is based on the following three assumptions. The first assumption provides - geometrical and the second - static linearity, whereas the third assumption provides physical linearity in solving the tasks of structural statics. The second order theory rejects only the second of the mentioned assumptions, and retains the first and the third assumption. The assessment of the stability criteria can be conducted on the basis of the experiments, and determine that system possesses a certain coefficient of safety only if during the test the designed load was actually achieved. Analytical methods are based on static, dynamic and energetic principle. In the application of the static criterion of stability, there are certain difficulties resulting from the neglecting the acceleration created after taking the system out of the equilibrium position. Applying the D’Alambert principle, the dynamic status can be replaced by the static status, treating in the further analysis additional inertial forces, as well. The system can be in the state of stable, neutral and critical equilibrium. Energy criterion of stability is based on the analysis of total potential energy of the system using Lagrange’s principles of virtual displacement.

In theory of structures, we are testing stability of engineering structures, that is, conditions which must be satisfied so that they would remain in stable equilibrium position. We will in practice use the simpler static criterion of stability (that is static definition of critical load), which is for the elastic systems loaded by conservative forces (and this is the most frequent load in practice) equivalent to dynamic criterion. The analytic definition of critical load is: “The critical load is the least value of the load at which the homogeneous system of linear equations has at least one solution different from the trivial”. The critical load is determined form the condition that the system determinant of the homogeneous system equals to zero. The *k* type and *g* type members are treated as a single type *k*, is semi-rigidly fixed on both ends, and the *s* type member is pinned at both ends.

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