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Study of Evolutionary Structural Optimization and Applications

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ABSTRACT

In this study, the evolutionary topological design of 2D structural systems was conducted by using ANSYS software. Since the material that is used in structures is not homogenous in general, the maximum and minimum principal stresses were taken into consideration in this study. Besides, these stresses were used as delimiters. The purpose of the designed algorithm, which was used in the analysis, was to gradually eliminate elements from the structure, to control the intense increase in the maximum stresses, and to optimize the structure by obtaining a homogenous stress distribution.
Keywords: evolutionary structural optimization (ESO), finite element analysis (FEA)

INTRODUCTION

The human being has been carrying the will of maximizing the mechanical productivity all over ages. Hence, it wasn't enough only to develop and use a system. Obtaining the best and the most appropriate systems has become a main goal. This main goal exposed the process named optimization. In optimization process, it is aimed to obtain the best result (the lightest, the most economical, the most productive, the speediest etc.) with regarding the limitations.

EVOLUTIONARY STRUCTURAL OPTIMIZATION (ESO)

The topological design of continuous structures is the major part of the structural optimization. It is possible to obtain more economical design via topological optimization [1]. Evolutionary optimization methods are leading to the new optimization techniques. Several algorithms have been developed for evolutionary optimization in recent ages. The basic principle of these algorithms is to harmonize with external conditions.

The solutions to the questions, such as how the objects will be shaped or what will be the best possible structural performance, have been found by using the evolutionary structural optimization (ESO) method. It has been presented by several researchers that the solutions to the size, shape, and topology optimization problems can also be found by using the ESO method [2].

The fully stressed design (FSD) concept was developed to resolve the unproductiveness in structural optimization. In FSD case, all of the members of the structure are exposed to the allowed maximum or minimum limits. The FSD is achieved when the stress in all members become approximately equal to each other. The FSD case is also named as equal stress case in the literature. Generally, the optimization algorithms consist of close loops to redesign in order to obtain fully stressed case.

ALGORITHM of the ESO

In general, the minimum weight or volume has been used as the goal function in engineering design. Diversity can be observed in restrictive. Behavioral quantities, such as stiffness, stress, thickness, frequency etc., can be used as restrictive depending on the purpose of the use, environmental conditions, and economical conditions. Although the design variables vary with the algorithm or the material, mostly the variables such as the number of elements, thickness of element, and the volume of element has been used.

In traditional ESO method, the von Mises stresses have been used as removal criteria (restrictive) [3]. However, in case of existence of tension and compression members at the same time in the structure, the use of principle stresses instead of von Mises stresses as removal criteria was found to be more appropriate [4]. Hence, the principle stresses are used as restrictive in this study to obtain optimized structures.

First of all, the principle stresses $\sigma^{e}{}_{11}$ and $\sigma^{e}{}_{22}$ are calculated for each element. The removal criterion is applied with respect to the compression and tension regions in the structure [5]. In the regions that the compression is dominating, members that are subjected to tension are removed ($\sigma_{22} \leq 0.0 \; ve \, |\sigma_{22}| >> |\sigma_{11}|$). In the opposite case, members that are subjected to compression are eliminated in the regions that the tension stress is dominating ($\sigma_{11} \geq 0.0 \; ve \, |\sigma_{11}| >> |\sigma_{22}|$). The element eliminating algorithm is the basis of the ESO method that is used in the presented study. The algorithm that is defined above can be mathematically expressed.

$$|\sigma_{11}{}^{e}| \leq RR_i \times |\sigma_{11,\max}| \qquad \sigma_{22}{}^{e} \leq 0.0 \qquad (1)$$

$$|\sigma_{22}{}^{e}| \leq RR_i \times |\sigma_{22,\max}| \qquad \sigma_{11}{}^{e} \geq 0.0 \qquad (2)$$

In the Eqn.1 and Eqn.2, $|\sigma_{11,\max}|$ and $|\sigma_{22,\max}|$ are the absolute maximum stresses of the member, RR_i is the ratio of removal that provides elimination of few elements. The loops of finite element analysis and the removal of elements continues until the Stable Case, at which no more element to eliminate is left. When the stable case is achieved, the evolutionary ratio (ER) is obtained and added to RR_i. The new loops for finite element analyses are set by using the new RR_{i+1} value (Eqn.3) and the elimination of elements continues until the new stable case is achieved.

$$RR_{i+1} = RR_i + ER \qquad i = 0,1,2,3,\ldots. \qquad (3)$$

In case of balance in the quantity of tension and compression elements, von Mises stresses (σ^{vm}) can also be used instead of principle stresses. The von Mises stresses in plane stress elements can be expressed as in Eqn.4.

$$\sigma^{vm} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \qquad (4)$$

where, τ_{xy} is the shear stress, σ_x and σ_y are the normal stresses in x and y directions, respectively. An algorithm that is similar the one for principle stresses is used, if the von Mises stresses are preferred in the analysis. At each loop, the value of σ_e^{vm} for each element is calculated and written to the database. Then, the elements that satisfies the comparison in between σ_e^{vm} and $\sigma_{\max}^{vm}$, as expressed below (Eqn.5), are eliminated from the system.

$$\frac{\sigma_e^{vm}}{\sigma_{\max}^{vm}} < RR_i \qquad (5)$$

where, the RR_i stands for the current removal ratio. The ER value that is used in this algorithm is same with the one for previous algorithm.

OPTIMIZATION with ESO METHOD

The application of the ESO method is exemplified by analyzing the frame with two members, which is commonly used in structural optimization problems. The structure is a cantilever beam that is restrained at left side and has a length of L and height of H. It is found analytically that the optimum height is twice the length of the beam (H=2L) (Figure 1) [2].

To obtain the example structure by using the ESO method, the model presented in Figure 2 is used. The model is chosen to be larger than 2L x L in dimensions; hence the coverage of the final design in dimensions by the model is taken into the consideration.

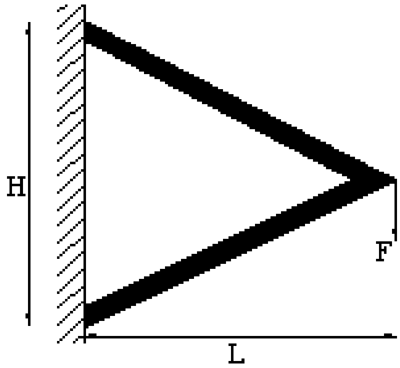


Figure 1 – Two member structural frame

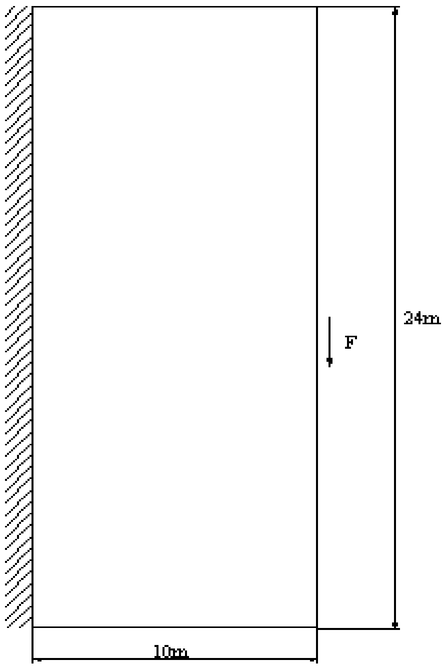


Figure 2 – Design area for two member structural frame

The design area of the model is meshed by 4-noded plane stress elements, which are in 0.4m x 0.4m in dimensions (Figure 3). The thickness of the plate, the modulus of elasticity, and the Poisson's ratio are chosen to be 1 mm, 100 GPa, and 0.3, respectively. A vertical load (525 kN) is applied to the middle of the right side.

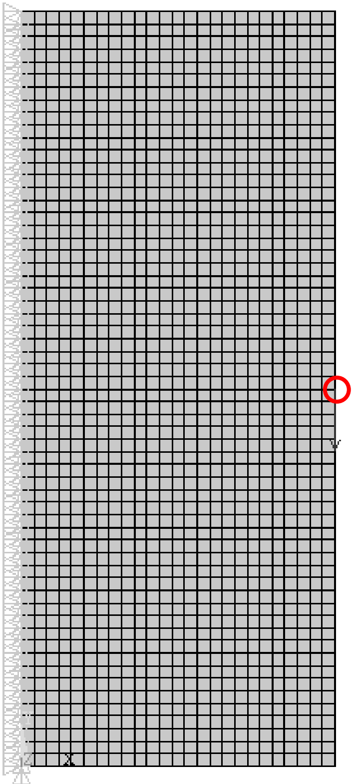


Figure 3 – Meshing profile of design area

In the optimization process, the number of iterations and the initial removal ratio (RR_0) is chosen to be constant. On the other hand, the evolutionary ratio (ER) varies with respect to the final removal ratio. The optimum structure is obtained after 50 iterations. The initial removal ratio RR_0 is chosen to be 0.1%, while, the value of ER is varied from 0.06% to 0.6% (Table 1).

The calculated stresses values of the initial and final design, the ratio of calculated stresses, and the volume for the analyzed model are compared in Table 2.

As it is presented in the Table 2, there is an extreme difference between the maximum and minimum von Mises stresses in the initial model. However, this difference remarkably decreases in the optimized structure. In fact, the final model reaches to the fully stressed design (FSD) status. However, the two elements that the load is applied deteriorate this uniform stress condition. The stress in these elements is twice the average stress. If the two elements that the load is applied are neglected, fully stressed design can be achieved.

When the value of RR reaches to a value in between 25-40%, the removal of element from the structure do not materialize. At RR values larger than 40%, all elements, except the ones that the load is applied, are removed from the structure. The removal of all elements states that a uniform structure is achieved.

Table 1 – The development process of design model to a truss structure

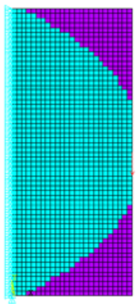
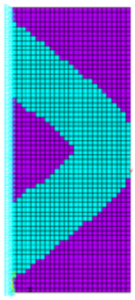
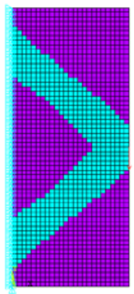
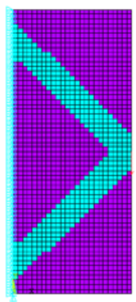
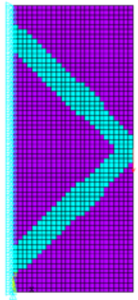
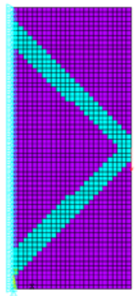
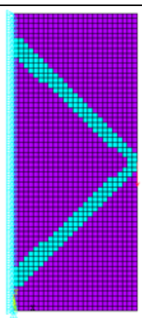
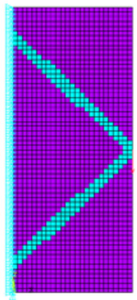
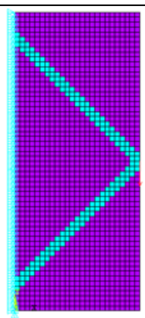
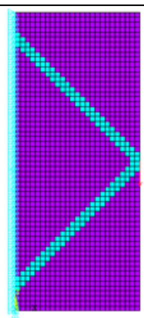
					
RR	3%	6%	9%	12%	15%
Max. Stress	0.8558	0.8653	0.8653	0.8797	0.8845
Min. Stress	0.0244	0.0343	0.0587	0.0830	0.0786
Volume Ratio	0.7585	0.4788	0.3178	0.2415	0.2119
					
RR	18%	21%	24%	27%	30%
Max. Stress	0.8940	0.9657	0.9657	0.9562	0.9610
Min. Stress	0.1125	0.1369	0.1805	0.3584	0.3636
Volume Ratio	0.1822	0.1271	0.1017	0.09746	0.09746

Table 2 – The comparison of the initial model and the optimized structure

	$\sigma_{\min}^{vm}$ (MPa)	$\sigma_{\max}^{vm}$ (MPa)	$\sigma_{\min}^{vm} / \sigma_{\max}^{vm}$	Volume (m^3)
Initial Model	0,0002	0,8602	0,0002	0,2400
Optimized Structure	0,3541	0,9606	0,3686	0,0237

CONCLUSION

The analysis by using the evolutionary structure optimization showed that remarkable decreases can be achieved in the volume of structures. The comparison of the initial model and the optimum design in this study presents a 98% decrease in the volume of the structure, while a 12% increase in the stress. It can be easily said that, this amount of decrease in the volume is more important than the increase in the stress.

Another advantage of the ESO method is that the establishment of new finite element network is not needed at each increment. Finally, it can be noted that a new and easy approach has been brought with ESO method.

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