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# The Necessity of Integration of Linguistic Competences in the Field of Mathematics for an Optimal Adoption of Mathematical Knowledge

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## Abstract

In the recent years, some studies have shown the dependence of mathematical performance to a large number of factors including until nowadays neglected extra-mathematical ones. In spite of a Platonist optics, for which the mathematical concepts exist in an independent way from the texts describing or from the persons using them, the language cannot be neglected or left aside. Given that thinking is a form of communication, it results due to perceptual factors also closely related to the quality of communication that in its turn is affected by the quality of language.

The linguistic communication is related to all professions stimulating the development of professional capacities of the individual in various fields of science. Only with such capacities, he is able to analyze, interpret and understand the complex phenomena of the contemporary world.

Under the light of modern theories, we have tried to highlight the relations and interaction between both social and scientific disciplines: linguistics and mathematics, in the best possible adoption of mathematical knowledge. The aspects of the relation, the necessity and the resulting problems provide a view of the actual situation of the interdisciplinary integration and interaction of both these disciplines.

**Key words:** *linguistic competence, mathematical knowledge, integration, adoption* etc.

## 1. Introduction

In the recent years, a considerable number of studies have researched the role of language in the mathematical communication. This attention may be attributed to some factors, such as the necessity to learn mathematics in groups of

students belonging to various ethnic and linguistic groups, or within the same group, with different linguistic competences. Some studies have observed the difficulties caused by the language in learning mathematics: some characteristics of mathematic language[33] (such as those of algebraic symbols) have been highlighted as potentially able to obstruct learning. Although some of these characteristics depend on traditions of teaching, and may be eliminated, the rest is not arbitrary, but constitute an answer to the actual needs of mathematics, especially in the implementation of the algorithms (such as the calculations in arithmetic or algebra).

The problem may be considered by the neglecting of the optimal use of linguistic competences, as it has occurred and still occurs in some cases (for instance, in the compilation of many mathematic texts), or trying to eliminate the specificity of mathematic language. In many books for mathematics is still used a pedant and unnecessary language; obstructing the development of mathematic thinking, neglecting thus the linguistic thinking. Nowadays, due to the increasing importance acquired by the artificial language, a special attention is requested for the rules governing a language. The aim is that the pupils or students understand that an algebraic expression is a linguistic phrase expressed through symbols and that in mathematics there may be "grammatical" errors.

Recent theories have laid importance of the fact that to learn and understand mathematics means to learn somehow to use the language.

We have based our study on two important issues: in first place the recognition of those characteristics of language that really serves to the needs of the discipline of mathematics, and in second place, the identification of linguistic competences necessary in different levels for the use of methods that support the development.

## 2. Language function

"The language is the most important medium of communication among people in the society. As the indispensable medium for exchanging of information and for the organization of all material and intellectual-spiritual activity, it is in itself a system of undersystems." (Saussure, F.)

[33] We shall use the term "mathematic language" in its broader meaning, including the verbal, symbolic and graphic components.

It gets developed on two basic thinking abilities: symbol and abstraction. In the language, we use the symbols and signs, while in mathematics are used numbers representing the corresponding ideas, for instance the number 10 (sign) showing 10 pen, 10 notebooks etc. disconnected by the physical characteristics of the objects (form, color). Every language has a system: i.e. a series of elements interconnected that compose a special connection to develop and utilize the synaptic potentials of the brain.

Every form of learning is based on these synaptic potentials, while the school curricula must be stimulated the whole time to realize what Halliday calls "potential of understanding" of the language (Halliday M.A.K. 1987).

Halliday describes three main functions of language:

a) Ideal function, which allows us the representation of our perceptions and experiences, our opinion on reality we live in, and the interpretation of what occurs around us.

b) Interpersonal function, through which we interact with the others, we communicate and exchange information, opinions, memoires and wishes, trying thus to influence, regulate and so on.

c) Textual function, which allows us to organize the discussion and to create a situation with everything connected together in a coherent way.

The textual function allows the development of two basic capacities: the ability of interpretation and the expressing ability – two very creative operations related to the reaching of "potential understanding" of language through the decomposition and composition of text. The text must be written or verbal. Both have characteristics and functions of great importance in the process of learning and intellectual development.

In fact, very often the formulation of scientific concepts in a learning context may be obstructed by the use of a language that students relate to the ordinary concepts developed in the daily context, and this may lead to semantic uncertainties. The problem arises related to both natural and symbolic language. For instance, the words and numbers in their phonic and graphic representation do not possess any direct relation with what they express. Symbols such as +, -, x, :, do not have any similarity with the transactions they refer to. For analogy, also in the language, words such as prepositions, if used in various contexts, express various meanings, for example, in the case of the preposition "**by**". In the sentence "*I stand by my*

*friends*", the preposition "**by**" provides the idea of accompanying. While in the sentence "*I travel by car*", the preposition "**by**" is related to the vehicle. For these reasons, in the mathematical education, the recognizing, methodological, linguistic and communicative abilities must be intertwined in a constant way.

The mathematics teachers must always keep in mind that mistakes deriving by the teaching of this subject are to be divided into two groups: mistakes deriving from the incorrect speaking, and mistakes out of the speaking area, i.e. related to the wrong way of thinking.

As a consequence, there arises the necessity of a serious work for the creation of habits and right use of linguistic knowledge. We must always keep in mind that teaching of mathematics constitutes at the same time the training of pupils or students at speaking.

**2. 1 Language and mathematics language**

Mathematics is also a language, although in some aspects very formalized. As the language, mathematics has its own fund of terms and symbols, in analogy with the natural language, such as: 0, 1, 2, 3, 4, 5, +, =, //, >, < etc. The terms composing the fund of mathematics as a science or school discipline are completely part of the lexicon of a language, while the rules of logic connection of terms in various judgments are part of the linguistic syntax. As a consequence, all this entirety of terms and rules of logical connections between them is a variation of the language, such as are also the dialects. As it occurs in the language, for the acquisition of mathematical language is requested in the first place the recognition of the meanings of symbols and formula (semantics of mathematic language) and secondly the structure of its inner construction (syntax of mathematic language).
It occurs often that pupils or students have acquired better the solution of equations even if they are complicated, while they may have not been able to provide a solution to easier exercises related to their lower abilities to provide a "translation" of the practical problem in the mathematic language, i.e. to construct the equation.

I. Newton, in his work "General Arithmetic", states: "To provide a solution for an issue related to numbers or to abstract relations of measures, the problem has to be returned only from the mother tongue to the language of algebra."

Often, in the process of mathematics learning at schools, we face such situations that the students says: "I know it, but I can't express it", or that he simply hesitates to start the answer, because he doesn't know quite well or is not able to attest the

theorem, and with the intervention, the guidance and the orientations of the teacher, he formulates correctly the answer in the syntactic as well as logical aspects, coming finally to the solution of the problem or attesting of the theorem.

In the end of the day, mathematic language is a result of perfecting the mother tongue. This kind of perfecting makes possible the neglecting of various interpretations that lead to chaos in the logical thinking.

Let's get an example of defining the product of two natural numbers. After giving correctly this definition, it has become a tradition to read the expression $7 \cdot 5$ as "shtatë herë pesë" (seven times five). This mathematical linguistic expression is not adequate with the scientific definition, because it creates indisputably by the largest part of the pupils the idea that 5 is taken as addable 7 times.

In spite of analogies mentioned above, this does not mean that mathematics has to be identified with the language and that it has to be learned by the use of the same methods as the language.

## 3. Mathematical education and the language

Recently has been stressed increasingly the importance of linguistic competence in mathematics teaching. Theories on the influence of language in mathematics teaching are focused progressively more in the relations between communication and acquiring of mathematical knowledge.

On one hand, teaching of mathematics in a multilingual community or in a milieu with individuals of different linguistic capacities has shown problems of a mathematical education that do not coordinate the construction of mathematical concepts with the language level.

On the other hand, teaching methods based on various forms of communication require a continuous increasing of linguistic competence.

Under this perspective, Sfard (2000) considers the language not only as a medium of communication, but also as a constructive medium of thinking.

Various studies (Sfard 2001; Blanton 2002) have highlighted the linguistic aspect as one of the difficulties in mathematics teaching for students of every schooling level, including also the university. Concretely, teaching of mathematics from high schools to universities has shed light on the following difficulties:

- Interpretation of isolated words, or key words instead of the whole text.
- Interpretation and production of texts in conformity with the typical ways of expression of the daily language, rather then specific language of science in general and especially of mathematics.

- Impossibility to use in a scientific milieu the linguistic abilities acquired in a linguistic-literary milieu: many students in possession of a distinguished baggage of linguistic competences do not use it even minimally in their scientific activities, especially for the solution of mathematical problems.

### 3.1 How can texts be interpreted?

Until now we talked about language learning during teaching of mathematics at school. It is necessary to study and use also the very important logical component of mathematic language. On the other hand, including of logical component in the mathematical language implies making known to students the elements of formal logic. Such a thing would require a serious training of teachers. The more rigorously is constructed the subject course, the more successfully may be used the linguistic-mathematical logic, and the more better may be used the logical mathematical symbols such as E, $\supset$, $\exists$, $\forall$, $\Rightarrow$, $\Leftrightarrow$, v etc.

Difficulties of interpretation of argumentative mathematical texts require the understanding of processes included in it (Radford, L.: 2000. As for the verbal texts, the linguists are divided among the advocates of the theories of code and those of inference. Simplifying further, it can be stated that the formers believe that the interpretation occurs substantially in texts in elementary units (through grammar) and using a code or a dictionary to translate the specific terms (nouns, verbs,…); relying on this point of view, the meaning is substantially included in the text and it must just be extracted of it. The latter are of the opinion that to interpret a text are necessary and fundamental also the inferences, i.e. reasoning, that it is at stake not only the grammar and the dictionary, but also the knowledge of the subject (his encyclopedia); according to this point of view, the meaning is not included in the text, but it depends on the subject interpreting it, on his initiative and culture. Although mathematical language, especially in its symbol components, seems in a first sight as a good model for the code theory, its inclusion in contexts of interpersonal communication causes inferencial processes that can not be controlled easily. In the didactic practice, for instance, it is necessary to describe mathematical ideas and relations and to communicate with the students as individuals. These can lead to the use of the same word in different meanings, for example: in the mathematics language, an **equilateral triangle** means every triangle with at least two equal angles. While in the communicational view, an equilateral triangle implies the three angles being equal. Let us look at an example taken from a book of mathematical exercises to see more clearly what was stated above.

Exercise
Complete the following text in order to give it a reasonable meaning, choosing among the measures listed below:
"There are four children playing. Arta is the tallest one: her body is 1 m and 10 cm tall. Also Genti is taller than a meter: the tallness of his body is _____ . Ira's tallness is anyway _____ . Ada is the one with the smallest body height. She is _____ tall. In spite of this, Ira is _____ tall. Ada, with the smallest body height, is _____ ."
1m and 20 cm 1m and 5 cm 90 cm 10 cm 85 cm

Exercises of this kind require a general text interpretation: to provide an answer is not enough to interpret only specific words or phrases, but it is necessary to coordinate the interpretation of phrases to come to the conclusion. Thus, it is the not necessarily deductive reasoning the medium to get an information that has not been given in an explicit way. This is especially important in the case of children that have to be able to control the meaning of mathematical actions.

Pupils are successful in giving a solution to a problem not only by using mathematical knowledge, but also the data deriving from general knowledge (for instance, there are no children taller than 10 cm), or the data of text leading to implicit suppositions for their cohesion; for instance, in the text has not been said that Gent has one of four, but this is implied, as there is a semantic connection between the five phrases composing it.

The ideas expressed above are not the only ones possible. There may be more ways and every teacher may concretize them also through examples extracted by the experience of teaching work.

## 4. The role of teacher between linguistic and mathematical education

Teachers have still a decisive role in the aspect of communication (Radford, L.: 2000). Them belongs the most complex duty to stimulate, lead and coordinate interactions between students, aiming to the encouragement of thought development. At any case, the request to make easier the communication in the classroom will be necessary, by using all linguistic mediums in order not to create unnecessary obstacles. In many books about mathematics is still in use an unnecessary and pedant language, obstructing the development of thought, paying attention to the mathematical thinking and neglecting the linguistic thinking. In the

present day, due to the increasing importance taken by the artificial languages, a special attention is required for the rules governing a language.

The intention is that pupils or students may be able to understand that an algebraic expression is a linguistic sentence expressed through symbols and that in mathematics books may be found also "grammar" errors.

On the other hand, the logic-mathematical education is also of great importance, because it stimulates the sensitive-motor and psycho-movement development for a more exact and necessary orientation in space and time and for the coordination through the perception and movement. It also stimulates the construction of mental models allowing a correct knowledge of the reality, encouraging the critical abilities of the subject in front of the problematic situations, and at the same time it develops the linguistic abilities through communication. Thus, logical-mathematical education plays an important role.

Linguistic education, relying on the modern theories, aims to provide a communicative competence based on the capacities and information of the language itself, with the communication becoming a concrete and real activity. Communication must be suitable and in conformity with the rules defined in an effective and meaningful way. In this sense, the logic mathematical education may provide mediums of rational analysis in the structure of language.

## 5. Conclusions

In this article, we tried to present some considerations on the relation existing between both these school disciplines. Without claiming to have said everything, we judge in this context that the work of teacher and his creativity during the mathematics lesson makes possible the highlighting of such relations and the contribution of linguistic knowledge to an optimal acquiring of the discipline.

Generally in our research related to the teaching of mathematics at school, we thought to clarify the necessity of study and use of language, arguing that this issue belongs to the object of methodology. Thus, it is a methodological problem of teaching and self-learning of mathematics at school.

Thus, presenting of exercises of a semantic cohesion, avoiding in this way the use of phrases with different speech values, ore reconsidering the text content of learning for the formulation of these exercises, may be considered of the most efficient ways

for the realization of the sentence not only as a constructional unit, but also as e functional one.

Every discipline has its objectives and aims. While stating that relations have to be established between linguistics and mathematics, these must not be considered as unilateral relations.

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