Computer software in helping the solution of planar mechanisms synthesis

Enri Saliko[1], Dr.ing. Miranda Kullolli[1]

[1]*Department of Mechanical Engineering,* Universiteti Politeknik i Tiranës; Sheshi "Nënë Tereza"; Nr.4

ABSTRACT

Mechanisms are mechanical devices widely used in a large number of applications including household appliances, toys, automobiles and machines, are essential in the development and functioning of almost every machine.

Synthesis is the study of methods of creating mechanisms having a given motion. If the motion is given and the mechanism is to be found this is the essence of kinematic synthesis. In this work will be treated analytical synthesis of four-bar plan mechanisms with the help of complex numbers. Method that brings the problem into a mathematical model ready to solved and further to be stimulated on the computer, specifically the problem is solved mathematically in MathCad (or MATLAB), while simulation is done in SAM.

INTRODUCTION

In either version, the motion is given and the mechanism is to be found. This is the essence of kinematic synthesis. Thus kinematic synthesis deals with the systematic design of mechanism for a given performance.

The areas of synthesis may be grouped into two categories

1. Type synthesis. Given the required performance, what type of mechanism will be suitable? (Gear trains? Linkages? Cam mechanisms?) Also: How many links should the mechanism have? How many degrees of freedom are required? What configuration is desirable? And so on.
2. Dimensional synthesis seeks to determine the significant dimensions and the starting position of a mechanism of preconceived type for a specified task and prescribed performance.

Tasks of kinematic synthesis

In function generation rotation or sliding motion of input and output links must be correlated. For example, synthesize a four bar linkage to generate the function y=logx (figure 1) where x is the angle of the input crank, y is the angle of output rocker

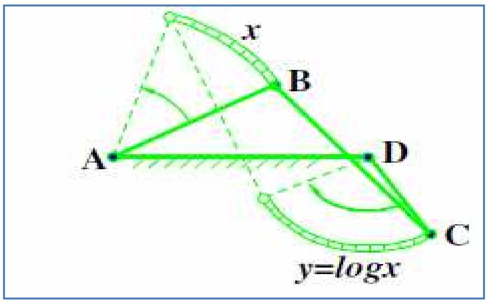

Figure 1 Function generation of a four bar linkage

In path generation is that a coupler point is to generate a path having a prescribed shape. For example the path of point E is a horizontal straight line (figure 2.a), the path of point E is a oval curve (figure 2.b).

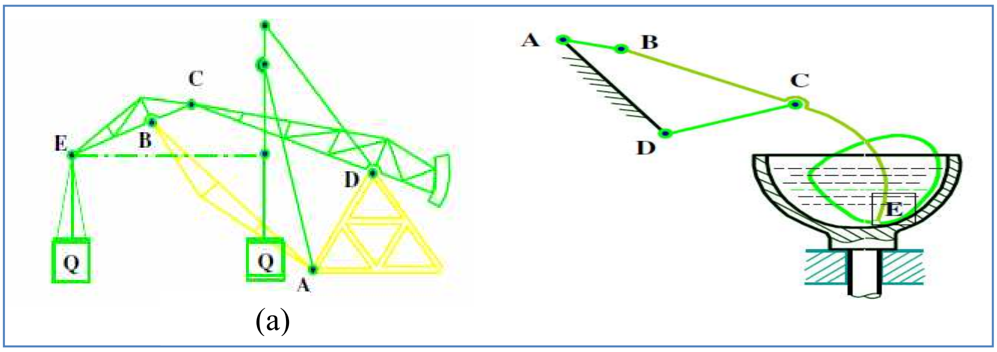

Figure 2 a) Path generation: the path of point E is a horizontal straight line; b) Path generation: the path of point E is a oval curve

In body guidance we are interested in moving an object from one position to another. For example, as for casting mould turnover, the mould must be moved from the horizontal position BC to the vertical positon B'C' (figure 3).

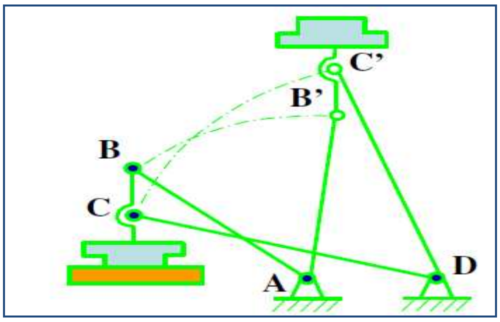

Figure 3 Body guidance

Algebraic methods of synthesis using complex numbers.

Algebraic methods of complex numbers are used in dimensional synthesis of mechanisms with levers, where transmission ratios are presented and intervention in them from mathematical via complex numbers. For each kinematic chain representing a closed contur write:

$$\sum_{k=1}^{n} z_{jk} = 0 \tag{1}$$

z – a complex number that represents a rib of vector polygon; **k** – expresses the number of polygon ribs; a particular position of kinematic chain during its motion. j- a particular positions of kinematics chain during its motion.

Equation (1) is expressed by complex numbers that give the initial position of vector contour.

$$\sum_{k=1}^{n} \epsilon_{jk} z_{jk} = 0 \qquad (2)$$

$\epsilon_{jk} = e^{i\varphi jk}$ - coefficient of rotation for complex number z_{1k}; φ_{jk} - Rotation z_{1k} from position 1 at position j in k (z_{jk}) and while i= -1.

This system of equation is used to determine the dimension of the mechanism through providing some positions of input linkage relatively output linkage, thus realizing approximation of the desired movement on precision points of the first order.

VELOCITY AND ACCELERATION SYNTHESIS BY COMPLEX NUMBERS

In the four-bar linkage O_AABO_B (fig.4), the frame (link 4) is stationary, but the other three links (1, 2, 3) possess angular velocities w_1, w_2, w_3 and angular acceleration ε_1, ε_2, ε_3. The problem considered in this section is to find the link lengths a_1; a_2; a_3; a_4; and the relative positions of the links satisfying angular velocity and acceleration specifications.

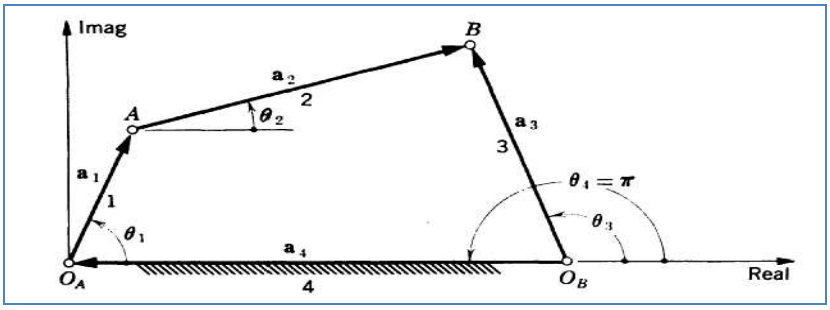

Figure 4 Four- bar linkage and vector polygon

The four bar linkage may be considered to be defined by four vectors a_1; a_2; a_3; a_4;, the linkage will now be taken to consist of a closed vector polygon, for which we may write:

$$a_4 + a_1 + a_2 = a_3$$
$$a_1 + a_2 - a_3 + a_4 = 0$$

Vector "a" may be written $a = ae^{i\theta}$, in which "a" is a distance and θ a counterclockwise angle measured from the real axis, the vector equation of the polygon may be written in complex-number form:

$$a_1 e^{-i\theta_1} + a_2 e^{-i\theta_2} - a_3 e^{-i\theta_3} + a_4 e^{-i\pi} = 0$$

On differentiating with respect to time, setting $\frac{d\theta}{dt} = \omega$

$$i(a_1\omega_1)e^{i\theta_1} + i(a_2\omega_2)e^{i\theta_2} - i(a_3\omega_3)e^{i\theta_3} - a_4(0) = 0$$

A second differentiation yields, after setting $\frac{d\theta}{dt} = \omega$; $\frac{d\omega}{dt} = \varepsilon$ and ordering:

$$i(a_1,\varepsilon_1)e^{i\theta_1} + i^2(a_1,\varepsilon_1)e^{i\theta_1} + i(a_2,\varepsilon_2)e^{i\theta_2} + i^2(a_2,\varepsilon_2)e^{i\theta_2} - i(a_3,\varepsilon_3)e^{i\theta_3} - i^2(a_3,\varepsilon_3)e^{i\theta_3} + a_4(0)$$

After actions have

$$\begin{cases} 1a_1 + 1a_2 - 1a_3 + 1a_4 = 0 \\ \omega_1 a_1 + \omega_2 a_2 - \omega_3 a_3 = 0 \\ (\varepsilon_1 + i\omega_1^2)a_1 + (\varepsilon_2 + i\omega_2^2)a_2 + (\varepsilon_3 + i\omega_3^2)a_3 = 0 \end{cases}$$

The solution may then be carried out by determinant D:

$$D = \begin{vmatrix} 1 & 1 & -1 \\ \omega_1 & \omega_2 & \omega_3 \\ \varepsilon_1 + i\omega_1^2 & \varepsilon_2 + i\omega_2^2 & \varepsilon_3 + i\omega_3^2 \end{vmatrix}$$

The unknowns $a_1; a_2; a_3;$ are expressed in complex-number. For facility a_4 is taken proportional to the determinant D; a_4=-D.

Problem 1. To determine the links of a four-bar mechanism that will in one of its positions satisfy the following specifications:

$$\omega_1 = 8\left[\frac{rad}{sek}\right];\ \omega_2 = 1\left[\frac{rad}{sek}\right];\ \omega_3 = -3\left[\frac{rad}{sek};\right]$$
$$\varepsilon_1 = 8;\ \varepsilon_2 = 20\left[\frac{rad}{sek^2}\right];\ \omega_3 = 0$$

We do automation in MathCad (fig.5.a). The vectors represented by the complex numbers are shown in fig.5.b. The relative lengths of the bars and their terminal points have been established as function of the specified "ω" and "ε" values. The bar a_4 must always be the fixed link, mechanism will defined when the vectors are assembled in order starting from a_4. After we have the defined mechanism stimulate it in SAM software, when can do further analysis for the defined mechanism

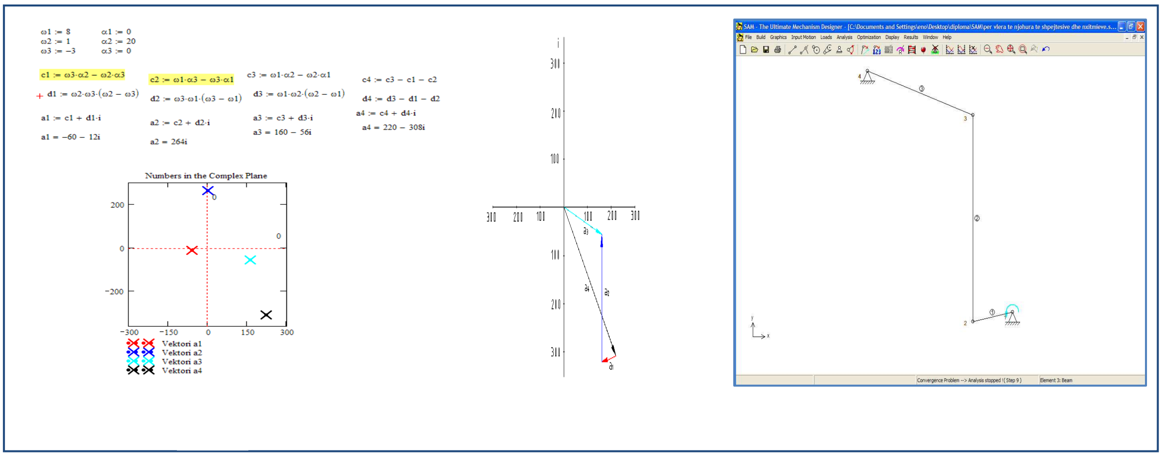

Figure 5 **a)** Automation in Mathcad; b)The vectors represented by the complex numbers; c)Mechanism stimulate in SAM software

COUPLER-CURVE SYNTHESIS

The problem to be considered in this section is the synthesis of a four-bar linkage (fig.6.a), that is to generate a coupler curve prescribed by means of the coordinates of specified accuracy points fig.6.b. As the coupler points passes through these accuracy points , the crank must rotate through prescribed angles $\varphi_2;\ \varphi_3;\ ....$ measured from position 1, fig.6.c.

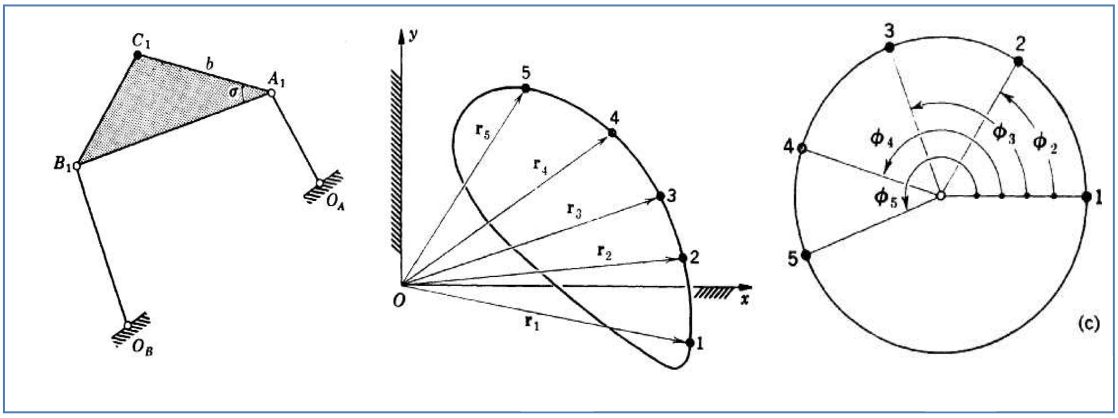

Figure 6 a) Four-bar linkage O_AABO_B with coupler-point C; b)Coupler-curve passing through five specified points to be used in synthesis, shown in position 1; c) Crank rotations correspond to coupler-point position

The design parameters to be used in this synthesis are: the link lengths $\boldsymbol{a_1; a_2; a_3; a_4;}$ the coordinates **(x,y)** of the point O_A with respect to O_A to a coordinate system Oxy; the angle θ between the line O_AO_B dhe aksit ox; the distance b and the axis Ox; the distance b and the angle $\boldsymbol{\sigma}$ defining the coupler point to be used; , φ_1 the initial crank angle. A total of 10 design parameters is thus at hand. Since each accuracy point is given by two coordinates, a maximum of five accuracy points may be specified on matching ten coordinates. For the first time this problem is has been developed by Freudenstein and Sandor.

Relying in their solution have designed computerized program for solving this problem. We give below the highlights of analytical solution. The radius vectors defining the accuracy points are denoted by the complex numbers:

$$r_j = r_j e^{i\theta_j} = r_{jx} + i r_{jy} \quad \text{where} \quad j = 1,2 \ldots 5$$

The configuration of the linkage in position 1, its location relative to the coordinate system oxy, and the location of the coupler point are defined by the complex numbers (fig.7.a)

$$z_k = z_k e^{i\beta_k} = z_{kx} + i z_{ky} \quad \text{where} \quad k = 1,2 \ldots 7$$

A rotation of the crank from position 1 to position j, already defined as φ_j, may be expressed by the complex number $\lambda_j = e^{i\varphi_j}$, thus $\lambda_j z_1$ the product denotes the crank in position j. Similarly, rotations γ_j and ψ_j of the coupler and follower from position 1 to position j may be expressed by the complex number: $v_j = e^{i\gamma_j}$ and $u_j = e^{i\psi_j}$

The radius vector of the coupler point in position 1 (fig.7.b) may now be expressed as a sum of vectors in two different ways as

$$r_1 = z_7 + z_5 + z_1 + z_2 = z_7 + z_4 + z_3$$

In position j fig.9 the radius vector may be expressed as

$$r_j = z_7 + z_5 + \lambda_j z_1 + \gamma_j z_2 = z_7 + u_j z_4 + v_j z_3$$

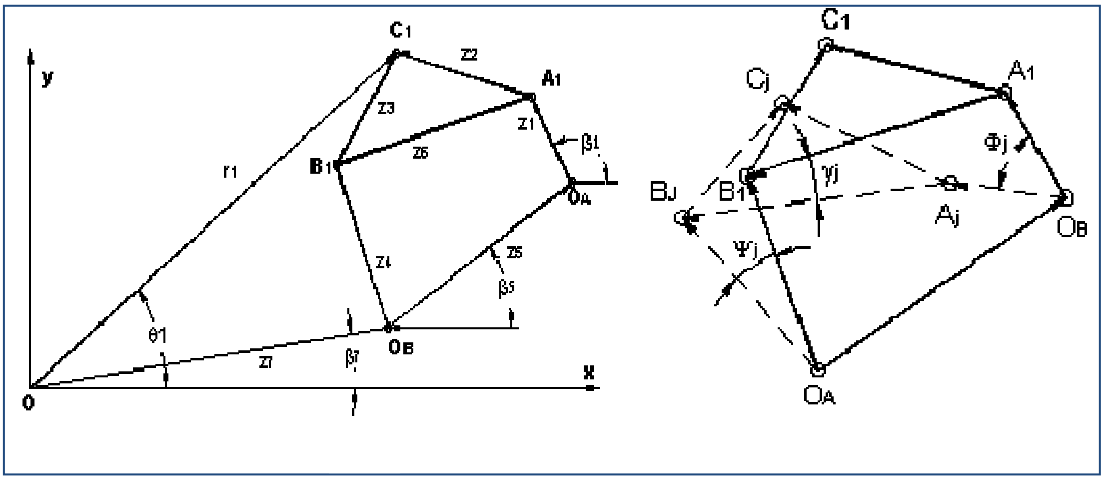

Figure7 a)Vectors to be used in coupler-curve synthesis; b)Displacement of linkage from position 1 to position j

If δ_j denotes the displacement of the coupler point from position 1 to position j, the vectors (or complex numbers) δ_j with j=2,3,4,5 are known and $\delta_j = r_j - r_1$

Or $\delta_j = z_1(\lambda_j - 1) + z_2(v_j - 1) = z_4(u_j - 1) + z_3(v_j - 1)$

With five accuracy points, these last equations must hold for j=2, 3, 4, 5. Since the vectors δ_j and coefficients λ are known, their condition yields two systems of equations. The two systems of equations are then

$$\begin{cases}(v_2 - 1)z_2 + (\lambda_2 - 1)z_1 = \delta_2 \\ (v_5 - 1)z_2 + (\lambda_5 - 1)z_1 = \delta_5\end{cases} \quad (1)$$

$$\begin{cases}(v_2 - 1)z_4 + (\lambda_2 - 1)z_3 = \delta_2 \\ (v_5 - 1)z_4 + (\lambda_5 - 1)z_3 = \delta_5\end{cases} \quad (2)$$

In summary, three cases may be considered:

- No real roots- the synthesis problem has no solution.
- Two real roots- there are two sets of solutions to the first pair $v_{2k} \cdots v_{5k}$ (k=1,2).
- The synthesis problem has 12 solutions.

Example 2. Determine the dimensions of a four-bar linkage to generate a curve passing through the five points shown in fig.10.

Note that points $c_2; c_3; c_4; c_5$ lie on a circle centered at the origin of the coordinate system; the desired coupler curve should approximate the circle as closely as possible between these points. The radius vectors of the five points are defined by their magnitudes and angles $r_j; \theta_j$ where $j = 1,2 \dots 5$. The crank rotations are defined by the angles φ_j where $j = 1,2 \dots 5$ see tab.2. The problem has 12 solutions two of these are considered in tab.3. Linkage 1,2 in which is used as solution of the first pair of compatibility equations and as solution of the second pair of compatibility equations. This linkage is shown in position 1 in fig.11 (SAM).

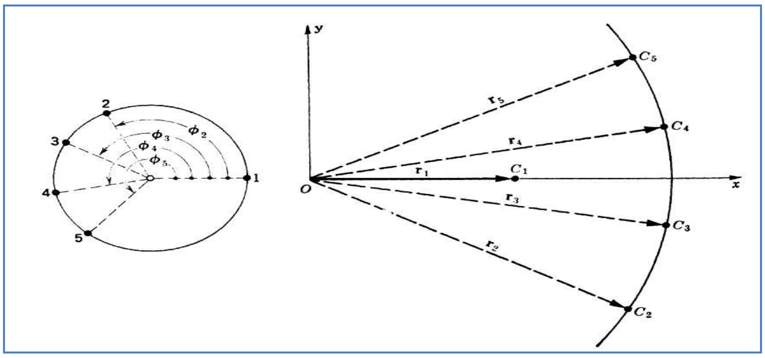

Figure 10 Example of coupler-curve synthesis, specification of crank rotations, and coupler-point positions

Table 2

pozicioni	φ_j	$r_j = \lvert r_j \rvert$	θ_j
1	-	1.0	0
2	117.0	1.740	-29.50
3	150.0	1.740	-10.70
4	191.0	1.740	10.30
5	228.0	1.740	25.90

Tabela 3

	Hallka (1,2)		Hallka (2,1)	
	Komponenti x	Komponenti y	Komponentix	Komponenti y
z_1	-0.48357	0.21780	-0.82939	-0.53627
z_2	1.15443	0.12010	0.96500	-0.04671
z_3	1.23473	-0.61734	0.02689	0.61689
z_4	0.02689	0.61689	1.23475	-0.61734
z_5	0.59077	-0.33925	1.12601	0.58253
z_6	0.08030	0.73745	0.93810	-0.66360
z_7	0.26163	0.00045	-0.26163	0.00045

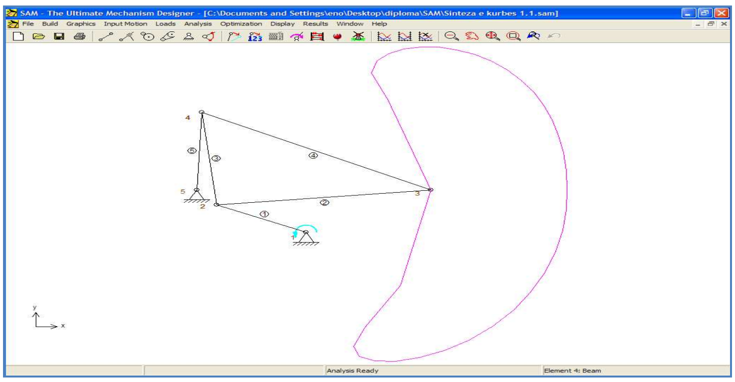

Figure 11 Mechanism stimulate in SAM software

REFERENCES

[1] Norton, R.L. “Design of machinery: an introduction to the synthesis and analysis of mechanisms and machines”. McGraw-Hill, 2003.

[2] Erdman, A.G. Sandor, G.N. Mechanism design. Analysis and synthesis. Prentice-Hall, 1997.

[3] Waldron K.J. Kinzel G.L. Kinematics, dynamics and design of machinery. Wiley, 1999.

[4] Erdman, Arthur G, Sandor, George N, Kota, Sridhar, Mechanism Design Analysis and

Synthesis, Vol 1, pp 514-647, 2001.

[5] Erdman AG, Riley DR (1981), Computer-aided linkage design using the lincages package. ASME Paper, no. 81-DET-121

[6] Jacques Denavit, Richard S.Hartenberg, Kinematic Synthesis of Linkages,Vol 1, pp 321-342, 1964.

[7] Norton R. L., Design of Machinery: “An introduction to the synthesis and analysis of mechanisms and machines”. 3nd edition, pp 188-232, 2008

[8] Xiguang HUANG, Qizheng LIAO, Shimin WEI, Qiang XU. “Five precision point-path synthesis of planar four-bar linkage using algebraic method”. Front. Electr. Electron. Eng. China 2008, 3(4): 470–474; DOI 10.1007/s11460-008-0063-x

[9] Kabileshkumar G. Cheetancheri, Harry H. Cheng, “Spreadsheet-basëd interactive design and analysis of mechanisms using Exceland Ch” Journal homepage: www.elsëvier.com/locate/advengsoft.

[10] Web-Basëd Mechanism Design and Analysis, Softintegration, Inc. [Online]. Available at http://softintegration.com/websër-vices/mechanism/

[11] Embedded Ch, Softintegration, Inc. [Online]. Available at

http://www.softintegration.com/products/sdk/embedd

http://www.artas.n

www.synthetica.eng.uci.edu