

On Intra-Regular Semihypergroups Through Intuitionistic Fuzzy Sets

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ABSTRACT

The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. In this paper, using Atanassov idea, we give some properties of intuitionistic fuzzy hyperideals and intuitionistic fuzzy bi-hyperideals in a semihypergroup. We use the intuitionistic fuzzy left, right, two-sided and bi-hyperideals to characterize the intra-regular semihypergroups, generalizing some known results of intra-regular semigroups.

1. INTRODUCTION AND PRELIMINARIES

Hyperstructure theory was born in 1934 when Marty [1] defined hyper groups, began to analysis their properties and applied them to groups, rational algebraic functions. Now they are widely studied from theoretical point of view and for their applications to many subjects of pure and applied properties. In 1965, Zadeh [2] introduced the notion of a fuzzy subset of a non-empty set X , as a function from X to $[0,1]$. After the introduction of the concept of fuzzy sets by Zadeh, several researches conducted the researches on the generalizations of the notions of fuzzy sets with huge applications in computer, logics and many branches of pure and applied mathematics. Fuzzy set theory has been shown to be a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a certain object belongs to a set. In 1971, Rosenfeld [3] defined the concept of fuzzy group. Since then many papers have been published in the field of fuzzy algebra. Recently fuzzy set theory has been well developed in the context of hyperalgebraic structure theory. A recent book [4] contains a wealth of applications. In [5], Davvaz introduced the concept of fuzzy hyperideals in a semihypergroup. But in fuzzy sets theory, there is no means to

incorporate the hesitation or uncertainty in the membership degrees. As an important generalization of the notion of fuzzy sets on a non-empty set X , in 1983, Atanassov introduced in [6] the concept of intuitionistic fuzzy sets on a non-empty set X which give both a membership degree and a non-membership degree. The only constraint on these two degrees is that the sum must be smaller than or equal to 1. The concept has been applied to various algebraic structures. Since then, the notion of Intuitionistic Fuzzy Sets has been explored by researchers and a number of theoretical and practical results have appeared. Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. The relations between intuitionistic fuzzy sets and algebraic structures have been already considered by many mathematicians. In [10], using Atanassov idea, Davvaz established the intuitionistic fuzzification of the concept of hyperideals in a semihypergroup and investigated some of their properties. Recently in [11], using the intuitionistic fuzzy left, right, two-sided and bi-hyperideals we gave a several characterizations of the regular semihypergroups.

In this paper, we will use the intuitionistic fuzzy left, right, two-sided and bi-hyperideals to characterize the intra-regular semihypergroups. Since intuitionistic fuzzy sets theory is a generalization of fuzzy sets theory, the fuzzy sets can be seen as a special situation of the intuitionistic fuzzy sets.

Recall first the basic terms and definitions from the hyperstructure theory.

Definition 1.1 A map $\circ: H \times H \rightarrow \mathcal{P}^*(H)$ is called hyperoperation or join operation on the set H , where H is a nonempty set and $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$ denotes the set of all nonempty subsets of H .

Definition 1.2 A hyperstructure is called the pair (H, \circ) where \circ is a hyperoperation on the set H .

Definition 1.3 A hyperstructure (H, \circ) is called a semihypergroup if for all $x, y, z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.$$

If $x \in H$ and A, B are nonempty subsets of H then

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\}, \text{ and } x \circ B = \{x\} \circ B.$$

Definition 1.4 A non-empty subset B of a semihypergroup H is called a sub-semihypergroup of H if $B \circ B \subseteq B$ and H is called in this case super-semihypergroup of B .

Definition 1.5 A nonempty subset I of a semihypergroup H is called a right (left) ideal of H if for all $x \in H$ and $r \in I$,

$$r \circ x \subseteq I (x \circ r \subseteq I).$$

A non-empty subset I of H is called a hyperideal (or two-sided hyperideal) if it is both a left hyperideal and right hyperideal.

We call a semihypergroup (H, \circ) a regular semihypergroup if for every $x \in H, x \in x \circ y \circ x$, for some $y \in H$. Hence every regular semigroup is a regular semihypergroup. We call a semihypergroup (H, \circ) an intra-regular semihypergroup if for every $x \in H, x \in y \circ x \circ x \circ z$, for some $y, z \in H$.

Atanassov introduced in [1, 2] the concept of intuitionistic fuzzy sets defined on a non-empty set X as objects having the form:

$$A = \{ \langle x, \sim_A(x), \}_A(x) \mid x \in X \},$$

where the functions $\sim_A : X \rightarrow [0,1]$ and $\}_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\sim_A(x)$) and the degree of non-membership (namely $\}_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \sim_A(x) + \}_A(x) \leq 1$ for all $x \in X$.

Obviously, each ordinary fuzzy set may be written as

$$A = \{ \langle x, \sim_A(x), 1 - \sim_A(x) \rangle \mid x \in X \}.$$

Let A and B be two intuitionistic fuzzy sets on X . The following expressions are defined in [7, 8, 9].

1. $A \subseteq B$ if and only if $\sim_A(x) \leq \sim_B(x)$ and $\}_A(x) \geq \}_B(x)$ for all $x \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $A^c = \{\langle x, \}_A(x), \sim_A(x) \mid x \in X \}$.
4. $A \cap B = \{\langle x, \min\{\sim_A(x), \sim_B(x)\}, \max\{\}_A(x), \}_B(x) \mid x \in X \}$.
5. $A \cup B = \{\langle x, \max\{\sim_A(x), \sim_B(x)\}, \min\{\}_A(x), \}_B(x) \mid x \in X \}$.
6. $\square A = \{\langle x, \sim_A(x), 1 - \sim_A(x) \mid x \in X \}$.
7. $\diamond A = \{\langle x, 1 - \}_A(x), \}_A(x) \mid x \in X \}$.

For the sake of simplicity, we use the symbol $A = (\sim_A, \}_A)$ for intuitionistic fuzzy set $A = \{\langle x, \sim_A(x), \}_A(x) \mid x \in X \}$.

Definition 1.6 Let A, B be two intuitionistic fuzzy sets in semihypergroup H , then

$$A \cap B = \{\langle x, \min\{\sim_A(x), \sim_B(x)\}, \max\{\}_A(x), \}_B(x) \mid x \in H \},$$

$$A * B = \{\langle x, \sim_{A*B}(x), \}_{A*B}(x) \mid x \in H \}$$

where

$$\sim_{A*B}(x) = \begin{cases} \sup_{x \in y \circ z} \{\min\{\sim_A(y), \sim_B(z)\}\} & \text{if } x \in y \circ z \\ 0 & \text{otherwise} \end{cases}$$

$$\}_{A*B}(x) = \begin{cases} \inf_{x \in y \circ z} \{\max\{\}_A(y), \}_B(z)\} & \text{if } x \in y \circ z \\ 1 & \text{otherwise} \end{cases}$$

Definition 1.7 Let H be a semihypergroup. An intuitionistic fuzzy set $A = (\sim_A, \}_A)$ in H is called an intuitionistic fuzzy semihypergroup in H if for all $x, y \in H$,

$$\inf_{z \in x \circ y} \{ \sim_A(z) \} \geq \min \{ \sim_A(x), \sim_A(y) \} \text{ and } \sup_{z \in x \circ y} \{ \}_A(z) \leq \max \{ \}_A(x), \}_A(y) \}.$$

Definition 1.8 Let H be a semihypergroup. An intuitionistic fuzzy set $A = (\sim_A, \}_A)$ in H is called a left (resp. right) intuitionistic fuzzy hyperideal of H if for all $x, y \in H$,

1. $\sim_A(y) \leq \inf_{z \in x \circ y} \{ \sim_A(z) \}$ (resp. $\sim_A(x) \leq \inf_{z \in x \circ y} \{ \sim_A(z) \}$).
2. $\sup_{z \in x \circ y} \{ \}_A(y) \leq \}_A(y)$ (resp. $\sup_{z \in x \circ y} \{ \}_A(z) \leq \}_A(x)$).

An intuitionistic fuzzy set A in H is called an intuitionistic fuzzy two-sided hyperideal of H if it is both an intuitionistic fuzzy left and an intuitionistic right hyperideal of H .

Definition 1.9 Let H be a semihypergroup. An intuitionistic fuzzy set $A = (\sim_A, \}_A)$ in H is called an intuitionistic fuzzy bi-hyperideal of H if for all $x, y, z \in H$,

$$\inf_{t \in x \circ y \circ z} \{ \sim_A(t) \} \geq \min \{ \sim_A(x), \sim_A(z) \} \text{ and } \sup_{t \in x \circ y \circ z} \{ \}_A(t) \leq \max \{ \}_A(x), \}_A(z) \}.$$

Let A be a subset of a semihypergroup H , then we denote

$$\tilde{A} = \{ \langle x, \sim_{\tilde{A}}(x), \}_A(x) \mid x \in H \}$$

where

$$\sim_{\tilde{A}}(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$$

$$\}_A(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise} \end{cases}$$

Obviously \tilde{A} is an intuitionistic fuzzy set in H . Obviously, semihypergroup H also can be seen as an intuitionistic fuzzy set $\tilde{H} = \{\langle x, 1, 0 \rangle \mid x \in H\}$. Throughout the paper we will use H represent H and \tilde{H} , it is easy to see their mean from context.

Lemma 1.10 [11] *Let $A = (\sim_A, \}_A)$ be a non-empty subset of a semihypergroup H , then*

1. *A is a subsemihypergroup of H if and only if \tilde{A} is an intuitionistic fuzzy semihypergroup of H .*

2. *A is a left (resp. right, two-sided) hyperideal of H if and only if \tilde{A} is an intuitionistic fuzzy left (resp. right, two-sided) hyperideal of H .*

Lemma 1.11 [11] *Let $A = (\sim_A, \}_A)$ be a non-empty subset of a semihypergroup H . A is a bi-hyperideal of H if and only if \tilde{A} is an intuitionistic fuzzy bi-hyperideal of H .*

Lemma 1.12 [11] *An intuitionistic fuzzy set A in a semihypergroup H is an intuitionistic fuzzy semihypergroup of H if and only if $A * A \subseteq A$.*

Lemma 1.13 [11] *For an intuitionistic fuzzy set A of a semihypergroup H , the following conditions are equivalent:*

1. *A is an intuitionistic fuzzy left hyperideal of H .*
2. *$H * A \subseteq A$.*

Lemma 1.14 [11] *For an intuitionistic fuzzy set A of a semihypergroup H , the following conditions are equivalent:*

1. *A is an intuitionistic fuzzy right hyperideal of H .*
2. *$A * H \subseteq A$.*

Lemma 1.15 [11] *For an intuitionistic fuzzy set A of a semihypergroup H , the following conditions are equivalent:*

1. *A is an intuitionistic fuzzy two-sided hyperideal of H .*

2. $H * A \subseteq A$ and $A * H \subseteq A$.

Lemma 1.16 [11] *For an intuitionistic fuzzy set A of a semihypergroup H , the following conditions are equivalent:*

1. A is an intuitionistic fuzzy bi-hyperideal of H .

2. $A * A \subseteq A$ and $A * H * A \subseteq A$.

Lemma 1.17 [11] *Let A be an intuitionistic fuzzy set in a semihypergroup H and B be any intuitionistic fuzzy bi-hyperideal of H . Then $A * B$ and $B * A$ are both intuitionistic fuzzy bi-hyperideal of H .*

Lemma 1.18 [11] *Every intuitionistic left (right) hyperideal of a semihypergroup is also an intuitionistic fuzzy bi-hyperideal.*

Theorem 1.19 [11] *Let H be a semihypergroup. Then the following statements are equivalent:*

1. H is regular;

2. $B * C = B \cap C$ holds for every intuitionistic fuzzy right hyperideal B and every intuitionistic fuzzy left hyperideal C of H .

2. CHARACTERIZATIONS OF INTRA-REGULAR SEMIHYPERGROUPS

In this section we shall give a characterization of an intra-regular semihypergroup by intuitionistic fuzzy right hyperideals and intuitionistic fuzzy left hyperideals.

Theorem 2.1 *Let H be a semihypergroup. Then the following statements are equivalent:*

1. H is intra-regular;

2. $A \cap B \subset B^*A$ holds for every intuitionistic fuzzy right hyperideal A and every intuitionistic fuzzy left hyperideal B of H .

Proof. Let us first assume (1) holds. Let A be an arbitrary intuitionistic fuzzy right hyperideal, B any intuitionistic fuzzy left hyperideal of H and a be an arbitrary element of H . Since H is intra-regular, there exist elements $x, y \in H$ such that $a \in x \circ a \circ a \circ y$. Then we have

$$\begin{aligned} \sim_{B^*A}(a) &= \sup_{a \in y \circ z} \{ \min \{ \sim_B(y), \sim_A(z) \} \} \\ &\geq \min \{ \inf_{t \in a \circ x} \sim_B(t), \inf_{s \in a \circ y} \sim_A(s), \} \\ &\geq \min \{ \sim_B(a), \sim_A(a) \} \\ &= \sim_{A \cap B}(a), \end{aligned}$$

$$\begin{aligned} \} _{B^*A}(a) &= \inf_{a \in y \circ z} \{ \max \{ \} _B(y), \} _A(z) \} \\ &\leq \max \{ \sup_{t \in a \circ x} \} _B(t), \sup_{s \in a \circ y} \} _A(s), \} \\ &\leq \max \{ \} _B(a), \} _A(a) \} \\ &= \} _{A \cap B}(a). \end{aligned}$$

Then we obtain $A \cap B \subseteq B^*A$.

Conversely, let P be any right hyperideal and Q be any left hyperideal of H , a be an arbitrary element of $P \cap Q$. Then by Lemma 1.10, \tilde{P}, \tilde{Q} are an intuitionistic fuzzy right hyperideal and an intuitionistic fuzzy left hyperideal of H respectively. So we have $\tilde{P} \cap \tilde{Q} \subset \tilde{Q}^* \tilde{P}$ and

$$\sup_{a \in y \circ z} \{ \min \{ \sim_{\tilde{Q}}(y), \sim_{\tilde{P}}(z) \} \} \geq \sim_{\tilde{Q}^* \tilde{P}}(a) \geq \sim_{\tilde{P} \cap \tilde{Q}}(a) = \min \{ \sim_{\tilde{P}}(a), \sim_{\tilde{Q}}(a) \} = 1,$$

$$\inf_{a \in y \circ z} \{ \max\{ \}_{\tilde{Q}}(y), \}_{\tilde{P}}(z) \} \leq \}_{\tilde{Q}^* \tilde{P}}(a) \leq \}_{\tilde{P} \cap \tilde{Q}}(a) = \max\{ \}_{\tilde{P}}(a), \}_{\tilde{Q}}(a) \} = 0.$$

This means that there exist elements $b, c \in H, a \in b \circ c$ such that

$$\sim_{\tilde{Q}}(b) = 1, \sim_{\tilde{P}}(c) = 1, \}_{\tilde{Q}}(b) = 0, \}_{\tilde{P}}(c) = 0.$$

Thus $a \in b \circ c \subseteq Q \circ P$, so we have $Q^* P \subseteq Q \circ P$. Since the converse inclusion always holds, we obtain that $Q \cap P = Q \circ P$. So, H is intra-regular.

Theorem 2.2 *Let H be a semihypergroup. The following statements are equivalent:*

1. H is both regular and intra-regular.
2. $A^* A = A$ for every intuitionistic fuzzy bi-hyperideals A of H .
3. $A \cap B \subset (A^* B) \cap (B^* A)$ for all intuitionistic fuzzy bi-hyperideals A and B of H .
4. $A \cap B \subset (A^* B) \cap (B^* A)$ for every intuitionistic fuzzy bi-hyperideal A and every left hyperideal B of H .
5. $A \cap B \subset (A^* B) \cap (B^* A)$ for every intuitionistic fuzzy bi-hyperideal A and every intuitionistic fuzzy right hyperideal B of H .
6. $A \cap B \subset (A^* B) \cap (B^* A)$ for every intuitionistic fuzzy right hyperideal A and every intuitionistic fuzzy left hyperideal B of H .

Proof. It is clear that (3) implies (4), (4) implies (6), (3) implies (5), (5) implies (6) and (3) implies (2). So we will prove that (1) implies (3), (6) implies (1), and (2) implies (1).

First, let us assume that (1) holds. In order to prove that (3) holds, let A and B be any intuitionistic fuzzy bi-hyperideals of H , and a an arbitrary element of H . Then, since H is regular, there exists an element $x \in H$ such that $a \in a \circ x \circ a \subseteq a \circ x \circ a \circ x \circ a$.

Since H is intra-regular, there exist elements $y, z \in H$ such that $a \in y \circ a \circ a \circ z$. Thus we have

$$a \in a \circ x \circ a \subseteq a \circ x \circ a \circ x \circ a \subseteq a \circ x \circ (y \circ a \circ a \circ z) \circ x \circ a = (a \circ x \circ y \circ a) \circ (a \circ z \circ x \circ a).$$

Since A and B are both intuitionistic fuzzy bi-hyperideals of H , we have

$$\inf_{t \in a \circ x \circ y \circ a} \sim_A(t) \geq \min\{\sim_A(a), \sim_A(a)\} = \sim_A(a),$$

$$\inf_{t \in a \circ x \circ y \circ a} \sim_B(t) \geq \min\{\sim_B(a), \sim_B(a)\} = \sim_B(a).$$

And

$$\sup_{t \in a \circ x \circ y \circ a} \}_A(t) \leq \max\{\}_A(a), \}_A(a)\} = \}_A(a),$$

$$\sup_{t \in a \circ x \circ y \circ a} \}_B(t) \leq \max\{\}_B(a), \}_B(a)\} = \}_B(a).$$

Thus we have

$$\sim_{A \circ B}(a) = \sup_{a \in p \circ q} \{\min\{\sim_A(p), \sim_B(q)\}\}$$

$$\geq \min\{\inf_{t \in a \circ x \circ y \circ a} \sim_A(t), \inf_{s \in a \circ z \circ x \circ a} \sim_B(s)\}$$

$$\geq \min\{\sim_A(a), \sim_B(a)\}$$

$$= \sim_{A \cap B}(a),$$

$$\}_A \circ B(a) = \inf_{a \in p \circ q} \{\max\{\}_A(p), \}_B(q)\}$$

$$\leq \max\{\sup_{t \in a \circ x \circ y \circ a} \}_A(t), \sup_{s \in a \circ z \circ x \circ a} \}_B(s)\}$$

$$\leq \max\{\}_A(a), \}_B(a)\}$$

$$= \} _{A \cap B}(a).$$

and so we have

$$A \cap B \subset A \circ B.$$

In similiar way it can be shown that

$$A \cap B \subset B \circ A.$$

Thus we obtain

$$A \cap B \subset (A * B) \cap (B * A).$$

and that (1) implies (3).

Let us now assume (6) holds. Let A and B be any intuitionistic fuzzy right hyperideal and any intuitionistic fuzzy left hyperideal of H , respectively. Then we have

$$A \cap B \subset (A * B) \cap (B * A) \subset (B * A).$$

Then it follows from Theorem 2.1 that H is intra-regular. On the other hand,

$$A \cap B \subset (A * B) \cap (B * A) \subset (A * B).$$

By Lemma 1.13 and Lemma 1.14 we obtain

$$A * B \subseteq A * H \subseteq A \text{ and } A * B \subseteq H * B \subseteq B,$$

and we have $A * B \subseteq A \cap B$. Thus we obtain that

$$A * B = A \cap B.$$

Thus it follows from Lemma 1.19 that H is regular. Thus we obtain that (6) implies (1).

Finally, let us assume that (2) holds. In order to prove that (1) holds, let C be any bi-hyperideal of H , and a any element of C . Since it follows from Lemma that \tilde{C} is an intuitionistic fuzzy bi-hyperideal of H , we have

$$\sup_{a \in p \circ q} \{ \min \{ \sim_{\tilde{c}}(p), \sim_{\tilde{c}}(q) \} \} \geq \sim_{\tilde{c} * \tilde{c}}(a) \geq \sim_{\tilde{c}}(a) = 1,$$

$$\inf_{a \in p \circ q} \{ \max \{ \}_{\tilde{c}}(p), \}_{\tilde{c}}(q) \} \leq \}_{\tilde{c} * \tilde{c}}(a) \leq \}_{\tilde{c}}(a) = 0.$$

This implies that there exist elements $b, c \in H$ with $a \in b \circ c$ such that

$$\sim_{\tilde{c}}(b) = \sim_{\tilde{c}}(c) = 1, \}_{\tilde{c}}(b) = \}_{\tilde{c}}(c) = 0.$$

Then we have

$$a \in b \circ c \subseteq C \circ C,$$

and so we have

$$C \subset C \circ C.$$

Since C is a bi-hyperideal of H , the converse inclusion always holds. Thus we obtain that

$$C \circ C = C.$$

Then it follows from Lemma 1.11 that H is both regular and intra-regular. Therefore we obtain that (2) implies (1). This completes the proof.

CONCLUSIONS

In this paper, continuing our study initiated in [11], we have characterized another class of semihypergroups, that of intra-regular semihypergroups through intuitionistic fuzzy left, right, two-sided and bi-hyperideals. The main results of the paper are those described in Theorems 2.1 and 2.2 generalizing some known results obtained for intra-regular semigroups directly in semihypergroups using intuitionistic fuzzy sets.

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