Dynamic Response of Structures with TMD

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ABSTRACT

A Tuned Mass Damper (TMD) is a device consisting of a mass, a spring and a viscous damper attached to a vibrating main system. The principles of it are classic but applying them has become possible only after the progress of other related fields like electronics, computer science, new materials and new technologies. Nowadays we have successful applications all around the world. Buildings are getting higher and higher and controlling the displacements would have been a lost match for a structural engineer without the devices of structural control on his side.

The main types of TMDs are presented in this paper followed by the main concepts and basic equations. The paper is focused on Passive Tuned Mass Dampers, which need no external source of energy to function.

A 20-storey reinforced concrete building is analyzed, first without TMD and then with a TMD installed on top. The most important results are obtained by Time History Analysis, although Linear Static Analysis, Modal Analysis and Response Spectrum Analysis have served to determine the basic characteristics of the structure. The Time History Analysis has produced meaningful graphs showing the displacements for different parameters of the TMD.

A TMD is known to be more effective in reducing the response of tall buildings under wind action. This paper gives some results for the response of the 20-storey building under seismic action, trying to find cases when the TMD can be effective under such actions. From harmonic time-history functions to completely irregular seismic time-history functions, they have been tested upon the Finite Element Method Structure of the building, giving the possibility to find the most effective cases of using a TMD.

1. INTRODUCTION

1.1 Tuned Mass Damper Systems

A Tuned Mass Damper (TMD) system consists of a mass, a spring and a viscous damper attached to the structure (usually on top of it, as shown in Figure 1.1) with the purpose to reduce its dynamic response.

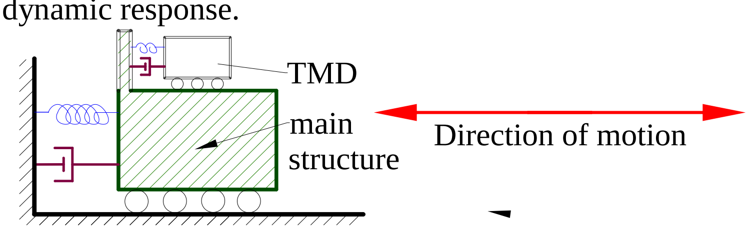


Figure 1.1 Model of a structure with a TMD

This system is most efficient when the frequency of the damper is tuned to a particular structural frequency so that when that frequency is excited, the damper resonates out of phase with the structure. There are different types of TMDs, with different schemes (pendulum TMD, compound pendulum TMD, mass on rubber bearings etc.), but almost all of them can be represented by a simple scheme like that of the Figure 1.1.

1.2 Basic equations of motion

To "tune" the frequency of a TMD means setting the frequency of it equal to the fundamental frequency of the main structure. The basic characteristics of the TMD systems have been studied by various authors using an idealized two degree of freedom system consisting of two masses (one of the structure and another of the TMD) connected by dash-pots and springs (like those on Figure 1.1). However, the model in this paper is a multi degree of freedom model (it is nowadays possible to easily analyze it with a Finite Element Method software such as Sap2000). According to Jerome J. Connor (see [1] in the references section), for the two degree of freedom structure under harmonic excitation, the response in the case of no TMD installed is given by:

$$u_{\max} = \frac{p_{\max}}{k} \cdot \left(\frac{1}{2\xi} \right) \quad (1)$$

where:

$u_{\max}$ - amplitude of displacements of the main structure

$p_{\max}$ - amplitude of the harmonic excitation

k - stiffness of the structure

ξ - damping ratio of the main structure

For the case of a structure with a TMD, the response is given by equation (2) below, using the concept of equivalent damping ratio ξ_e defined in equation (3) below (note that the index "d" is used for TMD parameters).

$$\begin{cases} u_{\max} = \dfrac{p_{\max}}{k} \cdot \left(\dfrac{1}{2\xi_e} \right) \\ u_{d,\max} = \dfrac{1}{2\xi_d} \cdot u_{\max} \end{cases} \quad (2)$$

$$\xi_e = \frac{\overline{m}}{2} \cdot \sqrt{1 + \left(\frac{2\xi}{\overline{m}} + \frac{1}{2\xi_d} \right)^2} \quad (3)$$

where:

$\overline{m}$ - mass ratio (mass of the damper/mass of the main structure, or m_d/m)

ξ_d - damping ratio of TMD

The phase shift of the tuned mass is 90º to the response of the main structure (not shown in the equations, see [1]). This difference in phase is responsible for the energy dissipation of such systems. Equation (3) shows the theoretical contribution of the damper parameters to the total damping. Theory shows that increasing the mass ratio gives more damping. On the other side, decreasing the damping ratio for the TMD also increases the damping. There are practical limits to these parameters, because the main structure has to resist bigger weight loads if we increase the mass of the TMD and more room is needed for the TMD to allow bigger displacements if we decrease the damping ratio of TMD. Design of these systems requires a compromise between these two constraints (the mass and the displacements), therefore various studies have been made regarding the optimal parameters of TMDs.

1.3 Optimal parameters

Den Hartog gave the optimal values of the damping ratio of TMD and the frequency ratio as a function of mass ratio $\overline{m}$ (see also [2-3] in the references section):

$$\rho_{opt} = \frac{\omega_d}{\omega} = \frac{1}{1+\overline{m}} \tag{4}$$

where ρ_{opt} is the optimal frequency ratio ;

$$\xi_{d,opt} = \frac{1}{2}\sqrt{\frac{\frac{3\overline{m}}{2}}{1+3\overline{m}}} \tag{5}$$

where $\xi_{d,opt}$ is the optimal damping ratio of the TMD.

2. STRUCTURAL ANALYSIS AND RESULTS

2.1 Structural analysis of a system without TMD

A 20-storey building is modeled and analyzed in this paper. The general plan of the structure is shown in Figure 2.1. The storey height is 3.5m, the concrete of the structural elements is grade C-30/37 and the reinforcement steel is grade S-500. The dead load on the floors is $5KN/m^2$, the dead load of the infill walls is $3KN/m^2$ and the live load on the floor is $3KN/m^2$. Seismic loads and other loads will be introduced further in the paper.

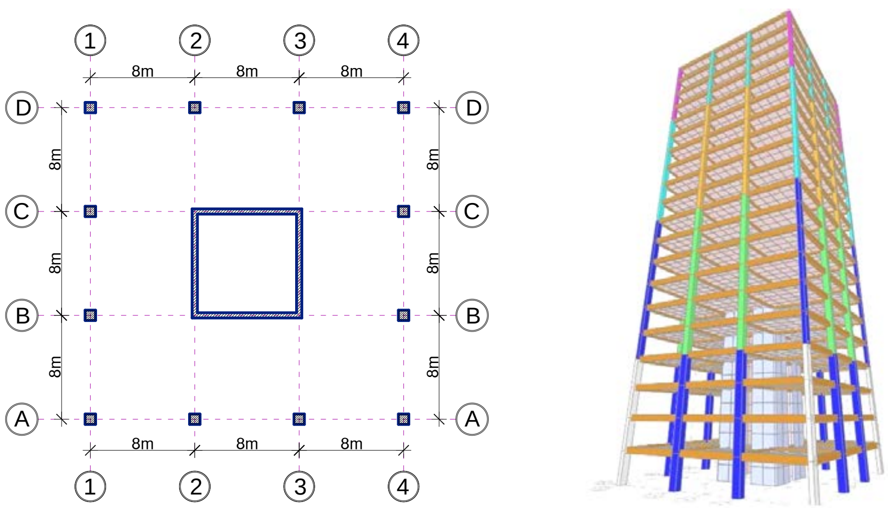


Figure 2.1 Plan view and 3D view of the model

The cross section of the beams is rectangular 40x70 cm, while the cross section of the columns varies from base to top, from square section 90x90 (base) to 60x60 (top).

Modal analysis results indicate that the fundamental period of the structure is T=2.009s and the fundamental frequency is □=3.128rad/s. The first vibration modes corresponding to a main axis of the structure are shown in Figure 2.2. Because of the symmetry, the first vibration modes are similar for the other axis.

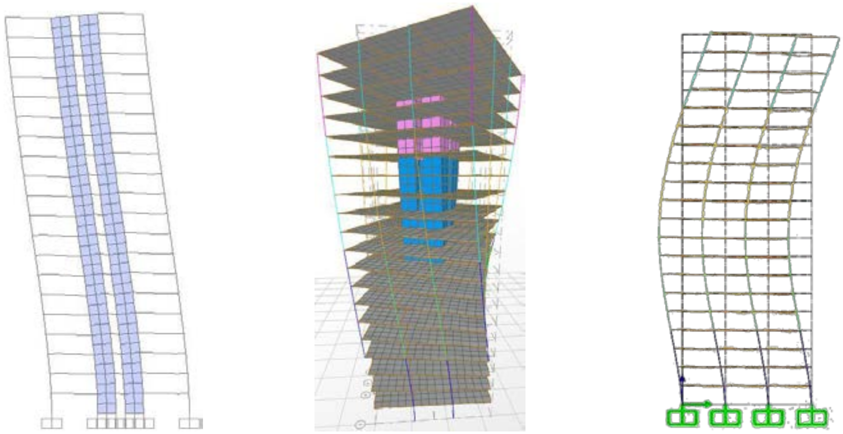


Fig. 2.2 – First modes, from left to right, mode 1, mode 3 and mode 4

2.2 Analysis of the Tuned Mass Damper system

The same structure is studied in the case of a TMD installed on top of it. The results of the modal analysis of the structure without TMD were used to calculate the damping system. The main structure damping is assumed $\xi = 5\%$, a typical value for a reinforced concrete building. The equations (2) to (5) may be used for a preliminary design of the TMD parameters. Of course this is not the case of a correct implementation of these equations for design as long as we do not have a two degree of freedom structure and we don't have a harmonic excitation. The value of ξ_e in equation (3) is taken equal to 0.1 (assuming that we aim to achieve a 10% damping). Therefore:

$$\frac{\overline{m}}{2} \cdot \sqrt{1 + \left(\frac{2 \cdot \xi}{\overline{m}} + \frac{1}{2\xi_d} \right)^2} = 0.1; \tag{6}$$

After entering the value of $\xi = 5\%$ and making further transformations, the equation is:

$$\frac{\overline{m}}{2} \cdot \sqrt{1 + \left(\frac{2 \cdot 0.05}{\overline{m}} + \frac{1}{2\xi_d} \right)^2} = \frac{\overline{m}}{2} \cdot \sqrt{1 + \left(\frac{0.1}{\overline{m}} + \frac{1}{2\xi_d} \right)^2} = 0.1 \tag{7}$$

Maximal relative displacement of the damper is given by equation (2). Therefore, the equations (8) and (9) are obtained:

$$2\xi_d = \frac{u_{\max}}{u_{\max,d}} \tag{8}$$

$$\frac{\overline{m}}{2}\cdot\sqrt{1+\left(\frac{0.1}{\overline{m}}+\frac{u_{\max,d}}{u_{\max}}\right)^2}=0.1 \tag{9}$$

In this preliminary design, we may accept $u_{\max,d}/u_{\max}=10$, which means that the displacements of the ideal two degree of freedom structure under harmonic excitation are 10 times smaller than the TMD displacements. The equation (10) is to be solved in order to obtain the mass ratio.

$$\frac{\overline{m}}{2}\cdot\sqrt{1+\left(\frac{0.1}{\overline{m}}+10\right)^2}=0.1 \tag{10}$$

The solution of equation (10) gives a mass ratio equal to 0.01. The mass of the main structure is m=17000 tons, therefore m_d=170 tons. The damping parameters are calculated below (note that c_d is the damping coefficient of the TMD):

$$\xi_d=\frac{1}{2}\cdot\left(\frac{u_{\max}}{u_{\max,d}}\right)=\frac{1}{2}\cdot\left(\frac{1}{10}\right)=0.05\ ; \tag{11}$$

$$c_d=2\xi_d\cdot\omega_d\cdot m_d=2\cdot\xi_d\cdot\frac{2\pi}{T_d}\cdot m_d=2\cdot\xi_d\cdot\frac{2\pi}{T}\cdot m_d=2\cdot 0.05\cdot\frac{2\pi}{2.0473}\cdot 170=$$
$$=52.17\frac{KN\cdot s}{m} \tag{12}$$

Equation (13) is derived from frequency tuning ($\omega=\omega_d$). Given that the stiffness of the main structure is k=165894 KN/m, the TMD stiffness is equal to:

$$k_d=\overline{m}\cdot k=0.01\cdot k=0.01\cdot 165894=1659\frac{KN}{m} \tag{13}$$

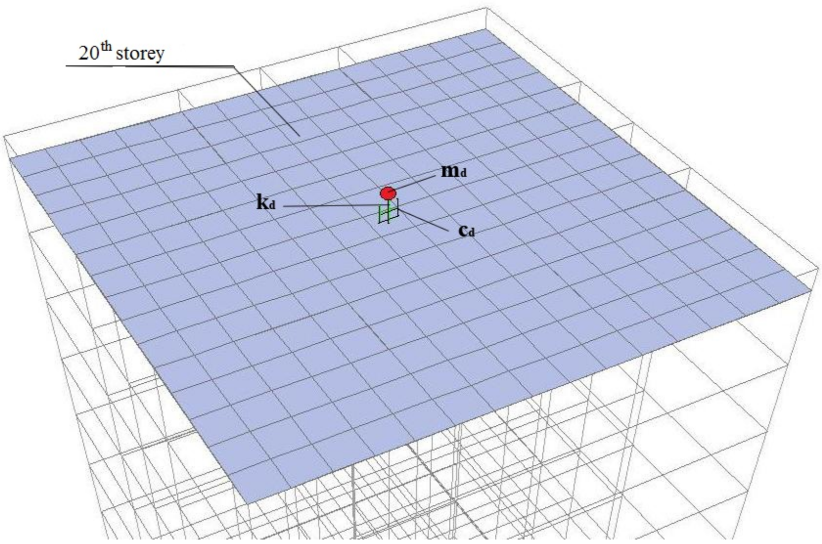


Figure 2.3 – “Link” element modeling the TMD on the 20th storey

The TMD is modeled as a “link” element in the 3D model in Sap2000. The element has two joints, the first one corresponds to the top floor of the structure and the other joint is above the first one, with a mass equal to m_d. The model is shown in Figure 2.3.

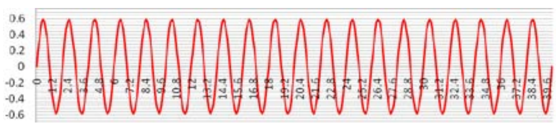
2.3 Structural response for various parameters of TMD and different excitations

Table 1 describes the parameters for mass ratios equal to 0.1%, 0.5%, 1% and 2%. The structure was analyzed for these different mass ratios.

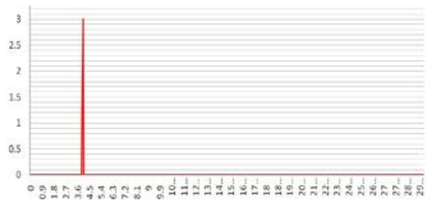
Table 1 – Parameters of the TMD for various mass ratios

Mass ratio $\overline{m}$	Mass of TMD m_d	Optimal frequency ratio ρ_{opt}	Optimal damping ratio of TMD $\xi_{d,opt}$	Frequency of TMD ω_d	Stiffness of TMD k_d	Damping coefficient c_d
$\frac{m_d}{m}$	$\overline{m} \cdot m$	$\frac{1}{1+\overline{m}}$	(see eq. (5))	$\rho_{opt} \cdot \omega$	$\omega^2 \cdot m_d$	$2\xi_{d,opt}\omega_d m_d$
0.001	17	0.999	0.019	3.137	167.341	2.062
0.005	85	0.995	0.043	3.124	831.283	22.829
0.010	170	0.990	0.060	3.108	1649.110	63.780
0.020	340	0.980	0.084	3.078	3245.050	176.082

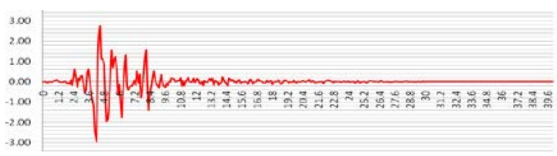
TMD structures will be studied for different excitations represented by their accelerograms shown in Figure 2.4 below (horizontal axis represents “time”, vertical axis represents the “acceleration”):



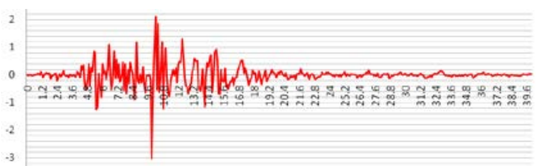
Sinusoidal excitation with T=2s (almost equal to the fundamental period of the main structure)



A single-shock excitation (an impact load or a very short idealized earthquake)



Lexington accelerogram, representing the Lexington earthquake, lasting only a few seconds



Santa Monica accelerogram, representing the Santa Monica earthquake, lasting longer than Lexington.

Fig. 2.4 – Various input accelerograms for the analysis of the structure

The response of the structure under each excitation mentioned above is plotted in the graphs shown in Figure 2.5 to 2.7. Figure 2.5 displays the horizontal displacements on top of the structure as a function of time, for different mass ratios, for the sinusoidal excitation and for the single shock excitation (theoretical accelerograms).

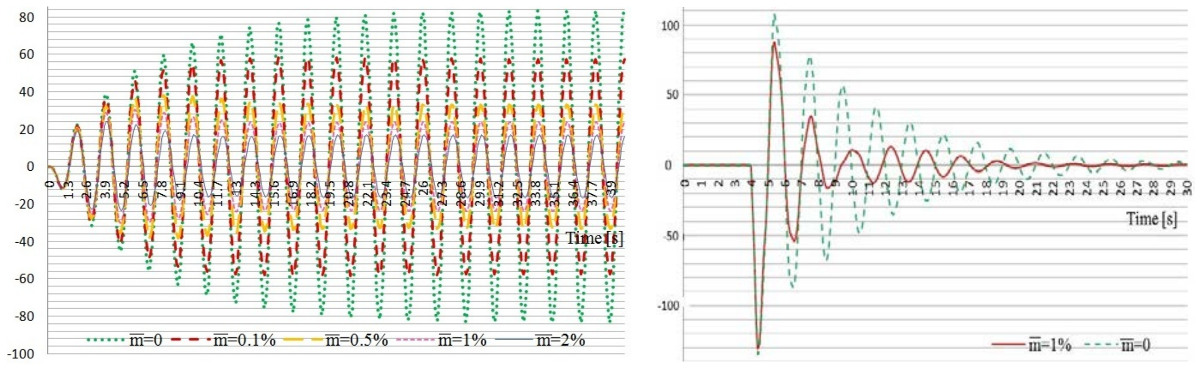


Figure 2.5 – Horizontal displacements on top of the main structure for sinusoidal accelerogram (left), single shock accelerogram (right).

The response reduction under the sinusoidal excitation is significant. Increasing the mass of the TMD increases the damping. In the case of a single shock, the TMD serves only to reduce the displacements after a few cycles (after one cycle, in this case). The maximum displacement is not reduced. This fact limits the use of the TMDs under such excitations.

Figure 2.6 shows the horizontal displacements for the structure under Lexington earthquake loading and Figure 2.7 corresponds to Santa Monica earthquake loading.

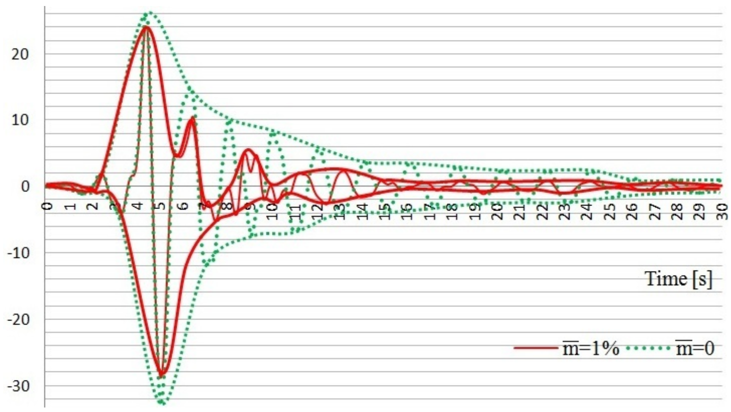


Figure 2.6 – Horizontal displacements on top, Lexington accelerogram

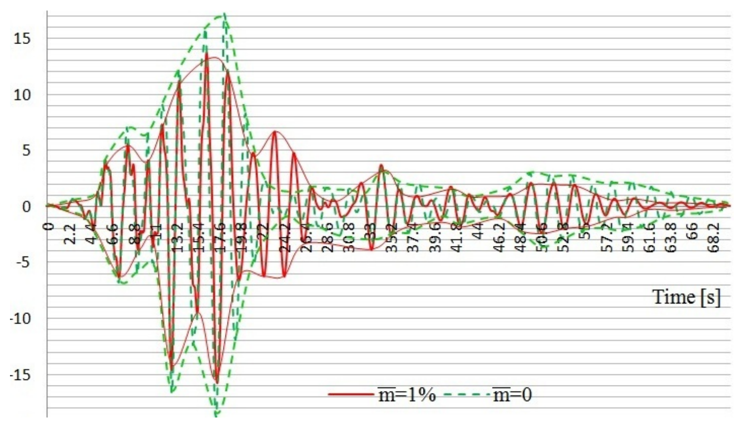


Figure 2.7 – Horizontal displacements on top, Santa Monica accelerogram

The peak values are joined together in Figures 2.6 and 2.7 with a line to make the view more clear. The graphs for other mass ratios are not shown in these figures (only mass ratio 1% is shown). Comparing Figure 2.5 with Figures 2.6 and 2.7, the response in the case of Lexington accelerogram is similar to the response in case of a single-shock accelerogram. Santa Monica accelerogram produces an irregular response, with some values higher than the values in case of no TMD. However, the maximum value is slightly reduced.

3. CONCLUSION

This paper highlights some cases when a TMD is effective or not. It has been shown that the case where the TMD system is most effective is the case of a harmonic excitation with a period close to the fundamental period of the structure. This might be the case of wind excitation to tall buildings or sometimes the case of distant earthquakes and with long duration in time. The TMD did not reduce the maximum value of horizontal displacements in the building subjected to “single shock” earthquakes or other types of similar accelerograms, but it did help reducing the displacements after the “shock”.

REFERENCES

[1]	Jerome J. Connor	Introduction to Structural Motion Control
[2]	H. Iemura	Principles of TMD and LTD – Basic principles and design procedure
[3]	T.T.Soong , M.C. Costantinou	Passive and active structural vibration control in civil engineering