The Computational Modeling of the Non-Constraint Mechanical Systems

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ABSTRACT

This study has contained the computer programming models which have been considering the equations of the motion in the Lagrange dynamics. The obtained equations for a mechanical system with sliding coulomb friction have been presented. Furthermore, the equations of a motion with sliding coulomb friction have been simulated by using MATLAB-SIMULINK program. Finally, some results have been obtained from the computer programming models of the non-constraint mechanical systems in Lagrange dynamics.

Keywords: Analytical Dynamics, Lagrange Dynamics, Coulomb Frictions, Sliding Frictions, MATLAB-SIMULINK, Computer Modeling, non-constraint mechanical systems.

INTRODUCTION

One of the central problems in the field of the mechanics is the determination of motion pertinent constrained and non-constrained systems. The problem dates at least as far back a Lagrange(1787), who devised the method of Lagrange multipliers specifically to handle constrained motion has since been worked on intensively by numerous scientists, including Volterra, Boltzmann, Hamel, Novozhilov, Whittaker, and Synge, to name a few.

About 100 years after Lagrange, Gibbs (1879) and Appell (1899) independently devised what is today known as the Gibbs-Appell method for orbiting the equations of motion for constrained and non-constrained mechanical systems whit non integrable equality constraints. The Gibbs-Appell approach relies on choosing certain quasicoordinates and eliminating others, thereby falling under the general category of elimination method [4].

More recently, an explicit equation describing constrained motion of both conservative and nonconservative dynamical systems within the confines of classical mechanics was developed by [3]. They used as their starting point Gauss's principle (1829) and considered general bilateral constraints that could be both nonlinear in the generalized velocities and displacements and explicitly dependent on time. Furthermore, their result does not require the constraints to be functionally independent [5]-[6].

In the Analytic Mechanics, the higher order Lagrange and Hamilton Equations of Mechanical systems on the Extended Vector Bundles were established by [1].

Moreover, the computer modeling of some mechanical systems were studied with using the some computer programs by [2].

In this paper, the programming in the MATLAB-SIMULINK of the explicit equations of motion general have been set up, conservative and nonconservative, dynamical systems under the influence of a general class of nonideal bilateral constraints.

LAGRANGE MECHANICAL SYSTEMS

A mechanical system depends on the some differential geometric structure: a phase space, configuration space and their mechanical coordinates $q_1, q_2, ..., q_n$ and $\dot{q}_1, \dot{q}_2, ..., \dot{q}_n$. If T and V is represented the kinetics energy and the potential energy o f a mechanical system respectively then the Lagrange function of the mechanical system is denoted by $L = T - V$. Let t be time parameter. Then we can write the Lagrange equations of the mechanical system as below;

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \qquad (i = 1,2,3,\ldots,\mathrm{n} \qquad (1)$$

Now we can make return Cartesian coordinates $x_1, x_2, ..., x_n$ then the Lagrange equations (1) will be rewritten as below;

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right)-\frac{\partial L}{\partial x_i}=0, \qquad (i=1,2,3,\ldots,n) \tag{2}$$

AN EXAMPLE OF A LAGRANGE MECHANICAL SYSTEM

Here we want to determine the equations of a mechanical model: Consider a particle of unit mass constrained to move in a circle in the vertical plane on a circular ring of radius *R* under the action of gravity.

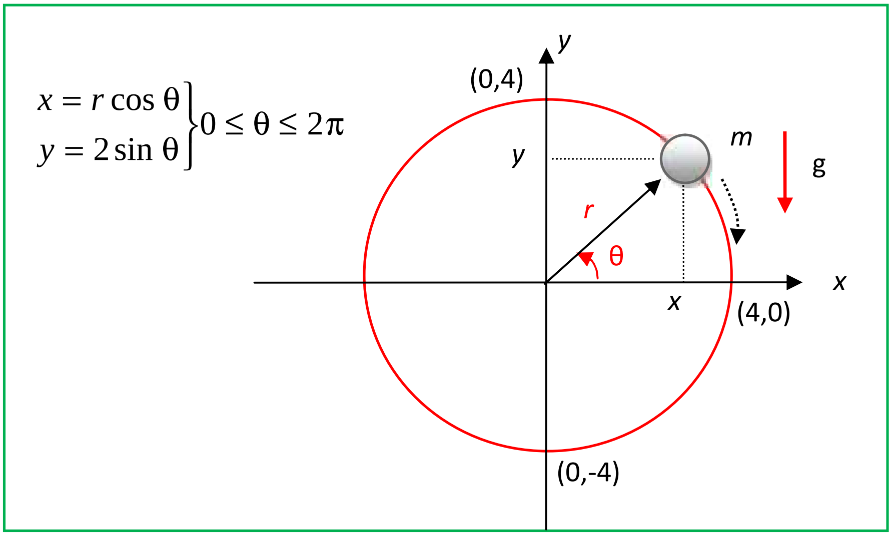

Figure 1. The moving object *m* on the circle.

We are aim to construct the Lagrange equations of this mechanical system. We supposed that a mass moves on the circle with radius *R* by effecting gravitational force *g* in the non-constrained case. Thus the kinetics energy and the potential energy o f this mechanical system can be written the following equations, respectively;

$$T=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right) \tag{3}$$

$$V=mgy \tag{4}$$

Furthermore, if we make the polar coordinate substitution $x = r\cos$¯ , $y = r\sin$¯ in the kinetics and potential energy of the mechanical system (3)-(4) then the we will obtain the following equalities;

$$T=\frac{1}{2}m\left((\dot{r}\cos\theta-r\dot{\theta}\sin\theta)^2+(\dot{r}\sin\theta+r\dot{\theta}\cos\theta)^2\right)=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right) \qquad (5)$$

$$V = mgr\sin\theta \qquad (6)$$

Hence we can get the Lagrange energy function of the mechanical system as below;

$$L = T - V = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - mgr\sin\theta \qquad (7)$$

Thus we prepare the Lagrange equations of the mechanical system (7)

$$\begin{aligned} &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \end{aligned} \qquad (8)$$

$$\begin{aligned} &\frac{d}{dt}\left(\frac{1}{2}m2\dot{r}\right) - \left(\frac{1}{2}m2r\dot{\theta}^2 - mg\sin\theta\right) = 0 \\ &\frac{d}{dt}\left(\frac{1}{2}m2r^2\dot{\theta}\right) - \left(-mgr\cos\theta\right) = 0 \end{aligned} \qquad (9)$$

From some operations we can obtain the following equations.

$$\begin{aligned} &m\ddot{r} - mr\dot{\theta}^2 + mg\sin\theta = 0 \\ &mr^2\ddot{\theta} + 2m\dot{r}\dot{\theta} + mgr\cos\theta = 0 \end{aligned} \qquad (10)$$

THE COMPUTER MODELING OF THE PREVIOUS EXAMPLE

In this section we will set up the modeling of mechanical system in the previous Example. If the modeling of the given mechanical system is prepared and run with respect to MATLAB-SIMULINK then the following results will be obtained. In these models, we have considered the following values;

The gravitational force $g = 9.81$

The initial position point of the mass with respect to polar coordinate: (r_0, θ_0)

The initial velocity components of the mass with respect to polar coordinate: (V_r, V_θ)

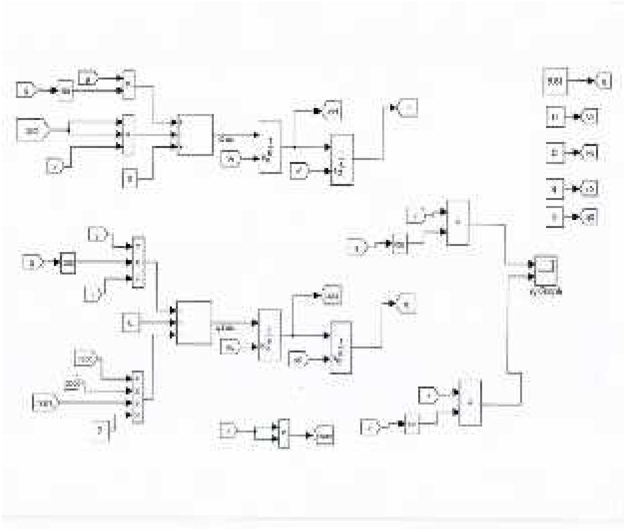

Figure 2: MATLAB-SIMULINK modeling of the mechanical system.

Friction forces have been neglected, and object models without any action begun before the first rate given the speed of modeling and then formed. These models have included in the model the peak in the presence of Coulomb sliding non-friction force, starting from object motion models have been created by changing the speed values and constants. The reaching times of the maximum velocity of the object have been determined.

Table 1: Computing some values of a particle of unit mass constrained to move in a circle in the vertical plane on a circular ring of radius *R* under the action of gravity.

	g	state		Velocity (m/s)		The reaching time of the maximum velocity of the object (second)
		r_0	θ_0	V_r	V_θ	
Figure 3	**9.81**	**4**	**0**	**0**	**0**	**100.1263676142237**
Figure 4		**4**	**0**	**0.5**	**0**	**85.76071912003**
Figure 5		**4**	**0**	**0.5**	**0.5**	**77.95831378207**
Figure 6		**4**	**0**	**0.5**	**0.6**	**47.45476591762**
Figure 7		**4**	**0**	**0.8**	**0.6**	**88.51907527025**

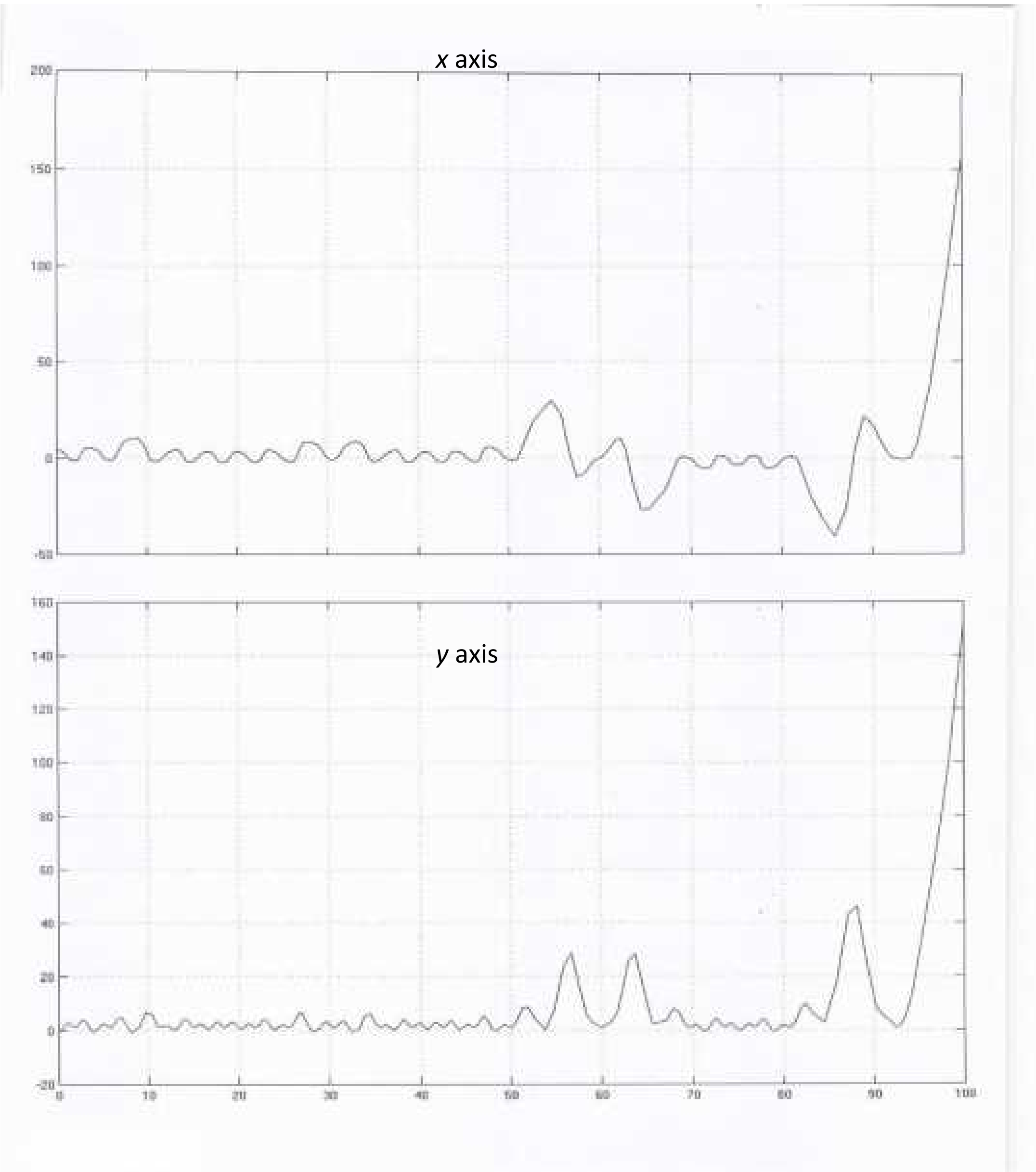

Figure 3: (Model 1) The graph belongs to the mechanical system when the velocity is applied to object without the initial velocity at the point (4,0).

In Model 1 (Figure 3) the friction force has been assumed to be zero. At the starting point of (4,0) and t=0, the initial speed of the mass is zero. The velocity of the mass decreases and increases in the different periods along the x and y axes. From the simulation, we can understand that the mass moves increasing or decreasing velocity and the mass have reached the maximum velocity at about 100.12 seconds on the circle.

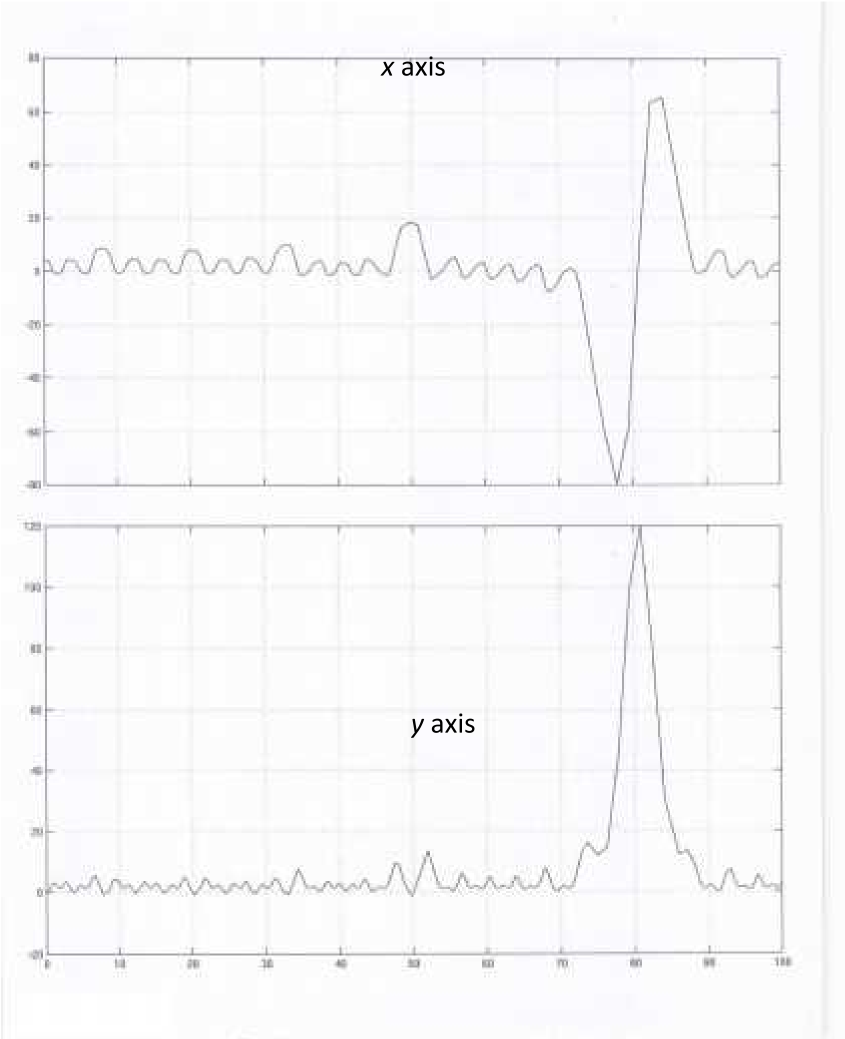

Figure 4: (Model 2) The graph belongs to the mechanical system when the velocity is applied to object only in the direction of the *x* axis at the point (4,0).

In Model 2 (Figure 4) the friction force has been assumed to be zero. At the starting point of (4,0) and $t=0$, the speed applied to the mass is $0.5 m/s$ in a direction parallel to the *x*-axis. The velocity of the mass then decreases and increases along the *x* and *y* axes in the different periods. In the simulation, the mass have reached the maximum velocity at about 80 seconds along the *y* axis and the maximum velocity at about 85 seconds along the *x* axis on the circle.

time

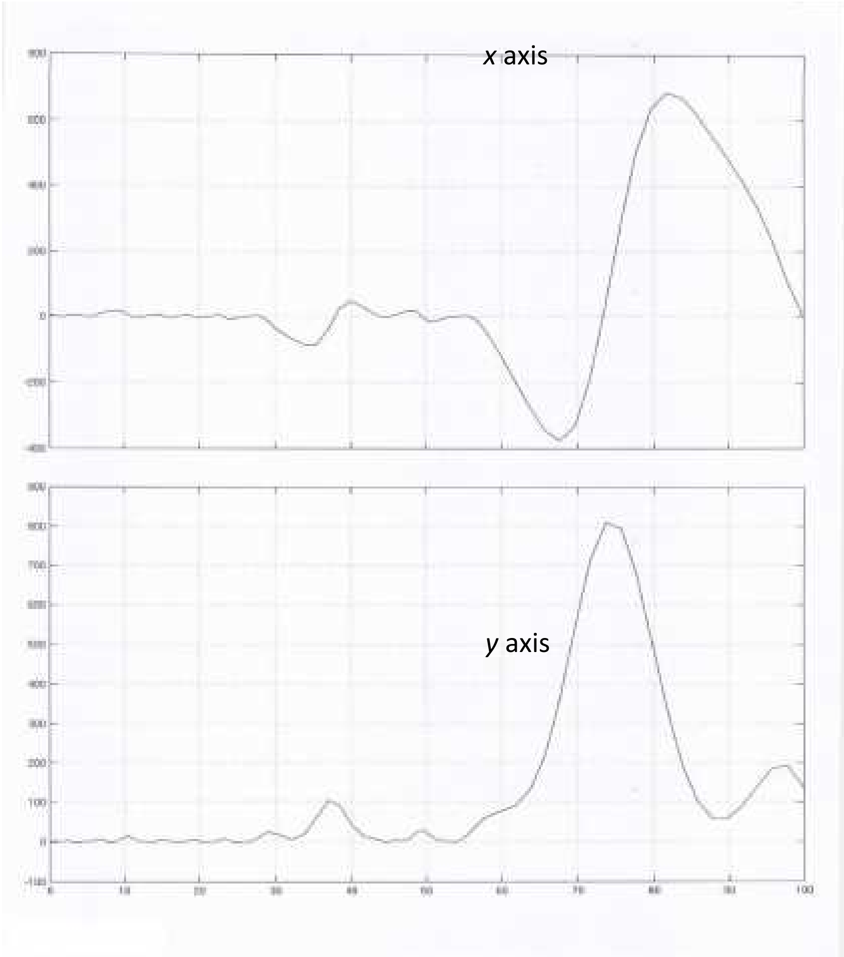

Figure 5: (Model 3) The graph belongs to the mechanical system when the velocity is applied to object in the both direction of the *x* and *y* axes at the point (4,0).

In Model 3 (Figure 5) the friction force has been assumed to be zero. At the starting point of (4,0) and t=0, the speeds applied to the mass are $0.5\,m/s$ in both direction parallel to the *x*-axis and *y*-axis. The velocity of the mass then decreases after 55 seconds and it reached the minimum velocity at 65 seconds along the *x*-axis. After then it reached the maximum velocity along the *x*-axis at about 80 seconds. Moreover in the simulation, the mass have reached the maximum velocity at about 75 seconds along the *y* axis on the circle.

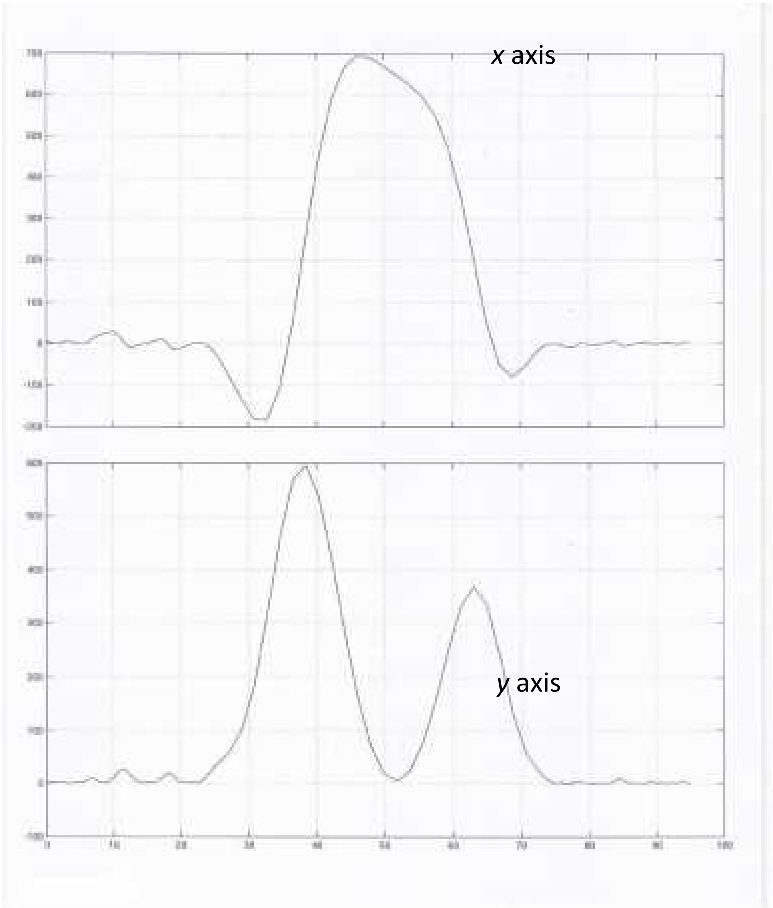

Figure 6: (Model 4) The graph belongs to the mechanical system when the velocity is applied to object in the both direction of the *x* and *y* axes at the point (4,0).

In Model 4 (Figure 6) the friction force has been assumed to be zero. At the starting point of (4,0) and $t=0$, the speeds applied to the mass are $0.5 m/s$ and $0.6 m/s$ in direction parallel to the *x*-axis and *y*-axis, respectively. From this simulation we can see that the mass has reached the minimum velocity and the maximum velocity at 32 seconds and 47 seconds along the *x*-axis, respectively. After then it reached the maximum velocity along the *y*-axis at about 38 seconds. Finally, the velocity of the mass has decreased and the movement of the mass has stopped at the 95 seconds.

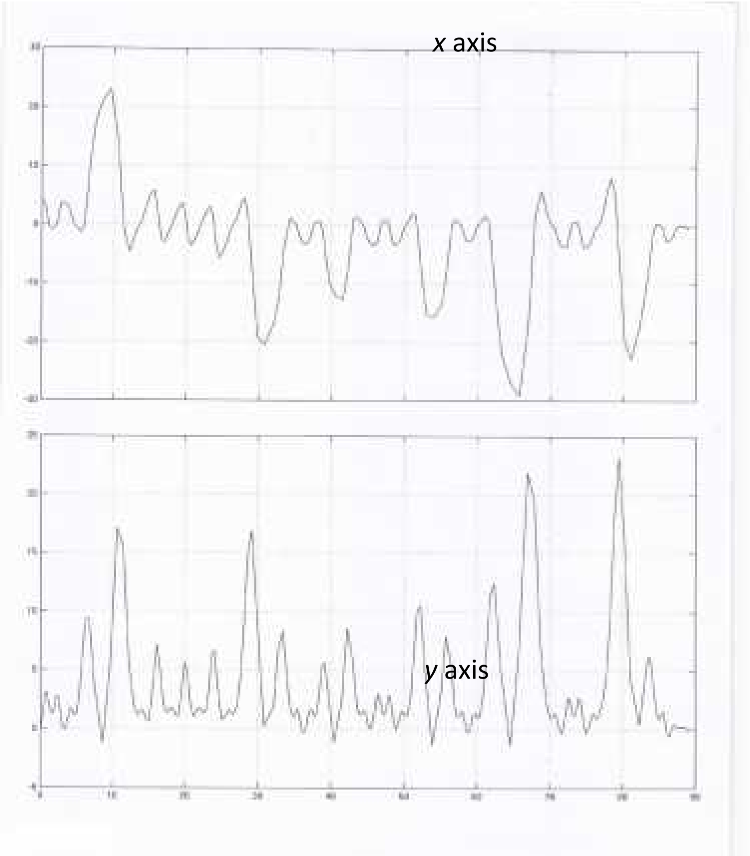

Figure 7: (Model 5) The graph belongs to the mechanical system when the velocity is applied to object in the both direction of the *x* and *y* axes at the point (4,0).

In Model 5 (Figure 7) the friction force has been assumed to be zero. At the starting point of (4,0) and t=0, the speeds applied to the mass are $0.8\,m/s$ and $0.6\,m/s$ in direction parallel to the *x*-axis and *y*-axis, respectively. From this simulation we can see that the mass has reached the maximum velocity at 29 seconds along the *x*-axis. After then the movement of the mass has decreased and finally the movement of the mass has stopped at about 88 seconds.

time

CONCLUSION

In this paper we have presented the generalization of the Lagrangian principle. The power of this principle is illustrated in the simplicity with which it yields a Lagrangian formulation of mechanics applicable to many systems with non-constraints.

Additionally, this article has revealed to be modeled the mechanical systems with the using some computer programs. Hence, in the example of the mechanical model: the circular ring of a rigid body movement in different positions on how the changes were reviewed. Especially starting in different locations act in a rigid body movement and the graph with a zero coefficient of friction was found.

Especially, in these models; when we have applied the different velocities to the object which has continued to decrease and increase, and has reached a maximum speed when it started to the movement at the point (4,0). The movement in the direction of the x and y axis has been seen that they were different each other. The most obvious difference in the resulting model, the same movement of the same objects the same starting position, movement ends in friction, but after the movement in the frictionless conditions continue, the stopping time of the movement have been calculated by the simulation.

In this study, the virtual modeling has helped us about the mechanical behavior of mechanical systems in real space. It has given new insights into the computational mathematical physics and computer modeling of non-constrained motions.

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